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Spacecraft Attitude Control Using Quaternion Parameters

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Abstracts This paper presents an attitude control method using quaternions as feedback attitude errors. The Euler's eigenaxis rotation provides the shortest angular path between two attitudes. This eigenaxis rotation can be achieved by using quaternions since quaternions are related with the eigenaxis. The suggested controller uses error quaternions and body angular rates and generates a decoupling control torque that counteracts the natural gyroscopic coupling torque. The momentum dumping strategy using the earth magnetic field is also applied in this paper to unload the angular momentum of the reaction wheels used in the attitude control.

Keywords quaternion, eigenaxis rotation, momentum dump, IGRF

1 Introduction

The problem of large angle reorientation/slew maneuvers is of considerable interest since many modern and future spacecrafts including LEO(Low Earth Orbit) satellites need large angle maneuvers for their variable missions. Conventional single-axis small angle feedback controls may not be adequate for this three-axis large angle maneuvers[1].

There have been many studies about large angle maneuvers. Among them, control algorithms using quaternion are now commonly applied since the quaternion method has no singularity and is well suited for onboard real-time computation. Moreover, as shown in this paper, quaternion can be used in eigenaxis rotation very easily.

According to the Euler's rotation theorem, the eigenaxis rotation is the minimum path-angle rotation.Although this rotation is not a time-optimal rotation but is considered to be generally "near" the time-optimal rotation[2]. The maneuver time of conventional successive rotations about each body axis is longer than that of a single maneuver about the eigenaxis[3].

The spacecraft also requires controllers that dump the angular momentum of reaction wheels or control momentum gyro by using gravity gradient, aerodynamic forces, earth magnetic fields and so on.

In this paper, a control algorithm for the eigenaxis maneuver with error quaternion feedback is proposed and the momentum dump of reaction wheels using earth magnetic field is considered.

2 Eigenaxis Rotations via Error Quaternion Feedback

This section presents a control algorithm for eigenaxis rotation using error quaternion feedback. An ideal body-fixed control torquer is assumed.

2.1 Euler's Equation of Motion

In general cases in which the body-fixed control axes do not coincide with the principal axes of inertia, the Euler's equation which describes the rotational motion of a rigid body about body-fixed axes with origin at the center of mass can be written as [3]

$$J\dot{\omega} = \Omega J\omega + \mathbf{T_a} + \mathbf{T_m} \tag{1}$$

where $\omega = [p, q, r]^T$ is the angular velocity vector of the rigid body,$\mathbf{T_a} = [T_1, T_2, T_3]^T$ is the attitude control torque vector, $\mathbf{T_m}$ is the angular momentum dump control torque vector, J is the inertia matrix, and $\Omega = [-\omega\times]$ is a skew-symmetric matrix defined by

$$\Omega = -\begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \tag{2}$$

It is assumed that the angular velocity is measured by some devices.

2.2 Quaternions

When two coordinate systems (X, Y, Z) and (X', Y', Z') are related as shown in Fig.1, the quaternion that expresses the coordinate transformation from coordinate systems (X, Y, Z) to (X', Y', Z') is defined as [4]

$$\begin{aligned} q_0 &= \cos\frac{\mu}{2} \\ q_1 &= \cos\alpha\sin\frac{\mu}{2} \\ q_2 &= \cos\beta\sin\frac{\mu}{2} \\ q_3 &= \cos\gamma\sin\frac{\mu}{2} \end{aligned} \tag{3}$$

The direction of the eigenaxis is specified by q_1, q_2 and q_3.

