'95 KACC (1995. 10. 23 ~ 25)

Robust Control by Universal Learning Network

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Abstract: Characteristics of control system design using Universal Learning Network(U.L.N.) are that a system to be controlled and a controller are both constructed by U.L.N. and that the controller is best tuned through learning. U.L.N. has the same generalization ability as N.N.. So the controller constructed by U.L.N. is able to control the system in a favorable way under the condition different from the condition of the control system in learning stage. But stability can not be realized sufficiently.

In this paper, we propose a robust control method using U.L.N. and second order derivatives of U.L.N.. The proposed method can realize better performance and robustness than the commonly used Neural Network. Robust control considered here is defined as follows. Even though initial values of node outputs change from those in learning,the control system is able to reduce its influence to other node outputs and can control the system in a preferable way as in the case of no variation. In order to realize such robust control, a new term concerning the variation is added to a usual criterion function. And parameter variables are adjusted so as to minimize the above mentioned criterion function using the second order derivatives of criterion function with respect to the parameters. Finally it is shown that the controller constructed by the proposed method works in an effective way through a simulation study of a nonlinear crane system.

Keywords Robust control , Neural networks, Second order derivative

1. Introduction

Universal Learning Network(U.L.N.) and a computing method for its higher order derivatives have been proposed,[1],[2] which can be used as a fundametal tool in modelling and control of large-scale complicated systems such as economic, social and living systems as well as industrial plants.

Each nodes in U.L.N. is allowed to have any nonlinear functions. In case of designing a control system using U.L.N., the system to be controlled and the controller are both constructed by U.L.N., and the controller is best tuned through learning to minimize a criterion function which is assumed to be function of the target value of system node output, actual value of system node output and output of the controller. U.L.N. has the same generalization ability as Neural Network(N.N.). So the controller constructed by U.L.N. is able to control the system in a favorable way under the conditions different from those of the control system in learning stage. But stability can not be realized sufficiently under the conditions much different from those in learning stage.

Robust control considered here belongs to the control which is able to control the system stably under the conditions much different from those of the control system in learning stage. The difference in conditions considered here is the difference between initial values of node outputs in control stage and those of node outputs in learning stage. A robust control design method is proposed where the parameters of the control system are tuned to minimize a criterion function, using second order derivatives of the criterion function with respect to parameters, which consists of two terms,one is a usual criterion function and the other is a new term which evaluates the influence of the above differences for all node outouts of the system. Finally it is shown that the controller constructed by the proposed method works in an effective way through a simulaton study of a nonlinear crane system.

2. Structure of Universal Learning Network

Basic structure of U.L.N. which consists of nonlinearly operated nodes and branches that may have arbitrary time delays is shown in **Fig.1**.

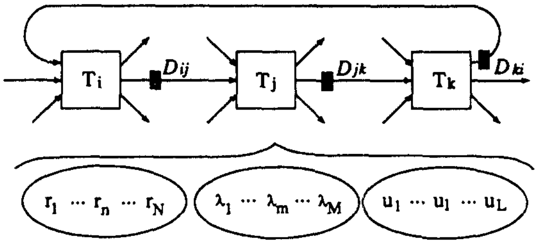

Fig.1 Structure of Universal Learning Network

Basic equation of U.L.N. is represented by Eq.(1):

$$h(T_j,t) = O_j(\{h'(T_j,t)\},\{r_n(t)\},\{u_l(t)\},\{\lambda_m(t)\}), \quad (1)$$

where

$h(T_j,t)$	:	Output of T_j node at time t,
$h'(T_j,t)$	:	Input of T_j node at time t,
$r_n(t)$	:	External input variable at time t,
$u_l(t)$	:	Control variable at time t,
$\lambda_m(t)$	:	Parameter variable at time t,
O_j	:	Nonlinear function of T_j node,

If T_i node is one of the nodes connected with input side of T_j node,then,$h'(T_j,t)$ via a branch with time delay D_{ij} is represented by Eq.(2):

$$h'(T_j,t) = h(T_i, t - D_{ij}) \quad (2)$$

Let a criterion function be written in Eq.(3):

$$E = E(\{h(T_k,s)\},\{u_l(s)\},\{\lambda_m(s)\}) \quad (3)$$

$k \in K_o$	K_o	:	Set of nodes related with evaluation,
$l \in L_o$	L_o	:	Set of control variables related with evaluation,
$s \in S_o$	S_o	:	Set of sampling times related with evaluation.

