ADAPTIVE PREDICTIVE CONTROL USING RHPC FOR ELECTRIC FURNACE

°Jin-Hwan Kim† and Uk-Youl Huh†
†Dept. of Elec. Eng. INHA Univ. Incheon, 402-751, KOREA
Tel: 82-32-860-7394, Fax: 82-32-864-6442 E-mail: uyhuh@dragon.inha.ac.kr

Abstracts This paper shows adaptive control using RHPC(Receding Horizon Predictive Control) with equality constraint which applied to Electric Furnace. The control strategy includes monotonic weighting (improving transient response) and pre-filtering (enhancing robustness), which is effective on real process. We can observe the performance of RHPC and confirm the practical aspect of RHPC with unmodelled dynamics through the experiment of Electric Furnace. Finally, this paper verifies the feasibility of RHPC to real process.

Keywords MBPC, RHPC, Robustness, Electric Furnace

1. INTRODUCTION

Many MBPC strategies which has been widely used in predictive control have been proposed since 1980's [1~3]. It can be called RHPC(Receding Horizon Predictive Control) since MBPC uses the RHC(Receding Horizon Control). RHPC may not get a sufficient stability analysis in spite of good performance because RHPC has many tuning knobs. Also, RHPC for robustness enhancement has been issued since 1991[4]. The remaining works of RHPC focus on stability analysis and robustness enhancement.

In this paper, we apply RHPC with equality constraint to the inlet temperature control of Electric Furnace. Generally, the process has the time delay, nonlinearities, and variable operating point. It is difficult to be controlled by PID and linear control with nearly perfect modelling because of large time delay and various dc-gain according to operating temperature.

This paper shows following expermental results - Electric Furnace control by RHPC with equality constraint, prefiltering to get over the unmodelled dynamics, and monotonic weighting for improving the transient reponse. Finally, we can confirm the feasible aspect of RHPC for real process through experimental results of Electric Furnace.

2. RECEDING HORIZON PREDICTIVE CONTROL

2.1 *Plant description and Control strategy*

Consider the following CARIMA(Controlled Auto-Regressive Intergrated Moving- Average) model.

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t-1) + \frac{T_c(q^{-1})}{A(q^{-1})\Delta}\xi(t) \qquad (1)$$

where $\xi(t)$ is the disturbance signal, $A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_{na}q^{-na}$, $B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + \cdots + b_{nb}q^{-nb}$, $T_c(q^{-1}) = 1 + t_1 q^{-1} + t_2 q^{-2} + \cdots + t_{nt}q^{-nt}$, $\Delta(q^{-1}) = 1 - q^{-1}$

Now, the following cost function is used for obtaining optimal control input. Equation (2) should be minimized with respect to $\Delta u(t)$ and $e_c(t)$.

$$J(t) = \left\{\sum_{j=1}^{N_y}\mu(j)(r(t+j)-\hat{y}(t+j))^2 + \sum_{j=N_y+1}^{N_2} \cdot (r(t+j)-\hat{y}(t+j))^2 + \sum_{j=1}^{N_u}\rho(j)\Delta u(t+j-1)^2\right\} \qquad (2)$$

where $\Delta u(t+j) = 0$ for $j \geq N_u$, $y(t+N_y+j) = r(t+N_y+j)$ for $j = 1, \cdots, N_2 - N_y$, and second term is terminal equality constraint.

For getting a future output prediction, use the following equations.

$$T_c(q^{-1}) = E_j(q^{-1})A(q^{-1}) + q^{-j}F_j(q^{-1}) \qquad (3)$$

$$E_j(q^{-1})B(q^{-1}) = G_j(q^{-1})T_c(q^{-1}) + q^{-1}H_j(q^{-1}) \qquad (4)$$

$$\hat{y}(t+j|t) = \frac{G_j(q^{-1})}{T_c(q^{-1})}\Delta u(t+j-1) + \frac{H_j(q^{-1})}{T_c(q^{-1})}\Delta u(t) + \frac{F_j(q^{-1})}{T_c(q^{-1})}y(t) \qquad (5)$$

