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Design of Fuzzy PID Controller for Based on PI and PD Parallel Structure

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Abstracts In this paper, a new PID fuzzy controller(FC) based on parallel operation of PI and PD fuzzy control is presented. First, two fuzzy rule bases are constructed by separating the linguistic control rule for PID FC into two parts : one is e-Δe part, and the other is $\Delta^2 e$ - Δe part. And then two FCs employing these rule bases indivisually are synthesized and run in parallel. The incremental control input is determined by taking weighted mean of the outputs of two FCs. The proposed PID FC improves the transient response of the system and gives better performance than the conventional PI FC.

Keywords Fuzzy control, PID, Parallel operation, Weighted average, Transient response

1. Introduction

Although the majority of industrial process nowadays are regulated by PID controllers, PID controller has some disadvantages on performance. It shows the limits on the speed and the stability, and it is sensitive to the change of the parameters of process and the control environment. Also, it doesn't give good performance for complex and/or nonlinear processes in general[1]. Thus fuzzy control may be used to overcome these situations.

Fuzzy control has created lots of interest in recent years. In fuzzy control, the linguistic descriptions of control laws are represented as fuzzy rules of relations, and this rule base is used by an inference mechanism to determine the control action. So it can reflect and implement the experience of human expert, and it doesn't need the mathematical modelling of controlled process[2-5].

Since most of fuzzy controllers adopt the linguistic control rules related to the output error e and the incremental change of output error Δe, they have the similarity to conventional PID controllers. But there exists the important and decisive difference between them. Conventional PID controller generates the control action linearly related to these variables. On the other hand, fuzzy controller(FC) determines the control action nonlinearly related to these variables due to the fuzzy rule base and the inference mechanism. Therefore it can cover broader range of operational conditions than conventional PID controller, and give better and robust performance [3,6-8].

Even though the details may differ with each other, the fuzzy controllers may be classified into two groups by and large: one is so-called PD FC which generates directly the control input u from e and Δe, and the other is PI FC which generates the incremental control input Δu from e and Δe instead of u. From the viewpoint of implementation of human expert's knowledge, PI FC is known to be more feasible than PD FC. PI FC gives good performance in steady state, but gives poor performance in transient state. One natural approach to overcome this situation is to adopt the change of output error rate $\Delta^2 e$ as well as e and Δe in the linguistic control rules, like PID controller. If so, the fuzzy controller becomes PID FC[9,10]. But it is difficult to implement PID FC because the fuzzy control rules related to $\Delta^2 e$ can hardly be constructed from the experience and the knowledge of human expert.

So we present a new PID FC based on parallel operation of PI and PD fuzzy control. Instead of finding the linguistic control rules in which e, Δe, and $\Delta^2 e$ are simultaneously taken into consideration, fuzzy rule bases are constructed by separating the linguistic control rule for PID FC into two parts : one is e-Δe part corresponding to PI control, and the other is $\Delta^2 e$ - Δe part corresponding to PD control. And then two FCs employing these rule bases indivisually are synthesized and run in parallel. The incremental control input Δu is determined by taking weighted mean of the outputs of PI FC and PD FC. The proposed PID FC improves the transient response of the system such as overshoot and rise time, and gives better performance than the conventional PI FC.

2. PID FC based on parallel operation of PI and PD fuzzy control

PID controller, as well known, generates the control input as a linear combination of output error(a component reflecting present status of output error), its derivative(a component reflecting future trend of output error), and its integral(a component reflecting past history of output error). The position type PID control law is as follows[1].

$$u(t)=K_p \left[e(t)+\frac{1}{T_i}\int_0^t e(t)dt + T_d\frac{de(t)}{dt} \right] \quad (1)$$

Discretizing eq.(1) by first order approximation of derivative and integral terms in consideration of sampling time T_s, the velocity type PID control law is obtained.

$$\Delta u(k) = K_p \left[\Delta e(k) + \frac{T_s}{T_i} e(k) + \frac{T_d}{T_s} \Delta^2 e(k) \right]$$
$$= K_1 e(k) + K_2 \Delta(k) + K_3 \Delta^2 e(k) \quad (2)$$

As shown in eq.(2), conventional PID control law determines Δu linearly with respect to e, Δe, and $\Delta^2 e$.

Instead of mathematical control rule such as eq.(2), a set of linguistic control rules expressed as "IF --- THEN ---" statement is used in fuzzy controller. The fuzzy controller determines Δu nonlinearly with respect to e, Δe, and $\Delta^2 e$ due to the rule base and the inference mechanism. In general, the fuzzy rule base of PID FC consists of the linguistic control rules of the the following type.