In the following chapters,a computing method for derivatives of criterion function E with respect to parameter variable $\lambda_m(t_0)$ is presented,which is essential to design a robust control system using U.L.N..[2]

3. Computation of First Order Derivative

First order derivative of E with respect to parameter $\lambda_1(t_0)$ can be written in the form of Eq.(4),assuming t_0 to be designated sampling time,

$$\frac{dE}{d\lambda_1(t_0)} = \sum_{k\in K_o}\sum_{s\in S_o}\Big(\frac{\partial E}{\partial h(T_k,s)}\frac{dh(T_k,s)}{d\lambda_1(t_0)}\Big) + \frac{\partial E}{\partial\lambda_1(t_0)} \quad (4)$$

As $\frac{\partial E}{\partial h(T_k,s)}$ and $\frac{\partial E}{\partial \lambda_1(t_0)}$ can be calculated easily from Eq.(3),it is matter of importance to calculate $\frac{dh(T_k,s)}{d\lambda_1(t_0)}$.

$\frac{dh(T_k,t)}{d\lambda_1(t_0)}$ can be transformed into Eq.(5).

$$\frac{dh(T_k,t)}{d\lambda_1(t_0)} = \sum_{j\in J}\Big(\frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}\frac{dh(T_j,t-D_{jk})}{d\lambda_1(t_0)}\Big) + \frac{\partial h(T_k,t)}{\partial\lambda_1(t_0)} \quad (5)$$

where

J : Set of nodes connected with input side of T_k node.

Putting $P_1(T_k,t,\lambda_1(t_0)) = \frac{dh(T_k,t)}{\lambda_1(t_0)}$, iterative equation of P_1 by forward propagation can be obtained from Eq(5).

$$P_1(T_k,t,\lambda_1(t_0)) = \sum_{j\in J}\Big[\frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}P_1(T_j,t-D_{jk},\lambda_1(t_0))\Big] + \frac{\partial h(T_k,t)}{\partial\lambda_1(t_0)} \quad (6)$$

$k = 1,2,.....,R,$
$t = 1,2,.....,T,$

where

R : Number of nodes of the system,
T : Number of sampling times.

Since $h(T_j,t_0-1)$ does not depend on $\lambda_1(t_0)$, initial value of Eq.(6),

$$P_1(T_j,t_0-1,\lambda_1(t_0)) = 0, \quad j = 1,2,.....,R. \quad (7)$$

4. Computation of Second Order Derivative

Second order derivative of E with respect to parameter variables $\lambda_1(t_0)$, $\lambda_2(t_0)$ can be obtained by differentiating Eq.(4) with respect to $\lambda_2(t_0)$,

$$\frac{d^2E}{d\lambda_1(t_0)d\lambda_2(t_0)} = \sum_{k\in K_o}\sum_{s\in S_o}\Big[\frac{d\Big(\frac{\partial E}{\partial h(Tk,s)}\Big)}{d\lambda_2(t_0)}\frac{dh(T_k,s)}{d\lambda_1(t_0)} + \frac{\partial E}{\partial h(T_k,s)}\frac{d^2h(T_k,s)}{d\lambda_1(t_0)d\lambda_2(t_0)}\Big] + \frac{d(\frac{\partial E}{\partial\lambda_1(t_0)})}{d\lambda_2(t_0)} \quad (8)$$

