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Construction of a Robust Control System for a Plant with Time Delay

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Abstracts In this paper, though Smith controller is also used, we propose a new system configuration which can be regarded an SISO continuous nth-order plant with time delay of k-times of a sampling period as a linear discrete $(n+k)$th-order plant of which all state variables can be available. Consequently, we can apply linear control system design techniques which do not consider the existence of time delay to the proposed system.

Keywords Time delay, Smith controller, Robustness, H_∞ control theory,

1. Introduction

Smith method is well known as a feedback design technique for a plant with time delay[1]. We, however, can't specify both of an input-output property and stability because the system designed by Smith method is with one degree of freedom.

In this paper, though Smith controller is also used, we propose a new system configuration which can be regarded an SISO continuous nth-order plant with time delay of k-times of sampling period as a linear discrete $(n+k)$th-order plant of which all state variables can be available.

Consequently, we can apply linear control system design techniques which do not consider the existence of time delay and need all state variables of the plant to the proposed system.

2. Smith controller

We consider an SISO nth-order plant with time delay which is asymptotically stable and whose input-output pulse transfer function is given by $M(z)(1+\Delta(z))Z^{-k}$,where $M(z)Z^{-k}$ represents the nominal model of the plant, $\Delta(z)$ represents uncertainty, and time delay is equal to k-times of a sampling period.

First, we state how to design the compensator which is an element in Smith controller. When we construct the system shown in Fig.1 for the plant with time delay, the relationship between $J(z)$ and $F(z)$ can be written as Eq.(1) .

$$F(z) = M(z)(1+\Delta(z)Z^{-k})\{-J(z)\} + \tilde{D}(z) \qquad (1)$$

where $\tilde{D}(z) = -D(z) - M(z)\Delta(z)Z^{-k}V(z)$, and $D(z)$ represents an external disturbance.

Furthermore, we construct the feedback system shown in Fig.2 by introducing an appropriate compensator $K(z)$ between $J(z)$ and $F(z)$, which are regarded as an input and output, respectively.

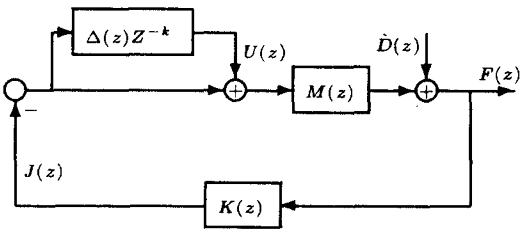

Fig.2 The considered feedback system.

The pulse transfer function from $U(z)$ to $J(z)$, which is called a complementary sensitivity function, is given by Eq.(2) .

$$T(z) = \frac{J(z)}{U(z)} = \frac{M(z)K(z)}{1+M(z)K(z)} \qquad (2)$$

When $\Delta(z) = 0$, the pulse transfer function from $\tilde{D}(z)(= -D(z))$ to $F(z)$, which is called a sensitivity function, is given by Eq.(3) .

$$S(z) = \frac{F(z)}{\tilde{D}(z)} = \frac{1}{1+M(z)K(z)} \qquad (3)$$

It follows from Eqs.(2) and (3) that if we can obtain the compensator $K(z)$ as a solution of the mixed sensitivity problem in the H infinity control theory under the performance index of Eq.(4) for the nominal model $M(z)$ without time delay, low sensitivity and robust stability are guaranteed in the system shown in Fig.2.

$$\left\| \begin{matrix} W_S(z)S(z) \\ W_T(z)T(z) \end{matrix} \right\|_\infty \qquad (4)$$

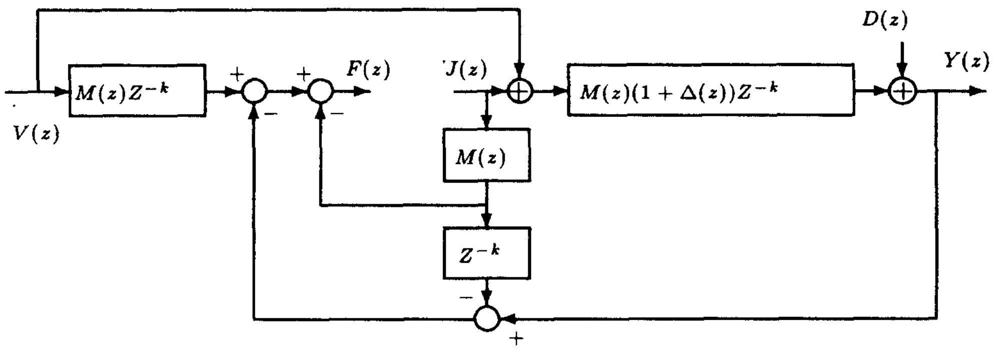

Fig.1 The considered system.