"IF e is A and Δe is B and $\Delta^2 e$ is C, THEN Δu is D" (3)

However, it is very difficult to construct the linguistic control rules like (3) in which e, Δe, and $\Delta^2 e$ are simultaneously taken into consideration. So many fuzzy controllers adopt the PI type rules because it is easier and more feasible. But PI FC gives poor performance in transient state.

To overcome this situation, we separate the linguistic control rule (3) into two parts: one is "IF e is A and Δe is B, THEN Δu is C", and the other is "IF $\Delta^2 e$ is D and Δe is E, THEN Δu is F". Since, from eq.(1) and eq.(2), we know that e, Δe, and $\Delta^2 e$ in velocity type control law correspond integral term, proportional term, and derivative term respectively, we can interpret that the former is the rule corresponding to PI control, and the latter is the rule corresponding to PD control. Based on this idea, two fuzzy rule bases for PI FC and PD FC are constructed. And then two FCs employing these rule bases indivisually are synthesized and run in parallel. Finally Δu is determined by taking weighted mean of the outputs of two FCs. The block diagram of the proposed PID FC scheme is represented in Fig. 1

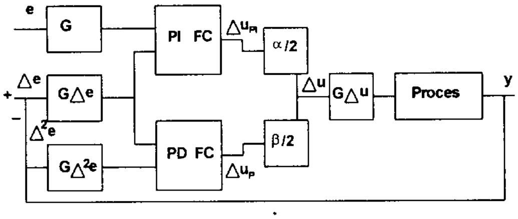

Figure 1. Configuration of PID FC

2.1 Synthesis of PI FC and PD FC

The fuzzy controller receives (crisp) quantitative numerical values as input and must provide actuators with (crisp) quantitative numerical values, while it internally handles fuzzy values described by membership functions. Also the fuzzy controller provides an algorithm which can convert a linguistic control strategy into an automatic control strategy. Therefore the fuzzy controller involves four parts: fuzzifier, rule base, inference engine and defuzzifier.

First of all, the fuzzifier maps crisp data to fuzzy variables characterized by a fuzzy set and a linguistic label. Then the inference engine applies an inference rule to the rule base to generate fuzzy values of the (incremental) control signal from the input facts of the controller. Before its application to the process, the fuzzy (incremental) control signal is defuzzified to provide a (crisp) numerical values of (incremental) control signal.

Let's consider the well known typical step response of the process. From this, we can obtain e, Δe, and $\Delta^2 e$, and plot them on e/N_e-$\Delta e/N_{de}$ plane and $\Delta^2 e/N_{sde}$-$\Delta e/N_{de}$ plane as shown in Fig.2, where N_e,N_{de} and N_{sde} are the normalization factors.

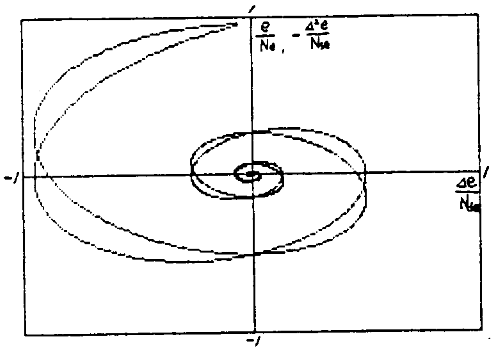

Figure 2. The chracteristic of the process in e/N_e-$\Delta e/N_{de}$, and $\Delta^2 e/N_{se}$-$\Delta e/N_{de}$ planes

Fig.2 reveals that the characteristics in e/N_e-$\Delta e/N_{de}$ plane is similar to that in $\Delta^2 e/N_{sde}$-$\Delta e/N_{de}$ plane. So the same control rule base can be applied to PI FC and PD FC. Here the well-known rule base for PI FC is used, which is represented in Table 1. The universes of discourse of e, Δe, $\Delta^2 e$ and Δu are partitioned into seven fuzzy subsets which are Negative Big(NB), Negative Medium(NM), Negative Small(NS), ZEro(ZE), Positive Small(PS), Positive Medium(PM), and Positive Big(PB). One example of the rules is as follows.

$R_{2,4}$: "IF e is ZE and Δe is NM, THEN Δu is NS"(PI FC)
"IF $-\Delta^2 e$ is ZE and Δe is NM, THEN Δu is NS"(PD FC)

TABLE 1. Linguistic control rule base for PI FC and PD FC.