$\frac{d^2h(T_k,s)}{d\lambda_1(t_0)d\lambda_2(t_0)}$ in Eq.(8) can be transformed into Eq.(9) by differentiating Eq.(5) with respect to $\lambda_2(t_0)$,

$$\frac{d^2h(T_k,t)}{d\lambda_1(t_0)d\lambda_2(t_0)} = \sum_{j\in J}\Big[\frac{d\big(\frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}\big)}{d\lambda_2(t_0)}\frac{dh(T_j,t-D_{jk})}{d\lambda_1(t_0)} + \frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}\frac{d^2h(T_j,t-D_{jk})}{d\lambda_1(t_0)d\lambda_2(t_0)}\Big] + \frac{d\big(\frac{\partial h(T_k,t)}{\partial\lambda_1(t_0)}\big)}{d\lambda_2(t_0)} \quad (9)$$

Putting $P_1(T_k,t,\lambda_1(t_0)) = \frac{dh(T_k,t)}{d\lambda_1(t_0)}$,and $P_2(T_k,t,\ \lambda_1(t_0),\ \lambda_2(t_0)) = \frac{d^2h(T_k,t)}{d\lambda_1(t_0)d\lambda_2(t_0)}$, as in the case of first order derivatives, iterative equation of P_2 by forward propagation can be obtained from Eq.(9),

$$P_2(T_k,t,\lambda_1(t_0),\lambda_2(t_0)) = \sum_{j\in J}\Big[\frac{d\big(\frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}\big)}{d\lambda_2(t_0)}P_1(T_j,t-D_{jk},\lambda_1(t_0)) + \frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}P_2(T_j,t-D_{jk},\lambda_1(t_0),\lambda_2(t_0))\Big] + \frac{d\big(\frac{\partial h(T_k,t)}{\partial\lambda_1(t_0)}\big)}{d\lambda_2(t_0)} \quad (10)$$

$k = 1,2,.....,R,$
$t = 1,2,.....,T.$

$$P_2(T_j,t_0-1,\lambda_1(t_0),\lambda_2(t_0)) = 0, \quad (11)$$

$j = 1,2,.....,R.$

$\frac{d\big(\frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}\big)}{d\lambda_2(t_0)}$, $\frac{d\big(\frac{\partial h(T_k,t)}{\partial\lambda_1(t_0)}\big)}{d\lambda_2(t_0)}$ in Eq.(10) can be calculated by the computation of first order derivatives putting $E = \frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}$, $E = \frac{\partial h(T_k,t)}{\partial\lambda_1(t_0)}$ respectively.

Substituting $\frac{dh(T_k,t)}{d\lambda_1(t_0)}$, $\frac{d^2h(T_k,t)}{\lambda_1(t_0)\lambda_2(t_0)}$ obtained from Eq.(6),(7) and Eq.(10),(11) respectively into Eq.(8), $\frac{d^2E}{d\lambda_1(t_0)\lambda_2(t_0)}$ can be calculated.

5. Robust Control Method

5.1 criterion for suppressing changes of nodes outputs

E is a usual criterion function, E_H is a new term which takes charge of suppressing changes of node outputs of the system caused by the changes of particular node outputs at time t_1.Then a new criterion function L is defined as follows:

$$L = E + E_H, \quad (12)$$

$$E_H = C_H\sum_{s=S_1}^{S_N}\sum_{r\in R_s}\Big(\sum_{i\in R}\frac{dh(T_r,s)}{dh(T_i,t_1)}\Delta h(T_i,t_1)\Big)^2 \quad (13)$$

R_s	: Set of nodes related with suppression,
$S_i(i=1,\cdots,N)$	: Set of sampling times related with suppression,
$C_H > 0$	: coefficient.

$\frac{dh(T_r,S_i)}{dh(T_i,t_1)}\Delta h(T_i,t_1)$ means the change of T_r node output in the case of a change of T_i node output at t_1, namely $\Delta h(T_i,t_1)$. Eq.(13) is the sum of those squared.

