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DESIGN OF CONTROLLER FOR NONLINEAR SYSTEM USING DYNAMIC NEURAL NETWORKS

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Abstract The conventional neural network models are a parody of biological neural structures, and have very slow learning. In order to emulate some dynamic functions, such as learning and adaption, and to better reflect the dynamics of biological neurons, M.M. Gupta and D.H. Rao have developed a 'dynamic neural model'(DNU). Proposed neural unit model is to introduce some dynamics to the neuron transfer function, such that the neuron activity depends. on internal states. Integrating an dynamic elementary processor within the neuron allows the neuron to act dynamic response Numerical examples are presented for a model system. Those case studies showed that the proposed DNU is so useful in practical sense.

Keywords Nonlinear System, Dynamic Neural Network, Sensitivity Model, Backpropagation Method Control

1. INTRODUCTION

It is currently understood that biological neuron provides two distinct mathematical operations distributed over the synapse and soma of neuron. These two neuronal mathematical operations are called synapse operations and somatic operations. From the biological point of view, the two operations are physically seprate, however, in the modeling of biological neuron, these operations have been combined, for example thresholding in the soma is transferred to the synaptic operation. At the macroscopic level, the dendrites of each neuron receive pluses at synapses and convert them to continuously variable dendritic current. For each neuron there is time varying nonlinear relationship between the pulse rate at the synapse and the amplitude of the dendritic current[1]. M.M.Gupta and D.H.Rao have proposed a different architecture to model the biological neuron named the 'dynamic neural model'[2]. In this paper, we propose new DNU structure by considering the idea that neuron activity depends on internal neuron states.

2. DYNAMICS OF AN ISOLTED DNU

The dynamic neural unit(DNU) proposed in M.M.Gupta and D.H.Rao[2], is a good dynamic neural model. The DNU comprises of memory elements, and feedforward and feedback synaptic weights. The output of this dynamic structure is to a time varying nonlinear activation function. Thus, the DNU performs two distinst operations. one for synaptic operation and the other for somatic operation. The first operation corresponds to the adaption of feedforward and feedback synaptic weights, while the second corresponds to the adaption of slope of the nonlinear activation function. The DNU consists of a linear structure having synaptic weights a_{ff} and b_{fb} representing a second order structure followed by a nonlinear activation function as shown in Fig. 1.

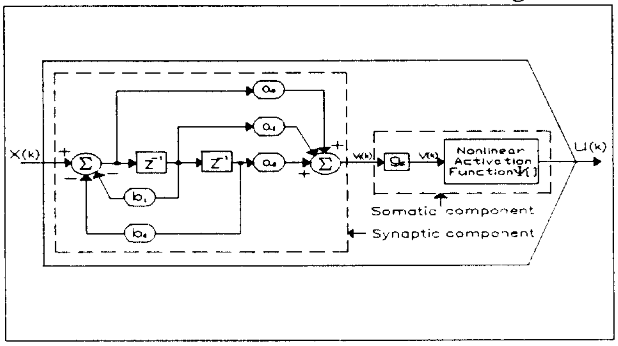

Fig. 1 Basic structure of DNU, which consists of synaptic and somatic components

3. PROPOSED DYNAMIC NEURAL UNIT MODEL

The dynamic structure of proposed DNU, as shown in Fig. 2, consists of synaptic component, dynamic elementary processor(DEP) and somatic component. A new structure of DNU accounts for synaptic adaption($w_1, w_2, \cdots w_p$), dynamic elementary adaption ($a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2, c_1, c_2$) and somatic adaption($g_s, \theta$). The DEP has a second order structure which can be described by the following state space representations.

