'95 KACC (1995. 10. 23 ~ 25)

Construction of a Robust Dead Beat Control System Considered a Transient Response

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Abstracts First, in this paper we propose a new dead beat control system design technique by which we can specify a transient response before the settling time. Though the resultant system has the same system configuration as Reference[1], that is realized by adopting the performance index which includes the term of the square of difference between specified and practical responses. Next, we state a technique which gives the dead beat control system robustness and construct a robust dead beat control system. Simulations of the proposed dead beat control and robust dead beat control systems show expected results.

Keywords Dead beat control, Transient response, Robustness, H_∞control theory, Model following system

1. Introduction

Many dead beat control system design techniques have been reported[1]-[4]. But a transient response before the settling time can not be specified by these techniques.

In this paper, first we propose a new dead beat control system design technique which can specify a transient response. Though the resultant system has the same system configuration as Reference[1], that is realized by adopting the performance index which includes the term of the square of the difference between specified and practical responses.

Next, we state a technique which gives the dead beat control system robustness and construct a robust dead beat control system. Simulations of the proposed dead beat control and robust dead beat control systems show expected results.

2. Dead beat control system

We consider a problem of designing a dead beat control system for a unit step input. Reference[1] proposed the system configuration shown in Fig.1 in order to solve this problem. The feature of the proposed system is that the coefficients in the numerator polynomial of the compensator is obtained as solution of the optimal problem, the coefficients in the denominator polynomial of the compensator and feedback gains are obtained from a linear algebraic equation. In this paper, we also adopt the system shown in Fig.1. It is assumed that the pulse transfer functions of the plant and the compentsator are given by eq.(1) and eq.(2), respectively.

$$P(z) = \frac{b_{n-1}z^{n-1} + \cdots + b_0}{z^n + a_{n-1}z^{n-1} + \cdots + a_0} = \frac{N_P(z)}{D_P(z)} \quad (1)$$

$$C(z) = \frac{d_m z^m + \cdots + d_0}{z^m + c_{m-1}z^{m-1} + \cdots + c_0} = \frac{N_C(z)}{D_C(z)} \quad (2)$$

And it is assumed that a specified error polynomial of the difference between the unit step input $R(z)$ and output $Y(z)$ is described as

$$E_s(z) = 1 + e_1 z^{-1} + e_2 z^{-2} + \cdots + e_{k-1} z^{-(k-1)} \quad (3)$$

where $k = n + m$ represents the settling step.

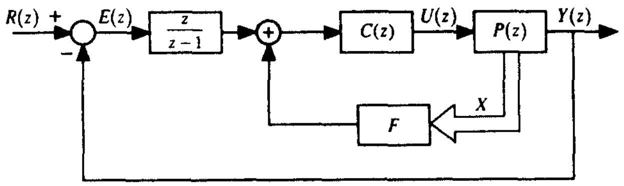

Fig.1. The dead beat control system.

The pulse transfer functions $G_{re}(z)$ from the input $R(z)$ to the error $E(z)$ and $G_{ru}(z)$ from $R(z)$ to the control input $U(z)$ can be obtained as

$$G_{re}(z) = 1 - \frac{zN_C(z)N_P(z)}{(z-1)(D_C(z)D_P(z) - N_C(z)N_F(z)) + zN_C(z)N_P(z)} \quad (4)$$

$$G_{ru}(z) = \frac{zN_C(z)D_P(z)}{(z-1)(D_C(z)D_P(z) - N_C(z)N_F(z)) + zN_C(z)N_P(z)} \quad (5)$$

$$\text{where} \quad N_F(z) = f_{n-1}z^{n-1} + \cdots + f_1 z + f_0 \quad (6)$$

$$F = \begin{pmatrix} f_0 & f_1 & \cdots & f_{n-1} \end{pmatrix} \quad (7)$$

The necessary and sufficient condition to realize dead beat control can be represented as

$$(z-1)(D_C(z)D_P(z) - N_C(z)N_F(z)) + zN_C(z)N_P(z) = z^{k+1} \quad (8)$$

Applying the final value theorem to the equation which is obtained by substituting eq.(8) into eq.(4), we can derive eq.(9) as a condition of no steady error

$$N_C(1)N_P(1) = 1 \quad (9)$$

When we define the following matrices and vectors, the vectors which consist of the coefficients in the numerator polynomials in eq.(4) and eq.(5) can be expressed by eq.(10) and eq.(11), respectively.

