'95 KACC (1995. 10. 23 ~ 25)

ON-LINE DYNAMIC SENSING OF SHIP'S ATTITUDE BY USE OF A SERVO-TYPE ACCELEROMETER AND INCLINOMETERS

Shogo Tanaka* and °Seiji Nishifuji*
Department of Electrical and Electronic Eng., Yamaguchi University, 2557 Tokiwadai, Ube 755 JAPAN
Tel.:81-836-35-9945; Fax: 81-836-35-9449; E-mail:nishifuj@sens.eee.yamaguchi-u.ac.jp

Abstract For an accurate on-line measurement of the ship's attitude the paper develops an intelligent sensing system which uses one servo-type accelerometer and two servo-type inclinometers appropriately located on the ship. By considering the dynamics of the servo-controlled rigid pendulums of the inclinometers, linear equations for the rolling and pitching of the ship are derived separately from each other. Moreover, one accelerometer is used for extracting the heaving signal. Through the introduction of linear dynamic models and the linear observation equations for the heaving, rolling and pitching, the on-line measurement of the three signals can be reduced to the state estimation of the linear dynamic systems. A bank of Kalman filters is adaptively used to achieve the on-line accurate state estimation and to overcome changes in parameters in the linear dynamic models.

Keywords Ship's Attitude, Accelerometer, Inclinometer, Linear Dynamics Model, Kalman Filter, Minimax Criterion

1. INTRODUCTION

In various marine constructions, seabed patterns are frequently searched with sonars. The data obtained with sonars are usually considerably affected by the heaving, rolling and pitching of the ship caused by surface waves. The exact measurement of the three signals is thus important in exact searching for the seabed patterns with sonars. On-line measurement of the signals is also required from the viewpoint of suppressing swings of the ship by waves.

From this point of view, the authors previously developed an automatic measurement system which processed outputs of several servo-type accelerometers appropriately located on the ship and estimated the ship's attitude[1]. This measurement system modeled the heaving and rolling signals as outputs of linear dynamic systems and derived observation equations for the two signals by adequate linear combinations of the sensors' outputs. The on-line measurement of the heaving and rolling was thus reduced to the state estimation of the linear dynamic systems. Moreover, for unknown periodic-lengths of dominant periodic-waves to each signal, adequate candidates were introduced and a bank of Kalman filters was used to execute the on-line state estimation. The system however ignored the pitching signal, because it was essentially developed for large ships. Moreover, dependently on the periodic-lengths of the dominant waves, the measurement accuracy rather deteriorated, because periodic-lengths of the candidates were not fully considered.

To overcome these difficulties, this paper proposes a new on-line sensing system which measures not only the heaving and rolling but also the pitching accurately. Furthermore, from the viewpoint of the minimax criterion, two methods for setting the periodic-length of the candidates for the Kalman filters are examined to improve the accuracy of the measuremen. Simulation and experimental results show that the proposed sensing system measures the ship's attitude accurately and minimizes the deterioration of the measurement accuracy for changes of the periodic lengths of the dominant waves.

2. LOCATION AND OUTPUTS OF SENSORS

2.1 Location of Sensors

Two inclinometers S_1, S_2 and one accelerometer S_3 are located at the point A of vertical distance L from O, the intersection of the rolling and pitching axes of the ship (Fig. 1). The inclinometers S_1, S_2 are directed in such a way that the rolling and pitching signals can be measured separately. The accelerometer S_3 is set upward and perpendicularly to the deck of the ship to get the heaving signal.

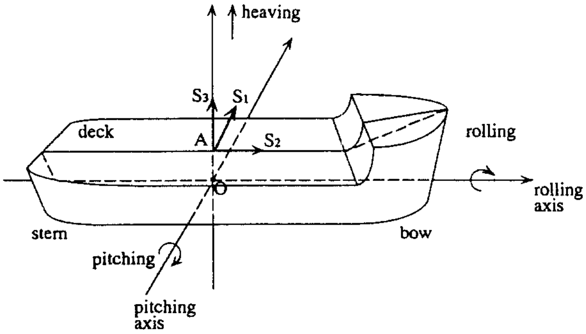

Fig. 1. Location of sensors (Entire view).