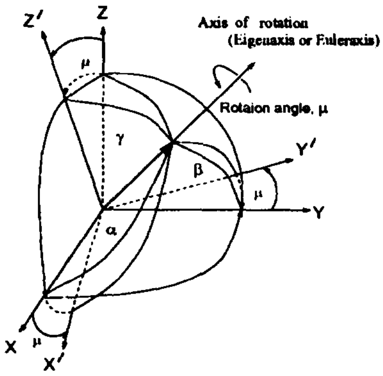

Fig. 1: (X, Y, Z) and (X', Y', Z') coordinates

The kinematic differential equations of the quaternion can be written as

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (4)$$

The control error quaternion expresses the coordinate transformation from the current attitude to the commanded attitude and can be written as [1]

$$\begin{bmatrix} q_{e0} \\ q_{e1} \\ q_{e2} \\ q_{e3} \end{bmatrix} = \begin{bmatrix} q_{c0} & q_{c1} & q_{c2} & q_{c3} \\ -q_{c1} & q_{c0} & q_{c3} & -q_{c2} \\ -q_{c2} & -q_{c3} & q_{c0} & q_{c1} \\ -q_{c3} & q_{c2} & -q_{c1} & q_{c0} \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (5)$$

where q_i, q_{ci} and $q_{ei} (i = 0, 1, 2, 3)$ are quaternions that express current attitude, commanded attitude and attitude error, respectively. For small attitude changes from the inertial reference frame ($q_1 = q_2 = q_3 \simeq 0, q_0 \simeq 1$), we have an approximation of Eq.(4) such that

$$2\dot{q}_1 = p, \quad 2\dot{q}_2 = q, \quad 2\dot{q}_3 = r \quad (6)$$

Hence, the angular body rates can be used for the rate feedback of quaternions.

The feedback controller proposed in this paper consists of linear error-quaternion feedback, linear body-rate feedback, and a nonlinear body-rate feedback term that counteracts the gyroscopic coupling torque[3]:

$$\mathbf{T_a} = -\Omega J\omega - D\omega - K\mathbf{q_e} \quad (7)$$

where D and K are 3×3 gain matrix and $\mathbf{q_e} = [q_{e1}, q_{e2}, q_{e3}]^T$. For simplicity, it is assumed that $\mathbf{q_e} = \mathbf{q}$ or the commanded quaternion is $[1, 0, 0, 0]^T$.

2.3 Eigenaxis Rotation

The Euler's rotation theorem says that a rotation about eigenaxis gives the shortest path angle between two orientations. This can be achieved by using a quaternion feedback of the form $kJ\mathbf{q}$, where k is scalar and J the inertia matrix. Since the vector $\mathbf{q}$ coincides with the spacecraft eigenaxis, the control torque $kJ\mathbf{q}$ produces an eigenaxis rotation. Wie *et al* have shown that an eigenaxis rotation for a rest-to-rest reorientation maneuver can be achieved with gain matrices $D = dJ$ and $K = kJ$ (d and k are scalar)[3].

3 Moment Dump via Magnetic Torquer

To attain high pointing accuracy, a control system employing three or more reaction wheels or control moment gyros is favored. However, the secular component of external disturbance torques can lead to the saturation of the momentum capacity of the reaction wheels, so the momentum dump systems using torques such as magnetic control torques are needed to dump excess reaction wheel angular momentum.

3.1 Magnetic Torquer

A magnetic moment $\mathbf{M}$ generated via electromagnets on board the spacecraft causes a magnetic control torque $\mathbf{T_m}$ given in body frame by

$$\mathbf{T_m} = \mathbf{M} \times \mathbf{B} \quad (8)$$

where $\mathbf{B}$ is the Earth's magnetic field vector.

For a planar, wire loop of N turns enclosing an area A through which a current I is flowing, the magnetic moment $\mathbf{M}$ is given by

$$\mathbf{M} = NIA\mathbf{n} \quad (9)$$

where $\mathbf{n}$ is a unit vector normal to the plane of the loop[5].

A magnetic torquer cannot produce a torque along the direction of the magnetic field vector. To generate control torques in any arbitrary directions, three magnetic coils orthogonal to each other are required. The Earth's magnetic field vector varies with the position of a satellite, so there is needed a magnetometer which senses the magnetic field.

3.2 Momentum Dump Algorithm

The attitude control system has a time constant on the order of seconds to minutes and the momentum dump control is on a scale of minutes or possibly hours. Justified by this difference in bandwidth, the torque by momentum dump control system can be considered as perturbations to the attitude control system and vice versa (See Fig.2)[6].