$$\mathbf{d} = \begin{pmatrix} d_m & d_{m-1} & \cdots & d_1 & d_0 \end{pmatrix}^T \in R^{m+1}$$

$$\mathbf{B} = \begin{pmatrix} b_{n-1} & 0 & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ b_0 & & \ddots & 0 \\ 0 & \ddots & & b_{n-1} \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & b_1 \end{pmatrix} \in R^{(k-1)\times(m+1)}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ a_{n-1} & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ a_0 & & & 1 \\ 0 & \ddots & & a_{n-1} \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & a_1 \end{pmatrix} \in R^{k\times(m+1)}$$

$$\mathbf{L} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix} \in R^{(k-1)\times(k-1)}$$

$$\mathbf{L}_u = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix} \in R^{k\times k}$$

$$N_C(z)N_P(z) \Rightarrow \mathbf{LBd} \tag{10}$$

$$N_C(z)D_P(z) \Rightarrow \mathbf{L}_u\mathbf{Ad} \tag{11}$$

Therefore, the vectors which consist of the coefficients in the polynomial of the difference between the input $R(z)$ and output $Y(z)$, and between the steady and transient input value to the plant can be represented by eq.(12) and eq.(13), respectively,

$$\mathbf{E} = \mathbf{1} - \mathbf{LBd} \tag{12}$$

$$\mathbf{E}_u = \mathbf{u}_s - \mathbf{L}_u\mathbf{Ad} \tag{13}$$

where $\mathbf{1} = \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \end{pmatrix}^T \in R^{k-1}$
$\mathbf{u}_s = \begin{pmatrix} u_s & u_s & \cdots & u_s & u_s \end{pmatrix}^T \in R^{k}$
$u_s = D_P(1)/N_P(1)$ is the steady input value to the plant.

Next, we consider the problem of minimizing the performance index* of eq.(14) under the constraint condition of eq.(9).

$$\mathbf{J} = (\mathbf{E}_s - \mathbf{E})^T(\mathbf{E}_s - \mathbf{E}) + \rho\ {\mathbf{E}_u}^T\mathbf{E}_u \tag{14}$$

where ρ is a non-negative number and $\mathbf{E}_s$ is given by eq.(15), which consists of the coefficients in the specified error polynomial.

$$\mathbf{E}_s = \begin{pmatrix} e_1 & e_2 & \cdots & e_{k-2} & e_{k-1} \end{pmatrix}^T \in R^{k-1} \tag{15}$$

The solution of the above optimal problem can be obtained as eq.(16) by Lagrange multiplier method.

$$\mathbf{d} = \delta^{-1}\varepsilon + (\mathbf{1}_d^T\delta^{-1}\mathbf{1}_d)^{-1}(\rho - \mathbf{1}_d^T\delta^{-1}\varepsilon)\delta^{-1}\mathbf{1}_d \tag{16}$$

where $\delta = 2(\mathbf{B}^T\mathbf{L}^T\mathbf{LB} + \rho\ \mathbf{A}^T\mathbf{L}_u^T\mathbf{L}_u\mathbf{A})$
$\varepsilon = 2\big(\mathbf{B}^T\mathbf{L}^T(\mathbf{1} - \mathbf{E}_s) + \rho\ \mathbf{A}^T\mathbf{L}_u^T\mathbf{u}_s\big)$

Consequently, the numerator $N_C(z)$ of the compensator was determined.

And then, coefficients in the denominator $D_C(z)$ of the compensator and feedback gains $(f_0, f_1, \cdots, f_{n-1})$ are obtained from eq.(17) which is derived from eq(8).