5.2.Learning Algorithm

The aim of the optimization learning in U.L.N. used here is to search for the parameters which make the above criterion function L minimal. (From now on the parameter variables are considered to be time invariant.)

The parameter variables in order to minimize Eq.(12) should be calculated by a gradient method.

$$\lambda_m \leftarrow \lambda_m - \gamma \frac{dL}{d\lambda_m} \qquad (14)$$

$$where \quad \frac{dL}{d\lambda_m} = \frac{dE}{d\lambda_m} + \frac{dE_H}{d\lambda_m},$$

$\gamma > 0$: coefficient.

Now,computation of $\frac{dE}{d\lambda_m}$ and $\frac{dE_H}{d\lambda_m}$ can be carried out by making use of the first and the second order derivatives in chapter.3,.4.

< Computation of $\frac{dE}{d\lambda_m}$ >

Putting $\lambda_1(t_0) = \lambda_m$, $\frac{dE}{d\lambda_m}$ is able to be computed using Eq.(4),(6)

< Computation of $\frac{dE_H}{d\lambda_m}$ >

First order derivative of E_H with respect to λ_m can be obtained by differentiating Eq.(13) with respect to λ_m,

$$\frac{dE_H}{d\lambda_m} = 2\, C_H \sum_{s=S_1}^{S_N} \sum_{r\in R_s} \Big[\Big(\sum_{i\in R} \frac{dh(T_r,s)}{dh(T_i,t_1)} \Delta h(T_i,t_1)\Big) \times \Big(\sum_{i\in R} \frac{d^2 h(T_r,s)}{dh(T_i,t_1)d\lambda_m} \Delta h(T_i,t_1)\Big)\Big] \qquad (15)$$

Now, $\frac{dh(T_r,s)}{dh(T_i,t_1)}$, $\frac{d^2h(T_r,s)}{dh(T_i,t_1)d\lambda_m}$ are needed in order to compute the $\frac{dE_H}{d\lambda_m}$.

[1] Computation of $\frac{dh(T_r,s)}{dh(T_i,t_1)}$

Putting E=$h(T_r,s)$, $\lambda_1(t_0)$=$h(T_i,t_1)$ and making use of the first order derivative in chapter.3, Eq.(16),(17) can be obtained,

$$\frac{dh(T_r,s)}{dh(T_i,t_1)} = P_1(T_r,s,h(T_i,t_1)) \qquad (16)$$

$$P_1(T_k,t,h(T_i,t_1)) = \sum_{j\in J}\Big[\frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})} P_1(T_j,t-D_{jk},h(T_i,t_1))\Big] + \frac{\partial h(T_k,t)}{\partial h(T_i,t_1)} \qquad (17)$$

[2] Computation of $\frac{d^2h(T_r,s)}{dh(T_i,t_1)d\lambda_m}$

Putting E=$h(T_r,s)$, $\lambda_1(t_0)$=$h(T_i,t_1)$, $\lambda_2(t_0) = \lambda_m$ and making use of the second order derivative in chapter.4, Eq.(18),(19) can be obtained,

$$\frac{d^2h(T_r,s)}{dh(T_i,t_1)d\lambda_m} = P_2(T_r,s,h(T_i,t_1),\lambda_m) \qquad (18)$$

$$P_2(T_k,t,h(T_i,t_1),\lambda_m) = \sum_{j\in J}\Big[\frac{d\big(\frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}\big)}{d\lambda_m} P_1(T_j,t-D_{jk},h(T_i,t_1)) + \frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})} P_2(T_j,t-D_{jk},h(T_i,t_1),\lambda_m)\Big] + \frac{d\big(\frac{\partial h(T_k,t)}{\partial h(T_i,t_1)}\big)}{d\lambda_m} \qquad (19)$$

The coefficient of P_1 in Eq.(19), $\frac{d\left(\frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}\right)}{d\lambda_m}$, can be calculated by computing the first order derivative of $E = \frac{\partial h(T_k,t)}{\partial h(T_j,t-D_{jk})}$ using Eq.(4),(6).