After (5) substitute to (2) and some manipulation, the optimal control increment is given the following equation by *inversion lemma* and *receding horizon strategy*.

$$\Delta u(t) = [10\cdots0][\mathbf{I} - \tilde{\mathbf{G}}\mathbf{G}_\mathbf{C}^T(\mathbf{G}_\mathbf{C}\tilde{\mathbf{G}}\mathbf{G}_\mathbf{C}^T)^{-1}\mathbf{G}_\mathbf{C}] \cdot\tilde{\mathbf{G}}\mathbf{G}_\mathbf{C}^T\mathbf{M}(\mathbf{w}-\mathbf{f})\tilde{\mathbf{G}}\mathbf{G}_\mathbf{C}^T\cdot(\mathbf{G}_\mathbf{C}\tilde{\mathbf{G}}\mathbf{G}_\mathbf{C}^T)^{-1}(\mathbf{w}_\mathbf{C}-\mathbf{f}_\mathbf{C}) \qquad (6)$$

where $\tilde{\mathbf{G}} = (\mathbf{G}_\mathbf{C}^T\mathbf{M}\mathbf{G}_\mathbf{C} + \Lambda_\mathbf{C})^{-1}$, λ is a lagrange multiplier vector, $\mathbf{G}_\mathbf{C}$ is a step response matrix for equality constraint, $\mathbf{M} = diag[\mu(N_1)\cdots\mu(N_2)]$, $\Lambda = diag[\rho(0)\cdots\rho(N_u-1)]$, $\mathbf{w}_\mathbf{C}$ is a future set point matrix, and $\mathbf{f}$ is $\sum_{j=1}^{N_y}\left\{\frac{H_j}{T_c}\Delta u + \frac{F_j}{T_c}y\right\}$, f_c is a matrix for equality constraint.

As well known, mean-level setting($N_u = 1, N_2 \to \infty$) is popular one for guaranteeing the stability in case of GPC. The following sufficients condition for stability of RHPC is referenced by some articles[5][6].

Theorem 1 : For any stabilizable and detectable system, RHPC stabilizes the system if
$N_u \geq n$, $N_y = N_u + 1$, $N_2 = N_y + n$
where $n = max(deg(B)+1, deg(A\Delta))$

Theorem 2 : For any stabilizable and detectable system, RHPC stabilizes the system if
$N_u \geq n_{a\Delta}$, $N_y \geq N_u + n - n_{a\Delta} + 1$, $N_2 = N_y + n_{a\Delta}$
where n_Δ *is the order of* $A\Delta$.

Theorem 1 and 2 are proven by using the monotonic decreasing property of the Riccati difference equation[10]. Theorem 2 has a less computational load than Theorem 1.

The use of monotonic weighting which is due to Bitmead[6] improves the transient response and the performance of system not controlled by GPC. The sequence of monotonic weighting is given by[7].

$$\mu(j) = \bar{\mu}\alpha^{-2j}, \quad \rho(j) = \bar{\rho}\alpha^{-2j} \tag{7}$$

where $\alpha \leq 1$, $0 < \bar{\mu} \leq 1$, $0 \leq \bar{\rho} \leq 1$.

Robustness problem is the main issue together with stability analysis since 90's. For improving the robustness of system with unmodelled dynamics, RHPC uses prefilter which is continued to next sub-section.

2.2 *Improvement of Robustness*

Generally, unmodelled dynamics is presented in high frequency range. So low-pass filter may get rid of unmodelled dynamics. Robinson[4] and Yoon[5] show the method of improving robustness using the following prefilter design, when the following additive uncertainty is considered. The robustness bound can be described by[5].

$$G^* = G^o + \tilde{G}, \quad \| \tilde{G} \| \leq \left\| \frac{P_c T_c}{A\,S} \right\|, \quad T_c = \hat{A}T^* \tag{8}$$

where G^*, G^0, and $\tilde{G}$ are true model, norminal model, and unmodelled dynamics, respectively. $\| \cdot \|$ is gain in the frequency domain, P_c is the characteristic equation, A is the output polynomial, and S is related to output for $R\Delta u(t) = T_c w(t) - Sy(t)$. $T^* = (1 - t_o q^{-1})^n$, $n \geq N_1 - deg(P_d)$, $P_d = 1$ in this paper.