$$\left(\begin{array}{cccc|ccc} a_{n-1} & 1 & & 0 & -d_m & & 0 \\ \vdots & a_{n-1} & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_0 & \vdots & & 0 & -d_0 & & \vdots \\ 0 & a_0 & \ddots & 1 & 0 & \ddots & 0 \\ \vdots & 0 & & a_{n-1} & \vdots & & -d_m \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & a_0 & 0 & & -d_0 \end{array}\right) \cdot \begin{pmatrix} c_{m-1} \\ \vdots \\ c_0 \\ \hline f_{n-1} \\ \vdots \\ f_0 \end{pmatrix} = \begin{pmatrix} -a_{n-1} + \beta_0 + \cdots + \beta_{k-2} \\ -a_{n-2} + \beta_0 + \cdots + \beta_{k-3} \\ \vdots \\ -a_1 + \beta_0 + \cdots + \beta_{m-1} \\ \beta_0 + \cdots + \beta_{m-2} \\ \vdots \\ \beta_0 \\ 0 \end{pmatrix} \tag{17}$$

where $\beta_0, \beta_1, \cdots, \beta_{k-1}$ express the coefficients in the polynomial $N_C(z)N_P(z) = \beta_{k-1}z^{k-1} + \cdots + \beta_1 z + \beta_0$.

3. Robust dead beat control system

We consider to give the proposed dead beat control system robustness. We construct the system shown in Fig.2 in which $P(z)$ is the nominal pulse transfer function of the plant, $\Delta(z)$ represents uncertainty and $K(z)$ is an appropriate compensator. This system is called the robust model following system[5].

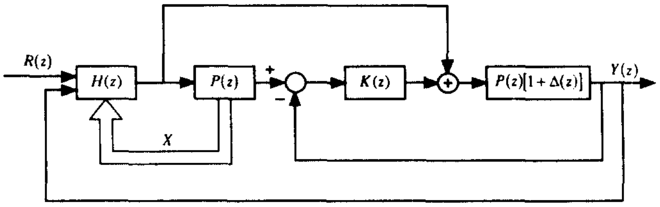

Fig.2. A robust model following system.

The property between the input and output in Fig.2 is given by the system shown in Fig.3 which is called a reference model, and a sensitivity and complementary sensitivity functions can be derived as eq.(18) and eq.(19), respectively.

$$S(z) = \frac{1}{1 + P(z)K(z)}\overline{M(z)} \tag{18}$$

$$T(z) = 1 - S(z) \tag{19}$$

where $\overline{M(z)}$ is the term determined by the reference model.

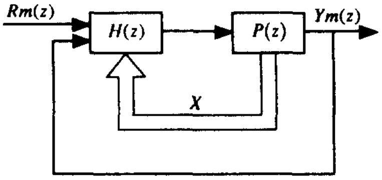

Fig.3. A reference model.

*The performance index in Reference[1] is given as follows.

$$\mathbf{J} = \mathbf{E}^T\mathbf{E} + \rho\ {\mathbf{E}_u}^T\mathbf{E}_u$$

Therefore, when we obtain the compensator $K(z)$ as a solution of the mixed sensitivity problem in $\mathbf{H}_\infty$control theory under the performance index of eq.(20) with appropriate weighting functions $W_S(z)$ and $W_T(z)$, low sensitivity and robust stability are guaranteed in the robust model following system.

$$\left\| \begin{array}{c} W_S(z)S(z) \\ W_T(z)T(z) \end{array} \right\|_\infty \tag{20}$$

Therefore, when we construct the system shown in Fig.4 which is obtained by uniting the systems in Fig.1 and Fig.2, it is appear from above discussion that the property between the input and output is given by that of the dead beat control system in Fig.1, and low sensitivity and robust stability are realized. Moreover all state variables can be extracted from $P(z)$ because the part enclosed by dotted line in Fig.4 is constructed in a digital computer. We call the system in Fig.4 the robust dead beat control system.

4. Simulations

We show a simulation result of the dead beat control system shown in Fig.1, where

$$P(s) = \frac{1}{s^2 + s + 1} \tag{21}$$

and it was discretized by using a sampler of which sampling period is 50(msec) and a zero-order hold device.

A specified error polynomial which corresponds to the output response shown in Fig.5 was chosen.

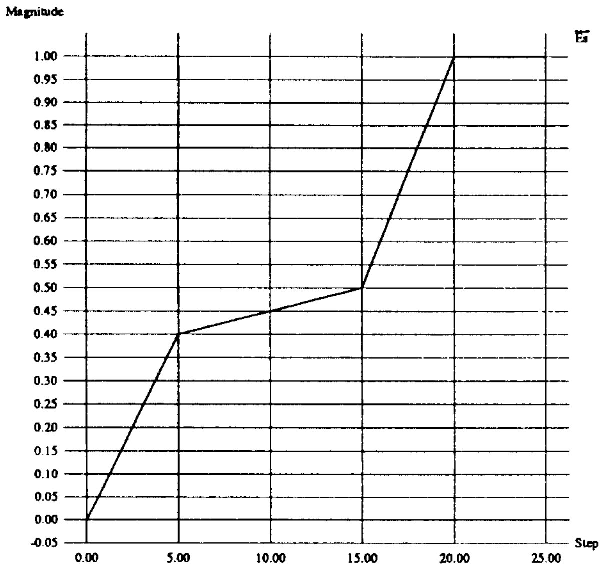

Fig.5. A specified output step response.