6. Numerical Example

6.1 Controlled System

The controlled system is a nonlinear crane system. A position of the crane stand, an angle between the rope and vertical line and a position of the load are represented as x, θ, ℓ respectively. Then the nonlinear crane system is described as follows:

$$\begin{aligned} \frac{d^2x}{dt^2} &= -\frac{mg}{M}\theta - \frac{D+G}{M}\frac{dx}{dt} + \frac{G}{M}u_d, \\ \frac{d^2\theta}{dt^2} &= -\frac{M+m}{\ell M}g\theta - \frac{D+G}{\ell M}\frac{dx}{dt} + \frac{G}{\ell M}u_d \\ \frac{d^2\ell}{dt^2} &= -\frac{C+G_m}{m}\frac{d\ell}{dt} + \frac{G_m}{m}u_m, \end{aligned} \qquad (20)$$

where, u_d, u_m are input voltage to a motor for moving the crane stand and to a motor for rolling up the load respectively,and C, G, G_m, D, M, m are appropriate system parameters.

Putting as follows,

$$h(T_1,t) = x(t),\quad h(T_2,t) = \dot{x}(t),\quad h(T_3,t) = \theta(t),$$
$$h(T_4,t) = \dot{\theta}(t),\quad h(T_5,t) = \ell(t),\quad h(T_6,t) = \dot{\ell}(t),$$

Eq.(20) can be transformed into discrete type equations as follows:

$$\begin{aligned} h(T_1,t) &= a_{11}h(T_1,\hat{t}) + a_{21}h(T_2,\hat{t}), \\ h(T_2,t) &= a_{22}h(T_2,\hat{t}) + a_{32}h(T_3,\hat{t}) + b_1u_d(\hat{t}), \\ h(T_3,t) &= a_{33}h(T_3,\hat{t}) + a_{43}h(T_4,\hat{t}), \\ h(T_4,t) &= a_{24}\frac{h(T_2,\hat{t})}{h(T_5,\hat{t})} + a_{34}\frac{h(T_3,\hat{t})}{h}(T_5,\hat{t}) \\ &\quad + a_{44}h(T_4,\hat{t})\frac{b_1}{h(T_5,\hat{t})}u_d(\hat{t}), \\ h(T_5,t) &= a_{55}h(T_5,\hat{t}) + a_{65}h(T_6,\hat{t}), \\ h(T_6,t) &= a_{66}h(T_6,\hat{t}) + b_2u_m(\hat{t}). \\ & \quad Where\ \hat{t} = t-1. \end{aligned} \qquad (21)$$

A recurrent type control model of the nonlinear crane system using U.L.N. is shown in **Fig.2**. Each control input u_d, u_m is constructed by two control nodes respectively ,one is the node with linear function,the other is the node with tanh function. (All branches have one sampling time delay.)

6.2 Criterion Function

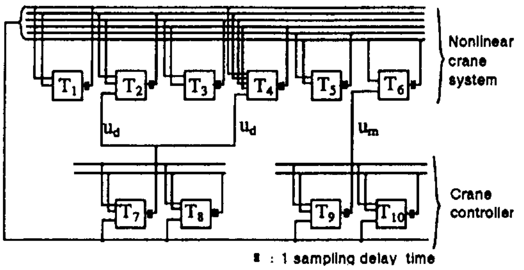

Fig.2 Recurrent type control model of a nonlinear crane system using Universal Learning Network

M=40[kg], D=300[kg/sec], G=700[N/V], m=2[kg], g=9.8[m/sec^2], G_m=0.98[N/V], C=0.42[kg/sec] are used, and reference of moving the crane stand(x_{ref}) is 1[m], reference of rolling up the load(ℓ_{ref}) is 0.5[m].