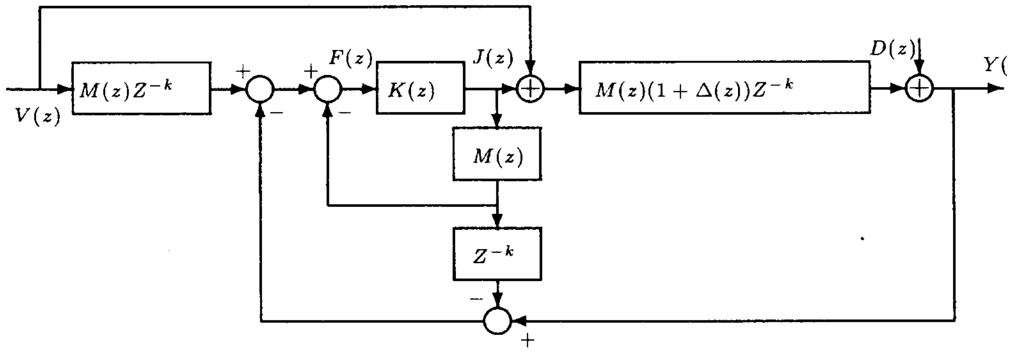

Fig.3 The system which can be regarded as the nominal model without time delay.

$$
\begin{aligned}
\frac{Y(z)}{V(z)} &= \frac{M(z)(1+\Delta(z))Z^{-k}(1+K(z)M(z))}{1+K(z)M(z)+K(z)M(z)\Delta(z)Z^{-k}} \qquad (5)\\
&= M(z)\left[1+\frac{\Delta(z)(1+K(z)M(z)-K(z)M(z)Z^{-k})}{1+K(z)M(z)+K(z)M(z)\Delta(z)Z^{-k}}\right]Z^{-k}\\
&= M(z)(1+\bar{\Delta}(z))Z^{-k}
\end{aligned}
$$

$$
\frac{Y(z)}{D(z)} = \frac{1+K(z)M(z)-K(z)M(z)Z^{-k}}{1+K(z)M(z)+K(z)M(z)\Delta(z)Z^{-k}}
$$

Therefore, when we construct the system shown in Fig.3 by using the compensator $K(z)$, robust stability are guaranteed in the feedback loop, low sensitivity is realized in a sense that the disturbance and uncertainty do not appear at $F(z)$. In the system shown in Fig.3, $(n+k)$ state variables in the nominal model $M(z)Z^{-k}$ are available because the nominal model is constructed in a computer.

The input-output pulse transfer function from $V(z)$ to $Y(z)$ in Fig.3 is given by Eq.(5) in which the disturbance and uncertainty remain at output $Y(z)$. We, however, can regard this system as the new linear $(n+k)$th-order plant whose input-output pulse transfer function is given by $M(z)(1+\bar{\Delta}(z))Z^{-k}$, and all state variables are available. This means that we can apply linear control system design techniques which need all state variables to the new linear $(n+k)$th-order plant.

3. LQ problem

As an example of applying a linear control system design technique to the proposed system, we consider the LQ problem.

The plant is expressed as

$$
M(s)(1+\Delta(s))e^{-sL} = \frac{1}{s^2+0.8s+0.8}e^{-20ms} \qquad (6)
$$

The nominal model of the plant is expressed as

$$
M(s)e^{-sL} = \frac{1}{s^2+s+1}e^{-20ms} \qquad (7)
$$

Choosing Eqs.(8) and (9) as the weighting functions, we obtained the compensater $K(s)$ by using MATLAB.

$$
W_S(s) = \frac{0.7(s+10)}{s+0.01} \qquad (8)
$$

$$
W_T(s) = \frac{0.01(s+100)}{s+10000} \qquad (9)
$$

Each element was discretized by using a sampler of which sampling period is 5msec and a zero-order hold device.

Consequently, the state equation of the nominal model was obtained as Eq.(10). We consider the LQ problem for the nominal model which minimizes the performance index of Eq.(11).

$$
J_d = \sum_{i=0}^{\infty}(x^T(i)Q_d x(i) + u^T(i)R_d u(i)) \qquad (11)
$$

$$
Q_d = \begin{pmatrix} 10 & 0 & 0 & 0 & 0 & 0\\ 0 & 10 & 0 & 0 & 0 & 0\\ 0 & 0 & 10 & 0 & 0 & 0\\ 0 & 0 & 0 & 10 & 0 & 0\\ 0 & 0 & 0 & 0 & 10 & 0\\ 0 & 0 & 0 & 0 & 0 & 10 \end{pmatrix} \qquad (12)
$$

$$
R_d = (1000) \qquad (13)
$$

$$
\begin{bmatrix} x_{1(k+1)}\\ x_{2(k+1)}\\ x_{3(k+1)}\\ x_{4(k+1)}\\ x_{5(k+1)}\\ x_{6(k+1)} \end{bmatrix} = \begin{bmatrix} 9.99987\times10^{-1} & 4.98750\times10^{-3} & 1.24791\times10^{-5} & 0 & 0 & 0\\ -4.98750\times10^{-3} & 9.95000\times10^{-1} & 4.98750\times10^{-3} & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1(k)}\\ x_{2(k)}\\ x_{3(k)}\\ x_{4(k)}\\ x_{5(k)}\\ x_{6(k)} \end{bmatrix} + \begin{bmatrix} 0\\0\\0\\0\\0\\1 \end{bmatrix} u_{(k)} \qquad (10)
$$