$$\begin{bmatrix} s_1(k+1) \\ s_2(k+1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} s_1(k) \\ s_2(k) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} x(k) \qquad (1)$$

$$v_1(k) = [\ c_1 \quad c_2\ ] \begin{bmatrix} s_1(k) \\ s_2(k) \end{bmatrix} + d_0 x(k) \qquad (2)$$

$$x(k) = w^T \cdot I(k) = \sum_{i=1}^{p} w_i I_i(k) \qquad (3)$$

where $I(k) \in R^p$ is the neural input vector, x(k) is the input of DEP, $v_1(k) \in R^1$ is the output of DEP, k is the dicrete time index, and w_i is the weight of the neuron input. Using the linear time shifting operator $q^{-1}[s(k)]=s(k-1)$ the neuron transfer function is described by

$$v_1(k)=\frac{Z(q)}{P(q)}[x(k)]=\frac{z_0+z_1q^{-1}+z_2q^{-2}}{1+p_1q^{-1}+p_2q^{-2}}[x(k)] \quad (4)$$

where z_0, z_1, z_2, p_1, p_2 are adaptable feedback and feedforward weights respectively.

The nonlinear mapping operation on $v_1(k)$ yields a neural output u(k) given by

$$u(k)=\Psi[g_s v_1(k), \theta] \quad (5)$$

where $\Psi[\cdot]$ is a nonlinear activation function of neuron with a threshold θ. In order to extend the mathematical operations on both the positive and negative neural outputs, and expand the neural activity for both the excitatory and inhibitory inputs, we can choose activation function to be a sigmoidal function defined as

$$\Psi[v(k)]=\tanh[g_s v_1]=\tanh[v] \quad (6)$$

where $v=g_s v_1$, and g_s is the somatic gain which controls the slope of the activation function.

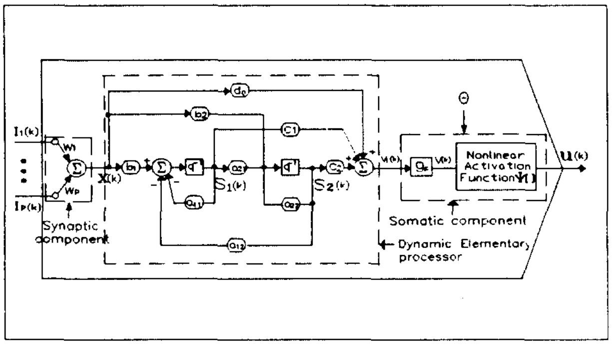

Fig. 2 Proposed Dynamic Neuron Model which is consists of synaptic, somatic components and elementary processor in state space represented with P inputs and 1 output

3.1 Adaptive Algorithm for Optimal Parameters

The Objective of the algorithm is to adjust the network parameters based on a given set of input-output pairs and to determine the optimal parameter set which minimizes the cost function J.

$$J=\frac{1}{2}E[(u_d(k)-u(k))^2] \quad (7)$$

where N is the training set size. The define error, e(k) is the difference between the desired response $u_d(k)$ and the actual neuron response u(k). Applying a gradient method, the optimal parameters are approximated by steepest descent rule:

$$\phi_{new}=\phi_{old}+\eta \cdot E[e(k)\frac{\partial u(k)}{\partial \phi}] \quad (8)$$

where ϕ is the parameter and η is the learning rate. It is obvious that

$$\frac{\partial u(k)}{\partial \phi}=\frac{\partial u(k)}{\partial v(k)}\frac{\partial g_s v_1(k)}{\partial \phi}=g_s\Psi'\frac{\partial v_1(k)}{\partial \phi} \quad (9)$$

Therefore, the activity function is to be differentiable.

Using the time shifting operator, five cases can be distinguished:

Case 1) ϕ is a DEP coefficient of the numerator P(q)

$$\frac{\partial[v_1(k)]}{\partial \phi}\Big|_{\phi=z_i}=[S_\phi(k)]=\frac{q^{-i}}{P(q)}[x(k)] \quad (10)$$

Case 2) ϕ is a DEP coefficient of the denominator Z(q)

$$\frac{\partial[v_1(k)]}{\partial \phi}\Big|_{\phi=p_i}=[S_\phi(k)]=\frac{-q^{-i}}{P(q)}[v_1(k)] \quad (11)$$

Case 3) ϕ is a neuron input weight

$$\frac{\partial[v_1(k)]}{\partial \phi}\Big|_{\phi=w_i}=[S_w(k)]=\frac{Z(q)}{P(q)}[I_i(k)] \quad (12)$$