When the parameter variables are tuned through learning, initial values of node outputs of the system are set up as follows:

$$h(T_5,0) = 1.0,\ h(T_i,0) \ = \ 0.0 \ \ (i = 1,2,3,4,6). \qquad (22)$$

In numerical example, a change of the initial position of the load ,namely $\Delta h(T_5, 0)$ is assumed.

The criterion E to achieve the desired dynamics of the system and E_H to achieve the suppressing the perturbation of the system caused by the change of the initial position of the load are defined respectively as follows,

$$\begin{aligned} E \ = \ & \frac{1}{2}\Big[\sum_{s \in S_o}\{Q_{11}(x_{ref} - h(T_1,s))^2\} + Q_{12}(h(T_2,t_f))^2 \\ & + \sum_{s \in S_o}\{Q_{13}(h(T_3,s))^2 + Q_{14}(h(T_4,s))^2\} \\ & + \sum_{s \in S_o}\{Q_{15}(\ell_{ref} - h(T_5,s))^2\} + Q_{16}(h(T_6,t_f))^2 \\ & + \sum_{s \in S_o}\{R_1(h(T_7,s))^2 + R_2(h(T_9,s))^2\}\Big] \qquad (23) \end{aligned}$$

$$E_H \ = \ C_H \sum_{r \in R}\Big(\frac{dh(T_r,s)}{dh(T_5,0)}\ \Delta h(T_5,0)\Big)^2 \qquad (24)$$

where, S_o : Set of all sampling times, t_f : final time, $Q_{11} \sim Q_{16} = 1.0$, $R_1 \sim R_2 = 0.001$, $R : 1,2,3,4,5,6.$ $s =$ 1.0 [sec] in Eq.(24).

Using these values , two cases of learning have been simulated,one is the case of using the criterion function E, the other is the case of using the criterion function L.

6.3 Simulation Results

Using the parameter variables obtained through learning for minimization of the criterion function E or $L(C_H = 100, \Delta h(T_5,0) = 1)$, simulation has been carried out for $\ell(0) = 2,3,4,\cdots$. In case of using the criterion function E ,when $\ell(0)$ becomes 10,namely the condition of operating the crane is very different from that in learning stage,the system was unstable.(See **Fig.3**.) Conversely, in case of $L(C_H = 100, \Delta h(T_5,0) = 1)$,the system operates stably.(See **Fig.4**.) ($\ell(0)$ means the initial position of the load in control stage)

Thus in the event of a large change of node outputs,the effect of E_H to stability of the system appears remarkably.

7. Conclusion

A robust control method using forward propagation U.L.N. is proposed,and through simulation study,it has been shown that the proposed method is very useful.

In numerical example, the controller obtained makes the system operate stably in the event of $\ell(0) = 20$. ($\ell(0) = 20$ is twenty times as large as $h(T_5,0) = 1$ in Eq.(22)). This means that the proposed method is the method for practical use.

In this paper we proposed the robust control method for a change of nodes outputs. The robust control method for perturbation of system parameter variables and for a change of external inputs can be derived in the same way as in the case of a change of node outputs.

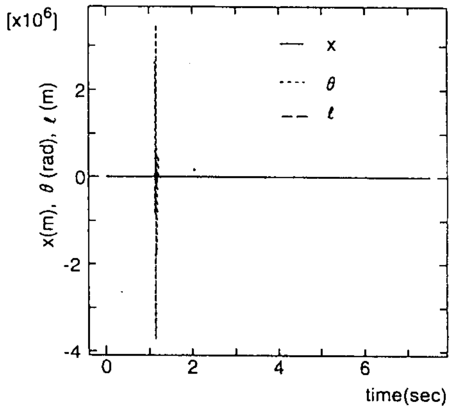

Fig.3 Results of control with the criterion E in case of $\ell(0) = 10$

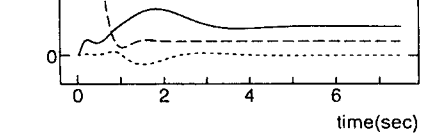

Fig.4 Results of control with the criterion L in case of $\ell(0) = 10$

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