(→ j; ↓ i)

i	Δe	e, $-\Delta^2 e$: NB	NM	NS	ZE	PS	PM	PB
	NB	NB	NB	NM	NM	NS	NS	ZE
	NM	NB	NM	NM	NS	NS	ZE	PS
	NS	NM	NM	NS	NS	ZE	PS	PS
	ZE	NM	NS	NS	ZE	PS	PS	PM
	PS	NS	NS	ZE	PS	PS	PM	PM
	PM	NS	ZE	PS	PS	PM	PM	PB
	PB	ZE	PS	PS	PM	PM	PB	PB

A fuzzy subset A is characterized by its membership function $\mu_A(\cdot)$. The membership functions of the fuzzy variables e, Δe, and $\Delta^2 e$ with respect to the liguistic variables in Table 1 are shown in Fig.3.

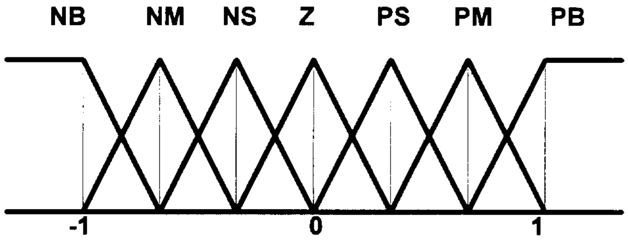

Figure 3. Membership function of e, △e, and $\triangle^2 e$

These membership functions have the following property.

$$\sum_{j=1}^{7} \mu_{Aj}(e) = \sum_{j=1}^{7} \mu_{Bj}(\Delta e) = \sum_{j=1}^{7} \mu_{Cj}(-\Delta^2 e)$$

$$= \sum_{j=1}^{7} \mu_{Tj}(\Delta u) = 1 \quad (4)$$

Using the membership functions in Fig.4, there are at most four linguistic control rules ($R_{i,j}$, $R_{i+1,j}$, $R_{i,j+1}$, $R_{i+1,j+1}$) which contribute to the output of FC at any time.

From the rule base in Table 1, the inference engine produces fuzzy value of Δu, and then crisp numerical value of Δu is obtained via defuzzification procedure. The most popular method on inference and defuzzification is Mamdani's max-min composition with center of gravity(COG) method. However, it is somewhat complex to implement from the view point of computational burdens. Thus we take the simplified method as the inference and defuzzification method[10].

In simplfied method, crisp value of the output of FC is computed by

$$\Delta u(k) = \frac{\sum_{i,j} \tau_{i,j} \Delta u_{i,j}(k)}{\sum_{i,j} \tau_{i,j}} \quad (5)$$

where $\tau_{i,j}$ is the firing level of i,j-th rule, $\Delta u_{i,j}(k)$ is the center point of the output fuzzy subset of the i,j-th rule as shown in Fig.3. The firing level of i,j-th rule $\tau_{i,j}$ is taken as

$$\tau_{i,j} = \mu_{Bi}(\Delta e(k)) \wedge \mu_{Aj}(e(k)) \quad \text{(PI FC)} \quad (6)$$
$$\mu_{Bi}(\Delta e(k)) \wedge \mu_{Aj}(-\Delta^2 e(k)) \quad \text{(PD FC)}$$

If minimum operator is chosen as $\wedge$ operator, $\tau_{i,j}$ becomes

$$\tau_{i,j} = \min\{\mu_{Bi}(\Delta e(k)),\ \mu_{Aj}(e(k))\} \quad \text{(PI FC)} \quad (7)$$
$$\min\{\mu_{Bi}(\Delta e(k)),\ \mu_{Aj}(-\Delta^2 e(k))\} \quad \text{(PD FC)}$$

And if product operator is chosen as $\wedge$ operator, $\tau_{i,j}$ becomes

$$\tau_{i,j} = \mu_{Bi}(\Delta e(k)) \cdot \mu_{Aj}(e(k)) \quad \text{(PI FC)} \quad (8)$$
$$\mu_{Bi}(\Delta e(k)) \cdot \mu_{Aj}(-\Delta^2 e(k)) \quad \text{(PD FC)}$$

Let the membership functions in i,j-th rule $R_{i,j}$ $\mu_{Bi}(\Delta e(k))$(or $\mu_{Bi}(\Delta e(k))$)=α and $\mu_{Aj}(e(k))$(or $\mu_{Aj}(-\Delta^2 e(k))$)=β.