Case 4) ϕ is a neuron threshold

$$\frac{\partial[u(k)]}{\partial \phi}\Big|_{\phi=\theta}=\frac{\partial \Psi}{\partial \theta} \quad (13)$$

Case 5) ϕ is a slope of the activation function

$$\frac{\partial[u(k)]}{\partial \phi}\Big|_{\phi=g_s}=\Psi' v_1(k) \quad (14)$$

$S_\phi(k)$ and $S_\phi(w)$ denote the parameter states. To determine the change of the neuron activity depending on a parameter, the gradient has to be filtered by the denominator of the used dynamics[3]. This has a further benefit in that the adaptation procedure is stabilized since the calculated parameter at time instant [k] takes account of the past change at [k-1] and [k-2].

3.2 Dynamic Multi Layer Perceptron

Now, to make use of the connective power of ANN, the DEP neurons can be distributed to build a dynamical muti layer perception(DMLP). Figure 3. represents such a three layer DMLP with 2 inputs and one output. It is important that the proposed DMLP does not require the past values of the process measurement. Instead, it processes the system measurements at current instant [k]. This reduces the dimension of the network input space. Equation(15),(16) and (17) described the inference in such a network beginning by the input layer <K> through the hidden layer <M> to the output layer <L> respectively[3].

$$\begin{aligned} x(k)^{<k>}&=W^{<k>}I(k)\\ v_1(k)^{<k>}&=\mathrm{diag}[\Gamma(k)^{<k>}Z^{<k>}]\\ u(k)^{<k>}&=\Psi(g_s v_1(k)^{<k>},\theta)\end{aligned} \quad (15)$$

$$\begin{aligned} x(k)^{<L>}&=W^{<L>}u(k)^{<k>}\\ v_1(k)^{<L>}&=\mathrm{diag}[\Gamma(k)^{<L>}Z^{<L>}]\\ u(k)^{<L>}&=\Psi(g_s v_1(k)^{<L>},\theta)\end{aligned} \quad (16)$$

$$\begin{aligned} x(k)^{<M>}&=W^{<M>}u(k)^{<M>}\\ v_1(k)^{<M>}&=\mathrm{diag}[\Gamma(k)^{<M>}Z^{<M>}]\\ u(k)^{<M>}&=\Psi(g_s v_1(k)^{<M>},\theta)\end{aligned} \quad (17)$$

where $x^{<J>}$ is the state vector of the layer, $W^{<J>}$ is the weight matrix of layer <J> conisting of the weight vectors w for each neuron, $I^{<J>}$ is the data matrix consisting of the signal vectors γ for each neuron in layer <J>, and $Z^{<J>}$ is the parameter matrix consisting of the parameter vector ξ for each neuron in layer <J>

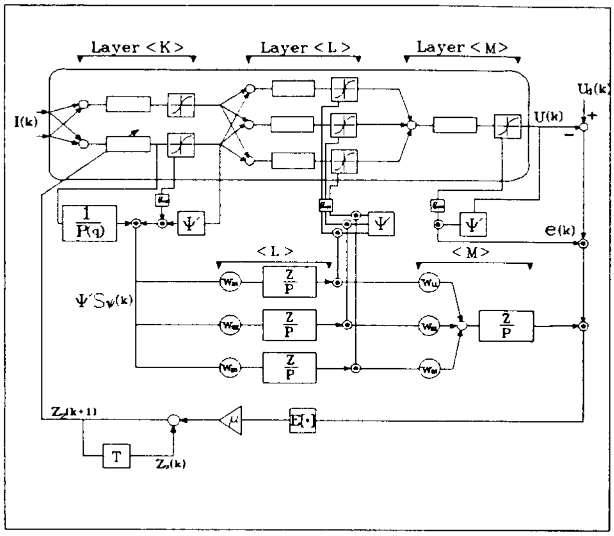

Fig. 4 Three layer DMLP with 6 DEP and with the adaption scheme of cofficient a_2 in the second DEP within the layer <k>

$$A=\begin{bmatrix} r\cos\theta & r\sin\theta \\ -r\sin\theta & r\cos\theta \end{bmatrix} \tag{22}$$