The theorical background of the control algorithm is shown in this section. In next sections, we would like to introduce the Electric Furnace control system and show several experimental results.

3. MATHMATICAL MODELLING OF THE ELECTRIC FURNACE

The modelling of thermal process can be developed by the basic mechanism of heat transfer, such as conduction, convection, and radiation. However, for simplicity, we describe the modelling of Electric Furnace only by convection. Now, the following equation can be drived by the energy balance equation using Laplace transformation.

$$Y(s) = \frac{1}{C(s + 1/\mathcal{R}C)} U(s) \tag{9}$$

where C is thermal capacitance, $\mathcal{R}$ is thermal resistance, $p(t)$ is supplied power by heater, $y(t)$ is inlet temperature of furnace, and $y_c(t)$ is circumambient temperature. $U(s)$ is $(P(s) + Y_c(s)/\mathcal{R})$, i.e., power by heater and circumambient air.

Finally, (9) can be transformed into CARIMA model by z-transformation.

4. EXPERIMENT

4.1 *Hardware of Electric Furnace Control system*

The hardward architecture of Electric Furnace control system is shown in Fig.1. It consists of five modules. The controlled process is Electric furnace which has nonlinearites such as variable operating point, and time delay. Sensor module uses $PT - 100\Omega$ thermistor. MCS-96 one chip controller generates PWM signal and run A/D conversion. SSR(Solid State Relay) by controlled PWM signal is used for on-off control of the heater. Control algorithm is implemented using PC-386. PC-386 captures the information of the control system and display the response of the current state. The period of PWM is 1 second and the resolution is 50000. The control algorithm is programmed by Borland C++ Ver. 3.1.

4.2 *Experiment Results*

The sampling time of experiment is 30 second. The number of I/O coeffcients in identification are $na = 1$ and $nb = 8$. The prefilter of identifier is set to low-pass filter such as $T_e = \Delta/T = (1 - q^{-1})/(1 - 0.9q^{-1})$. For guaranting the convergence of identifier, maximum PWM inputs to Furnace until 50°C. All graphs whose vertical unit is temperature and horizontal unit is samples. The weighting coefficient, $\bar{\mu}$ is 1 in all expermients.

Fig.2 shows the case of tuning knobs, ($N_1 = 1, N_u = 2, N_y = 9, N_2 = 11, \bar{\rho} = 0, \alpha = 1$), which is set by *Theorem 2*. The initial overshoot is about 1.3°C with set-point 60°C and undershoot is −0.67°C with 80°C. The rising times are 35 and 40 min, respectively. Also, each steady-state error is about ±0.5°C. However, the control input does not converge to steady state. This might be caused by the following reason. The response of Furnace is weak for noise since coefficients of $\hat{B}$ are laid on the range of $10^{-6} \sim 10^{-7}$. So, the control increment may be sensitive to noise.

Fig.3 shows the case of N_2 increasing to 15. It is improved the initial transient response with set-point 80°C. This result presents that larger N_2 enhance the preformance of response. The rising time and steady-state error are nearly same as Fig.2.

For reducing input sensivity, $\bar{\rho}$ is set to 1 in Fig.4. But the convergence of control input does not improved. It shows rather oscillation in the case of set-point 80 °C. It may be guessed that larger $\bar{\rho}$ becomes less sensitive to tracking error.

Now N_u is set to 8, which is shown in Fig.5. RHPC minimizes cost subject to $\Delta u(t + N_u - 1) = 0$. Therefore, as shown in Fig.5, large N_u has not better performance than low value in steady-state response.

Fig.6 presents the result of using monotonic weighting. The initial overshoot and steady-state error are 0.7°C and ±0.35°C, respectively. In comparison with Fig.2, performance is improved. we know that monotonic weighting enhances the transient response from this result.