From eq.(4) and eq.(7), it is easily verified that the firing levels of four rules $R_{i,j}$, $R_{i+1,j}$, $R_{i,j+1}$, and $R_{i+1,j+1}$ become

$$\begin{aligned}
\tau_{i,j} &= \alpha && \text{if } \alpha<\beta \\
& \beta && \text{if } \alpha>\beta \\
\tau_{i+1,j} &= 1-\alpha && \text{if } \alpha+\beta>1 \\
& \beta && \text{if } \alpha+\beta<1 \\
\tau_{i,j+1} &= \alpha && \text{if } \alpha+\beta<1 \\
& 1-\beta && \text{if } \alpha+\beta>1 \\
\tau_{i+1,j+1} &= 1-\alpha && \text{if } \alpha>\beta \\
& 1-\beta && \text{if } \alpha<\beta
\end{aligned} \quad (9)$$

2.2 Determination of △u by Weighted Average

When PI FC and PD FC produce the values of the incremental control inputs indivisually, the control input to the process must be determined from them.

We know that, in general, PD control increases the damping of the process and decreases the overshoot of its response. On the other hand, PI control Improves the steady state errors of the process, but has a disadvantage in the response speed. also we know from Table 1 that the control rules below a diagonal, R_{ij}, $j \geq -i+8$ play a role of accelaration because △u is positive, while those above a diagonal function as decelaration since △u is negative.

So the output of PI FC, $\triangle u_{PI}$ is heavily weighted in case of accelaration, and is lightly weighted in case of decelaration. That is, the more positive $\triangle u_{PI}$ is, a larger weight it has, and the more negative it is, a smaller weight it has. So far as PD FC is concerned, it is the very reverse.

Thus the output of PID FC, $\triangle u$ is determinated as follows.

$$\Delta u = (\alpha \Delta u_{PI} + \beta \Delta u_{PD}) / 2 \tag{10}$$

$$\alpha = \begin{cases} 1-\dfrac{\Sigma\,(8-i-j)}{14n} & \Delta u_{PI} < 0 \\ 1+\dfrac{\Sigma\,(i+j-8)}{14n} & \Delta u_{PI} > 0 \end{cases} \tag{11}$$

$$\beta = \begin{cases} 1+\dfrac{\Sigma\,(8-i-j)}{14m} & \Delta u_{PD} < 0 \\ 1-\dfrac{\Sigma\,(i+j-8)}{14m} & \Delta u_{PD} > 0 \end{cases} \tag{12}$$

where n and m are the number of control laws used in the datermination of $\triangle u_{PI}$ and $\triangle u_{PD}$ respectively.

3. Computer simulation

The computer simulations are performed to verify the effectiveness of the proposed PID FC.

example> 2nd order process

$$G(s)= \frac{1}{s\,(s+1)} \tag{13}$$

The step responses corresponding to the PID FC and the conventional PI FC are shown in Fig. 4.

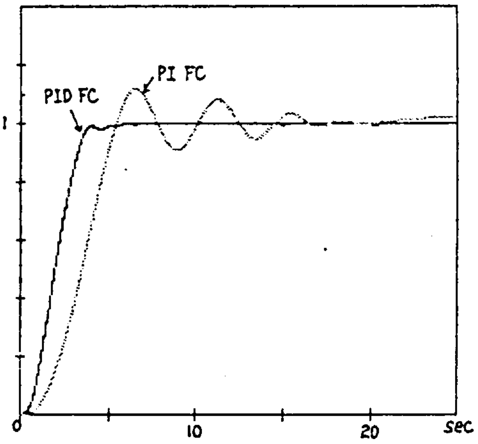

Figure 4. The unit step response of the plant by PID FC and PI FC

As shown in Fig.4, the response of the PID FC gives better performence-short rising time and small overshoot-than that of the conventional PI FC.

4. Conclusion

The commonly used PI FC gives good performance in steady state, but gives poor performance in transient state.

So we present a new PID FC based on parallel operation of PI and PD fuzzy control. Instead of finding the linguistic control rules in which e, Δe, and $\Delta^2 e$ are simultaneously taken into consideration, fuzzy rule bases are constructed by separating the linguistic control rule for PID FC into two parts : one is e-Δe part corresponding to PI control, and the other is $\Delta^2 e$ - Δe part corresponding to PD control. And then two FCs employing these rule bases indivisually are synthesized and run in parallel. The incremental control input Δu is determined by taking weighted mean of the outputs of PI FC and PD FC.

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