The optimal solution was obtained as

$$\begin{bmatrix} f1 \\ f2 \\ f3 \\ f4 \\ f5 \\ f6 \end{bmatrix}^T = \begin{bmatrix} 4.53165 \times 10^{-3} \\ 9.35179 \times 10^{-3} \\ 4.68193 \times 10^{-5} \\ 4.69394 \times 10^{-5} \\ 4.70590 \times 10^{-5} \\ 4.71781 \times 10^{-5} \end{bmatrix}^T \quad (14)$$

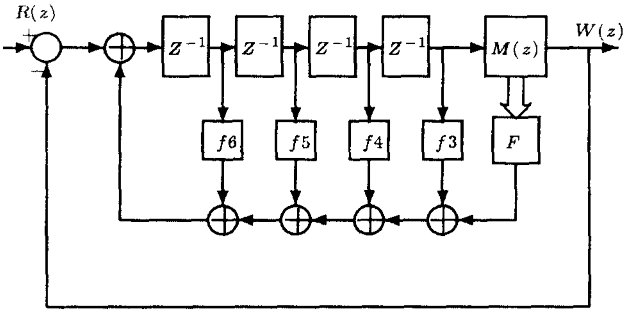

Fig.4 The optimal system for the nominal model.

Fig.4 shows the optimal control system for the nominal model, where $F = [f1 + 1, f2]$. In this case, we must represent explicitly the output feedback. We construct the system shown in Fig.5 by uniting the two systems in Fig.3 and Fig.4. In Fig.6, A and B show the step responses of the systems in Fig.4 and Fig.5, respectively. It follows from Fig.6 that we can apply linear control design techniques to the proposed system.

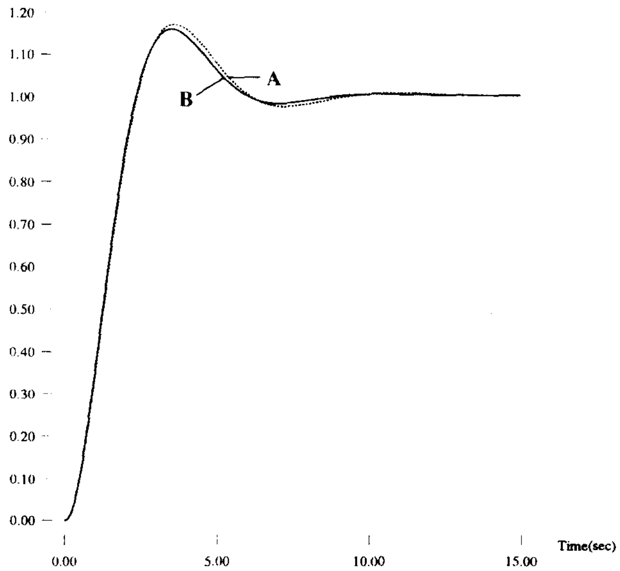

Fig.6 A and B show the step responses of the systems in Fig.4 and Fig.5, respectively.

4. Conclusions

In this paper, we proposed a new system configuration which can be regarded as an SISO continuous plant with time delay as a linear discrete plant of which all state variables can be available.

Moreover, we solved the LQ problem for the proposed system as an example.

Reference

[1] Keiji Watanabe: Control of systems with time delay, SICE in Japan(1993)

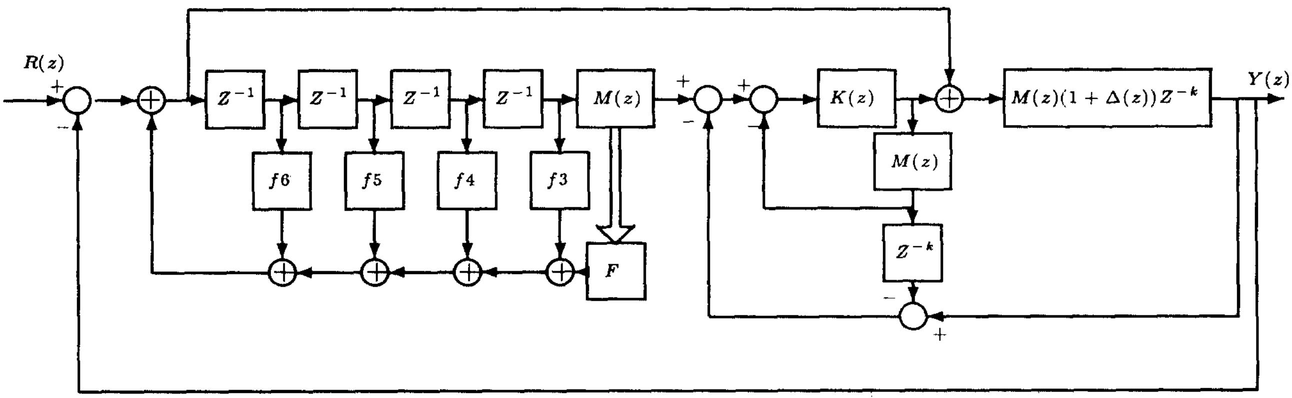

Fig.5 The optimal system for the plant with time delay.