The next result is the case of Furnace with unmodelled dynamics which has the lower order than sufficient order of plant. Fig.7(a),(b) show the estimated model gain with ($na = 1, nb = 6$) and ($na = 1, nb = 3$), boths are compared with sufficient model($na = 1, nb = 8$). Fig.7(a) shows that $na = 6$ has the nearly same steady-state gain as the case of sufficient order. However, as shown in Fig.7(b), the steady-state gain in case of ($na = 1, nb = 3$) cannot capture the steady-state gain of plant. So, This unmodelled dynamics in case of ($na = 1, nb = 3$) causes bad performance. As shown in Fig.8(b), Furnace cannot be controlled in spite of RHPC with the same tuning knobs as Fig.3. This result is re-observed in Fig.8(b). Now, for improving robustness, let's the pre-filter be set to $T_c = \hat{A}T = (1 - 0.98q^{-1})(1 - 0.7q^{-1})$. As result of Fig.9(a), closed-loop system becomes asymptotically stable. The modelling error is laid on below bound than robustness bound in Fig.9(b). Fig.9(b) confirms the role of prefilter for improving robustness with respect to unmodelled dynamics.

5. CONCLUSIONS

In this paper, RHPC is proposed for the Electric Furnace. From the results of experiment the practical aspect of RHPC was sufficiently confirmed. Especially, RHPC using low-pass prefilter could run effectively in case of the Furnace with unmodelled dynamics. So, we have following conclusions.

- RHPC with tuning knobs by *Theorem 1, 2* stabilizes the Electric Furnace. Especially, *Theorem 2* is the guide of saving the computational load.
- Although $\bar{\rho}$ is increased to 1 for converging the control input, control input does not converge to any value at steady-state. So, even if $\bar{\rho} = 0$, it does not concern to any performance in large time delay process.
- $N_u = 1$ is preferable than $N_u \geq 2$ for large time delay process.
- Monotonic weighting improves the performance. Initial overshoot and steady-state error are reduced as shown in Fig.6
- The prefilter by (8) enhances the robustness.
- The robustness of adaptive control system depends on the identified parameter of plant. So the robust identification method is needed. The robust identification method is parallel to robust control. This work is going to be continued in other reseaches[8].

This paper shows the feasibility of RHPC for real process - actually Electric Furnace. Futher reseach work is that we apply RHPC to mutivariable system. Also, remaning work is robustness enhancement problem through the pre-filter manipulation of identifier. This research will gaurantee the performance and robustness without trade-off.

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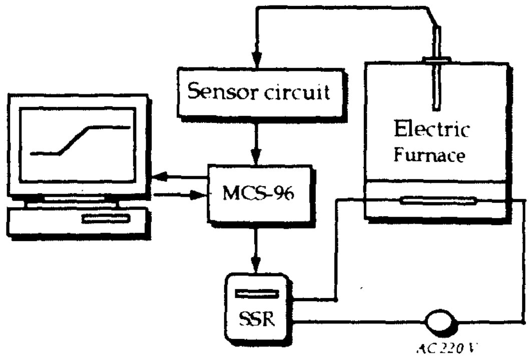

Fig. 1. Hardware architecture of Electric Furnace Control System

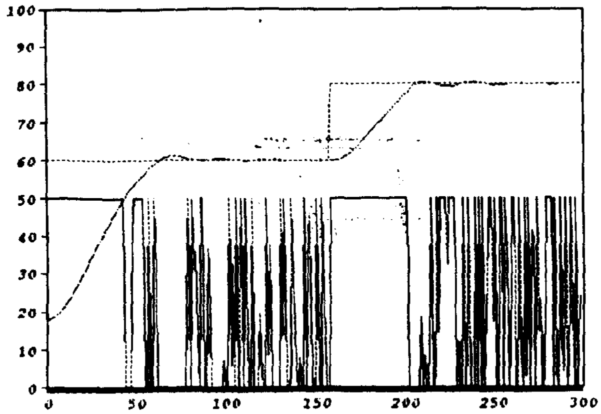

Fig. 2. Response of Electric Furnace for RHPC ($N_1 = 1, N_u = 2, N_y = 9, N_2 = 11, \bar{\rho} = 0, \alpha = 1$)