In the case of zero-bias reaction wheel, the angular momentum of the reaction wheel $\mathbf{h}$ is written as

$$\dot{\mathbf{h}} = -\mathbf{T_a} \quad (10)$$

A momentum dump control law seeks a torque $\mathbf{T_m}$ that makes the angular momentum $\mathbf{h}$ zero. This momentum dump strategy proposed in the 1960's by White *et al* can be written as[8]

$$\mathbf{M} = -k_m \mathbf{B} \times \mathbf{h} \quad (11)$$

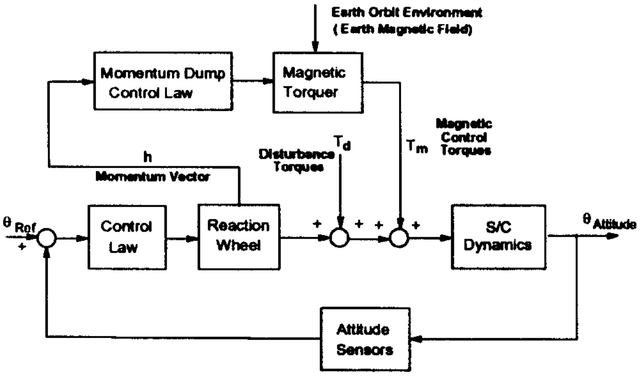

Fig. 2: Attitude and momentum dump control systems

It is easy to show that **h** converges to zero[7][8].

3.3 Earth Magnetic Model

Although a magnetometer is used to measure the earth magnetic field vector in practice, calculation of magnetic field vector is needed for some purposes such as simulation.

The Earth's magnetic field vector can be expressed as the gradient of a scalar potential V

$$\mathbf{B} = -\nabla V \tag{12}$$

The scalar potential V can be expressed as a spherical harmonic function:

$$V(r,\theta,\phi) = a\sum_{n=1}^{k}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}(g_n^m \cos m\phi + h_n^m \sin m\phi)P_n^m(\theta) \tag{13}$$

where a is the equatorial radius of the Earth and r,θ and ϕ are the geocentric distance, coelevation, and east longitude from Greenwich. g_n^m and h_n^m are called *Gaussian coefficients* and $P_n^m(\theta)$ are the associated Legendre functions[5]. The Gaussian coefficients of the International Geomagnetic Reference Field (IGRF(1990)) is shown in Table 1.

4 Simulation

An asymmetric rigid spacecraft with the following inertia matrix is considered.

$$J = \begin{bmatrix} 1000 & 100 & -200 \\ 100 & 2000 & 300 \\ -200 & 300 & 3000 \end{bmatrix} \mathrm{Kg}\cdot\mathrm{m}^2$$

The initial attitude Euler angles at $t = 0$ are given as

$$\theta_0 = -30^\circ, \quad \psi_0 = 60^\circ, \quad \phi_0 = 80^\circ$$

The feedback gains in this simulation are chosen as

$$K = 0.05J, \quad D = 0.3J, \quad k_m = 3\times 10^7$$

Table 1: IGRF Gaussian Coefficients for Epoch 1990.

n	m	g(nT)	h(nT)	$\dot{g}$(nT/yr)	$\dot{h}$(nT/yr)
1	0	-29775.0	–	18.0	–
1	1	-1851.0	5411.0	10.6	-16.1
2	0	-2136.0	–	-12.9	–
2	1	3058.0	-2278.0	2.4	-15.8
2	2	1693.0	-380.0	0.0	-13.8
3	0	1315.0	–	3.3	–
3	1	-2240.0	-287.0	-6.7	4.4
3	2	1246.0	293.0	0.1	1.6
3	3	807.0	-348.0	-5.9	-10.6
4	0	939.0	–	0.5	–
4	1	782.0	248.0	0.6	2.6
4	2	324.0	-240.0	-7.0	1.8
4	3	-423.0	87.0	0.5	3.1
4	4	142.0	-299.0	-5.5	-1.4

Fig.3 thru 6 show the simulation results. The time histories of Euler angles and quaternions (Figs.3 and 4) show that the proposed controller provides large angle maneuvers. Especially, Fig.5 indicates that the quaternion vector $\mathbf{q} = [q_1, q_2, q_3]^T$ coincides with the eigenaxis and the ratios of quaternions remain nearly constant (without momentum dump control, in fact, the ratios are perfectly constants.). This means that the control law provides the eigenaxis rotation.