From (22), a_{11} is equal to a_{22}, a_{12} is equal to $-a_{21}$, and $\lambda[A]$ is $a_{11} \pm ja_{12}$. To derive stability condition of DEP, we suppose $Q=I$. Solving (21), unknoun element of matrix P yields a simultaneous linear equation given by (23). From (23), if absolute value of r has

$$P=\frac{1}{1-r^2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{23}$$

less than 1, then P is positive definite. Therefore optimal parameters of DEP are stabilized during learning and adaption under condition $|r|<1$. As to the stability of the gradient computation, we note that the original feedback matrix is scaled by a constant $\Psi'[v]$. So as long as $|\Psi'[v]| \le 1$ the stability condition will remain intact. For the nonlinear activation function, $\Psi(v)=\frac{e^{g_s v}-e^{-g_s v}}{e^{g_s v}+e^{-g_s v}}$, this condtion can be met by requiring g_s to less than or equal to 1.

4. STABILITY ANALYSIS OF NEURAL MODEL

In order to stabilize the proposed dynamic neural model, it is necessary to derive the stability condition for parameters of DEP and to determine the range for slope of activation function. Equation (1) and (2) may be written in the compact form as

$$s_{k+1}=As_k+bx_k \tag{18}$$

$$v_{1k}=cs_k+dx_k \tag{19}$$

where x_k and v_{1k} are the scalar input and ouyput of DEP, and s_k is the second order state vector; matrices A,b,c, and d are 2×2, 2×1, 1×2, and 1×1 real constant matrices, respectively. Given (18) and (19), we obtain the necessary and sufficient condition that the state coefficient matrix A converges in the steady state as follows:

$$|\lambda_i[A]|<1 \quad ,i=1,2 \tag{20}$$

where $\lambda_i[A]$ is the i th eigenvalue of the matrix. If and only if the above equation holds, all the poles of a transfer function of DEP lie inside the open unit circle of the z plane. hence the roots of Z(q) are identical to the eigenvalues of A.

Applying the Lyapunov theorem, parameters of DEP [A,b,c,d] are stable, and update optimally. All eigenvalue of A have magnitudes less than 1 if and only if for any given positive definite hermitian matrix Q with the property $\{A, Q\}$ observable. The matrix equation

$$P-A^TPA=Q \tag{21}$$

has unique hermitan solution P and P is posive definite. Proposed DNU consist of DEP which have minimum sensitivity structure[4]. The minimum sensitivity structures can be composed by selecting a output feedback matrix A as follow.

5. SIMULATION RESULT AND REVIEW

In the case studies, we show the proposed DNU is so useful in practical sense. First, we demonstrate the functional approximation capability of the proposed DNU. Different arbitrary nonlinear functions were used to evaluate the function approximation capability of the DNU. Some of the functions and their approximations are shown in Fig. 5. To compare convention DNU, approximation result was presented in Fig. 4. and it was observed that proposed DNU could approximate more accurately. This approximation feature of proposed DNU is exploited to systhesize a controller for nonlinear dynamic systems as discussed in the following problem. Second, the problem to be addressed in the control paradigm consists of finding a control signal u(k) that will force the system output y(k) to track asymptotically the desired output $y_d(k)$; that is $\lim[y_d(k)-y(k)]=0$ as $k\rightarrow\infty$. In this example, a nonlinear dynamic system of the form

$$y(k)=\sum_{i=1}^{2}\alpha_i y(k-i)+\sum_{j=0}^{2}\beta_j u(k-j)+0.05\cdot f[y(k-i),u(k-j)] \tag{24}$$

where f[·] is arbitrary nonlinear function, was cascade with the dynamic neural model. The objective of this to demonstrate the adaptive tracking capability of the dynamic neural model based controller under the following situations: (i) time varying nonlinear functions, (ii) varying pattern of input signals, and (iii) perturbations in the plant parameters and changes in configuration of the plant dynamics. The nonlinear function used in this example was[5]:

$$f[\cdot]=e^{-(y(k-1)^2+y(k-2)^2)}+\sqrt{u(k)^2+u(k-1)^2+u(k-2)^2} \quad ,0<k\le 300$$

$$f[\cdot]=\frac{0.5-0.5\cos(7\pi(y(k-1)^2+y(k-2)^2))+e^{-u(k)}}{4+u(k-1)^2+u(k-2)^2} \quad ,300<k\le 1000$$