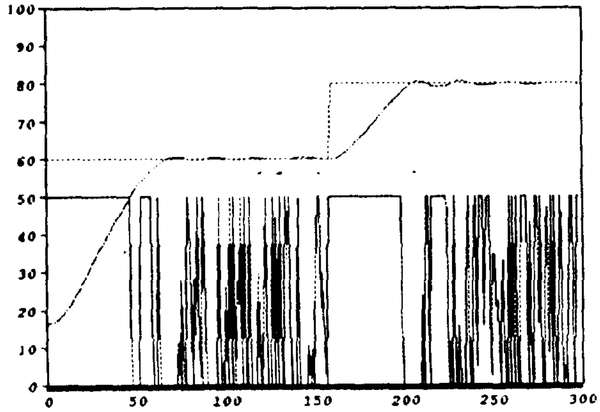

Fig. 3. Response of Electric Furnace for RHPC ($N_1 = 1, N_u = 2, N_y = 10, N_2 = 15, \bar{\rho} = 0, \alpha = 1$)

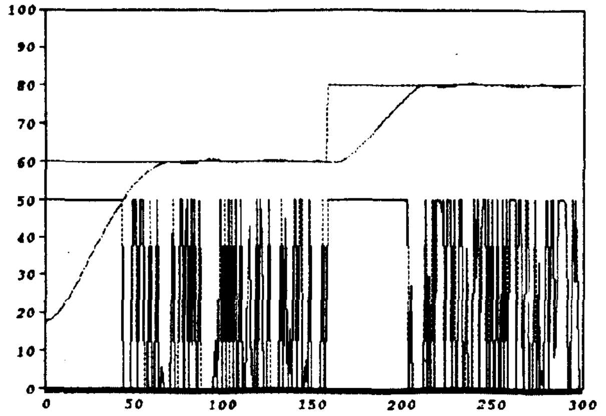

Fig. 4. Response of Electric Furnace for RHPC ($N_1 = 1, N_u = 2, N_y = 10, N_2 = 15, \bar{\rho} = 1, \alpha = 1$)

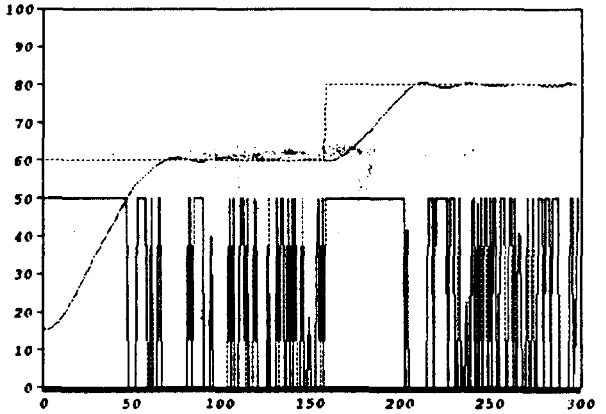

Fig. 5. Response of Electric Furnace for RHPC ($N_1 = 1, N_u = 8, N_y = 10, N_2 = 15, \bar{\rho} = 0, \alpha = 1$)

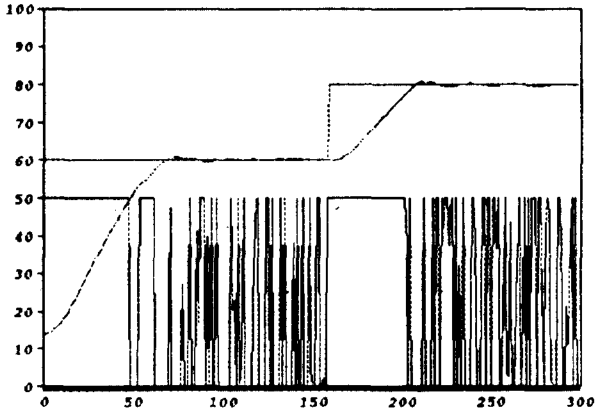

Fig. 6. Response of Electric Furnace for RHPC ($N_1 = 1, N_u = 2, N_y = 10, N_2 = 15, \bar{\rho} = 0, \alpha = 0.1$)