Fig.6 shows the effect of the momentum dump control. The angular momentums of reaction wheels remain some nonzero values without momentum dump control but the momentums approach to zero where the momenum dump control is applied.

5 Conclusions

We considered control algorithms for large angle maneuvers about eigenaxis and angular momentum dump. Eigenaxis rotations are simply achieved by error quaternion feedback. This rotation is the minimum path-angle rotation and can be applied in many cases where large angle maneuvers are needed.

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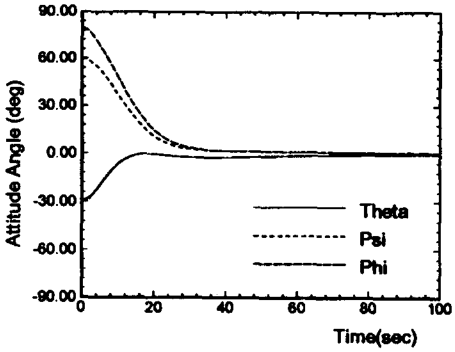


Fig. 3: Time histories of Euler angles

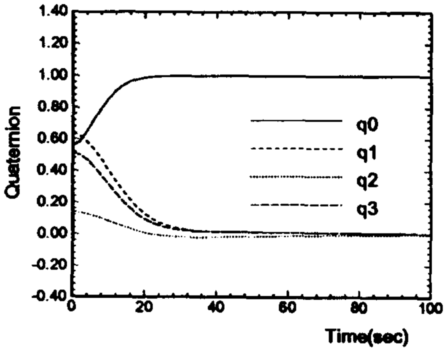


Fig. 4: Time histories of quaternions

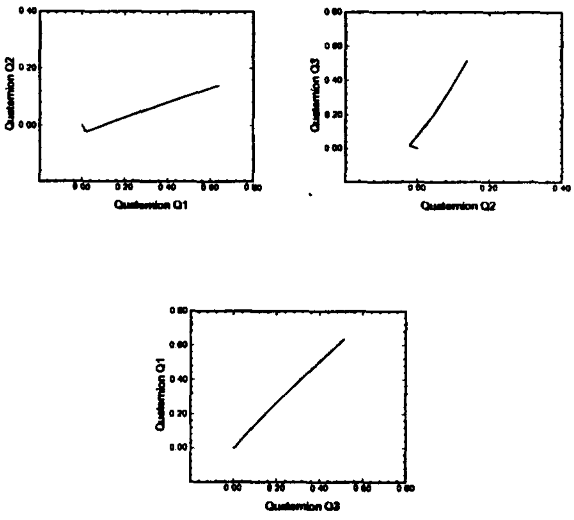
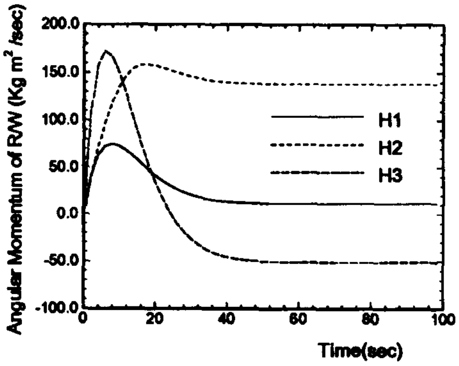
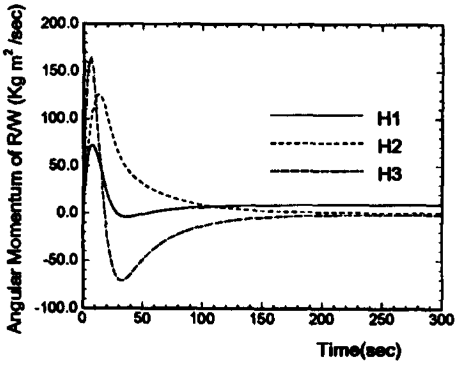


Fig. 5: q_i vs q_j plots



(a) W/O momentum dump



(b) W/ momentum dump

Fig. 6: Angular momentum of R/W