The simulation results of output responses and control inputs are shown in Fig.6 and Fig.7.

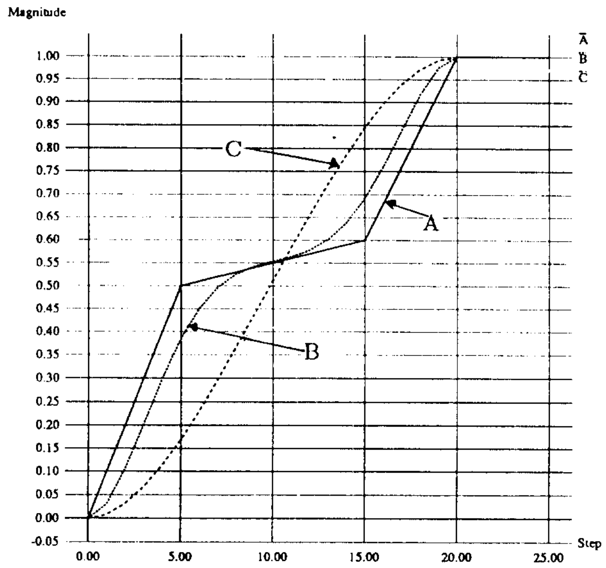

Fig.6. A, B and C show output responses in the case of $\rho = 0$, 10^{-4} and 10^{-2}, respectively.

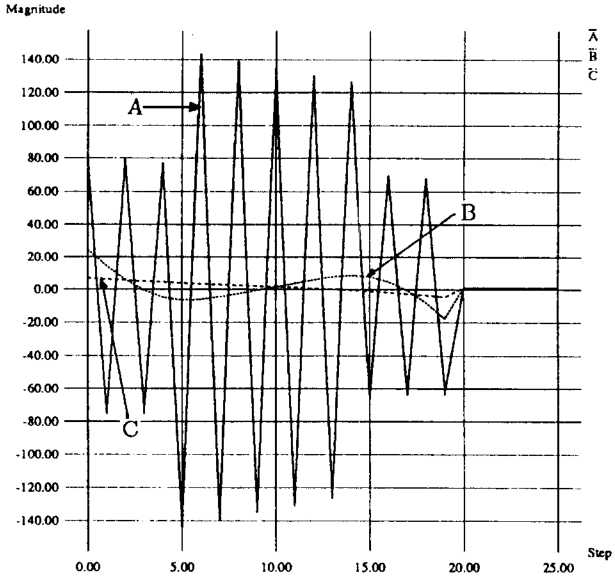

Fig.7. A, B and C show control inputs in the case of $\rho = 0$, 10^{-4} and 10^{-2}, respectively.

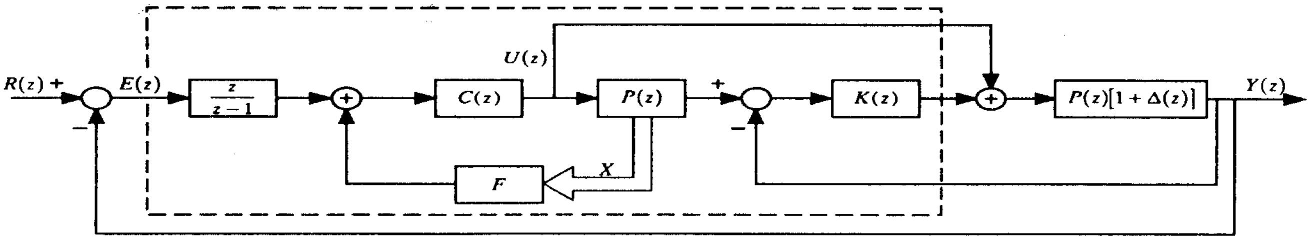

Fig.4. The robust dead beat control system.