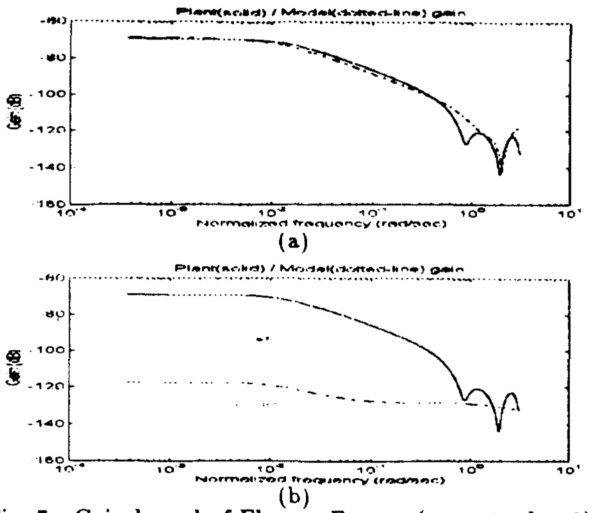

Fig. 7. Gain bound of Electric Furnace($na = 1, nb = 8$) and model (a) $na = 1, nb = 6$, (b) $na = 1, nb = 3$

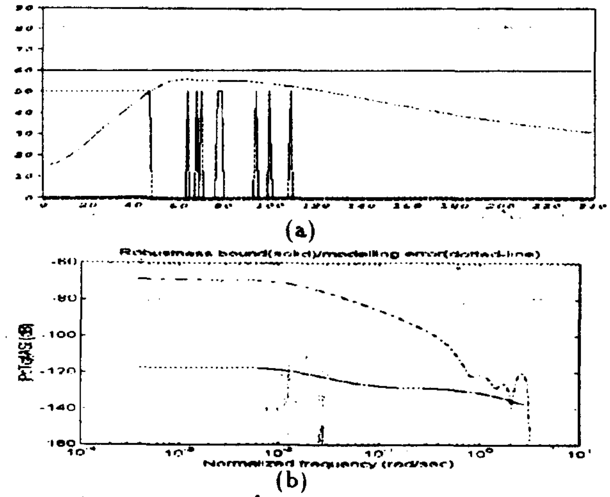

Fig. 8. ($T_e = (1 - 0.9q^{-1}), T_c = 1$), (a) Response of Electric Furnace for RHPC, (b) Robustness bound(solid-line) and modelling error(dotted-line)

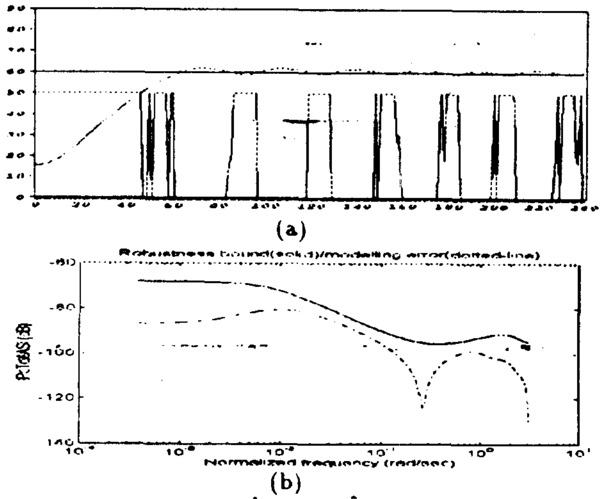

Fig. 9. ($T_e = (1 - 0.9q^{-1}), T_c = \hat{A}T^* = (1 - 0.984q^{-1})(1 - 0.7q^{-1})$) (a) Response of Electric Furnace for RHPC, (b) Robustness bound(solid-line) and modelling error(dotted-line)