System input was changed as follows:

$I(t)=\sin(2\pi k/200)$, $0<k\leq 400$
$I(t)=0.6$, $400<k\leq 500$
$I(t)=0.2$, $500<k\leq 600$
$I(t)=-0.2$, $600<k\leq 700$
$I(t)=-0.6$, $700<k\leq 800$
$I(t)=0.6\sin(2\pi k/200)$, $800<k\leq 1000$

The plant parameters were:

$\beta_{ff}=[1.2,1.0,0.8]$ $\alpha_{fb}=[1.3,0.9,0.7]$, $0<k\leq 400$
$\beta_{ff}=[1.2,1.0,1.4]$ $\alpha_{fb}=[1.3,0.9,0.7]$, $400<k\leq 850$
$\beta_{ff}=[1.2,1.0,0.0]$ $\alpha_{fb}=[1.0,0.9,0.0]$, $850<k\leq 1000$

As may be observed from the above details, the nonlinear plant undergoes both input signal variations and parameter perturbations during the time interval $400\leq k<800$. Futher, the plant structure was changed at k=850 from a second order to a first order system. The simulation results obtained for this example are shown in Fig. 4-Fig. 11. The results shown in Fig. 10 and Fig. 11 illustrate that proposed parallel connection DNU is less sensitive to variations in the plant parameters than convention parallel DNU. This example demonstrates the robustness of the proposed neural model for variations in nonlinearity characteristics, input signal, and for changes in the dynamic characteristics of the plant.

6. CONCLUSION

A new steucture of dynamic neural model has been proposed for control applications. The architectural and algorithm to update the adjustable parameters of DEP, which serve as the dynamic element in the proposed neural model, have been described. The conclusion obtained on studies summarized as follows:

1. The proposed DNU could be used to adaptively track nonlinear function.
2. This functional approximation capability of the nerural modelis employed to control unknown nonlinear systems.
3. The select of the optimal layer and the connection(parallel and cascade) of DNU will be study in future.

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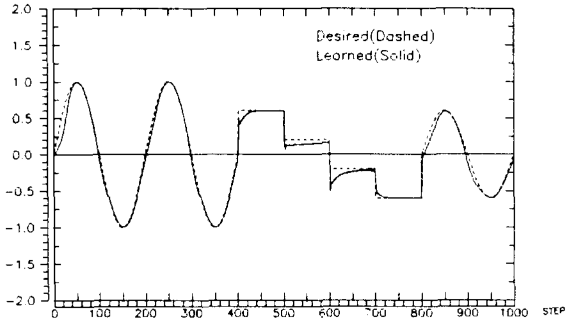

Fig. 4 Simulation results for nonlinear control system with parameter perturbations and structureal disturbation.(Convention DNU)

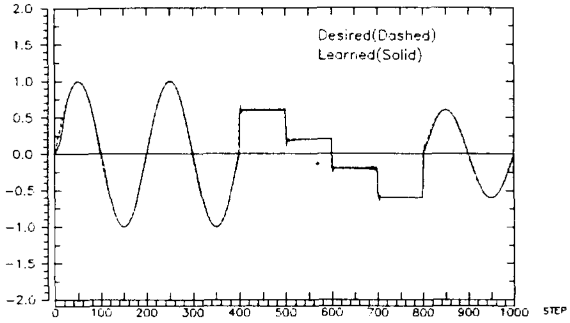

Fig. 5 Simulation results for nonlinear control system with parameter perturbations and structural disturbation.(Proposed DNU)

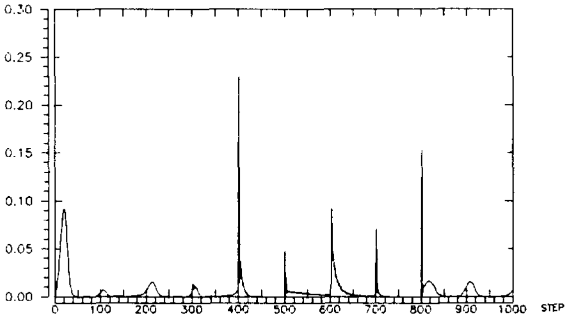

Fig. 6 Error square for convention DNU

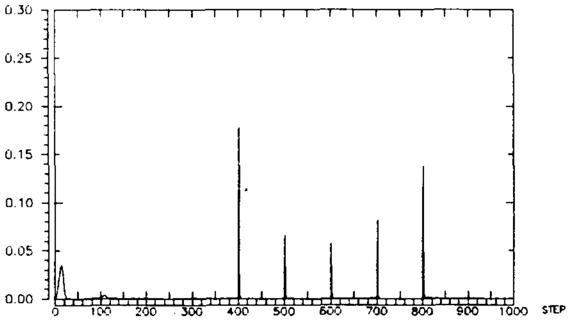

Fig. 7 Error square for proposed DNU

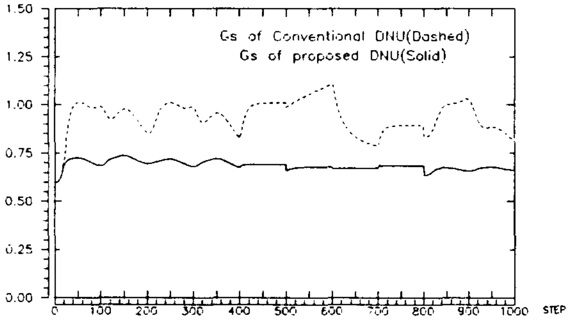

Fig. 8 Error respose for convention DNU(dased) and proposed DNU(solid)

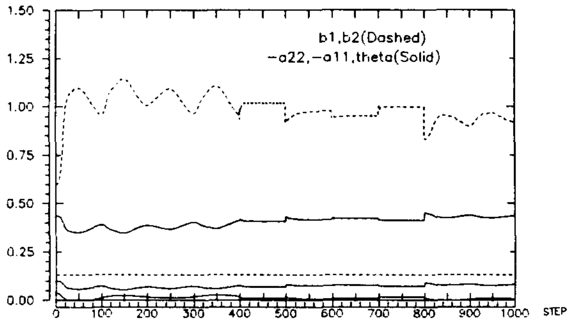

Fig. 9 Threshold and parameter adaption for proposed DNU

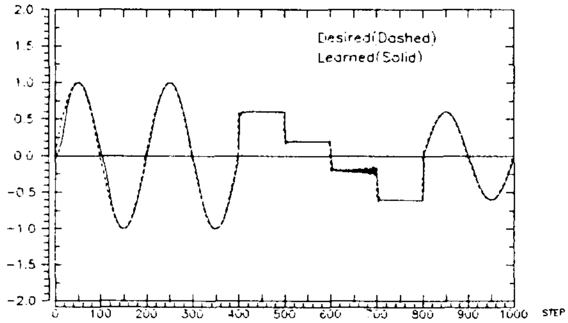

Fig. 10 Output response for parallel connection convention DNU

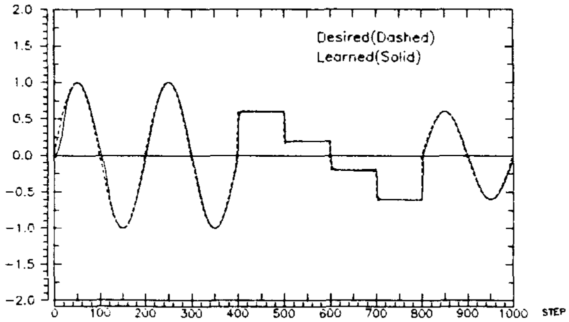

Fig. 11 Output response for parallel connection for proposed DNU