2.2 Outputs of Inclinometers

The output of an inclinometer is derived from the torque by the servomechanism applied to the rigid pendulum of the inclinometer S_1. By considering the dynamics of the rigid pendulum, the output of the inclinometer set on the ship, in Fig.1, is obtained as

$$z_1(t) = \theta(t) - \frac{L}{g}\ddot{\theta}(t) + v_1(t). \tag{1}$$

Here $z_1(t)$, $\theta(t)$, L, g and $v_1(t)$ denote the output of the inclinometer, the rolling angle, the distance from the intersection of the rolling and pitching axes to the supporting point of the rigid pendulum, the gravitational acceleration and the noise on the inclinometer, respectively. It is seen that the rolling acceleration causes the output to indicate the wrong inclination. Moreover, the output $z_1(t)$ is seen to be a linear equation regarding the rolling angle $\theta(t)$ and its acceleration $\ddot{\theta}(t)$. Similarly the output $z_2(t)$ of another inclinometer S_2 becomes

$$z_2(t) = p(t) - \frac{L}{g}\ddot{p}(t) + v_2(t) \tag{2}$$

where $p(t)$ and $v_2(t)$ represent the pitching angle and the noise as well as $v_1(t)$ in (1).

On the other hand, the output of the accelerometer S_3 reflects the effect of not only the heaving acceleration but also the rolling and pitching angles and becomes

$$z_3(t) = (g + \alpha(t))\cos\theta(t)\cos p(t) + v_3(t) \tag{3}$$

where $z_3(t)$, $\alpha(t)$ and $v_3(t)$ denote the output of the accelerometer, the heaving acceleration and the noise on the accelerometer, respectively. Since the rolling and pitching angles can be measured on-line as mentioned below, substitution of their estimates $\hat{\theta}(t)$, $\hat{p}(t)$ into (3) yields a new linear observation equation for the heaving

$$z_3(t) = (g + \alpha(t))\cos\hat{\theta}(t)\cos\hat{p}(t) + v_3(t). \tag{4}$$

3. ON-LINE SENSING OF SHIP'S ATTITUDE

3.1 Dynamics of the Heaving, Rolling and Pitching

The heaving, rolling and pitching of the ship are, respectively, known to be locally well modeled by composite waves of two sinusoidal waves[1][2]. One of the sinusoidal waves has a short periodic length of about 3 sec and the other a long one in the range of 6 to 10 sec. It is usual for the rolling and pitching to have biases, whereas not for the heaving. The heaving displacement can be locally modeled by

$$x(t) = a_1\sin(\omega_1 t + \varphi_1) + a_2\sin(\omega_2 t + \varphi_2) \tag{5}$$

where $\{\omega_i\}_{i=1}^2$ are unknown angular frequencies of the two sinusoidal waves. Of course $\{a_i\}_{i=1}^2$ and $\{\varphi_i\}_{i=1}^2$ are unknown. The $x(t)$ is seen to satisfy the differential equation

$$\frac{d^4x}{dt^4} + \left(\omega_1^2 + \omega_2^2\right)\frac{d^2x}{dt^2} + \omega_1^2\omega_2^2 x = 0. \tag{6}$$

Introducing the phase variables $x_n = dx^{n-1}/dt^{n-1}$ $(n = 1,\cdots,4)$, we obtain the linear dynamic equation for the heaving displacement $x(t)$

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} \tag{7}$$

where $\boldsymbol{x} \triangleq (x_1, x_2, x_3, x_4)^T$ and

$$A \triangleq \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2\omega_2^2 & 0 & -(\omega_1^2+\omega_2^2) & 0 \end{bmatrix}. \tag{8}$$

On the other hand, the rolling and pitching angles having biases, we have another model for the signals like

$$x(t) = a_1\sin(\omega_1 t + \varphi_1) + a_2\sin(\omega_2 t + \varphi_2) + b \tag{9}$$

satisfying the differential equation

$$\frac{d^5x}{dt^5} + \left(\omega_1^2 + \omega_2^2\right)\frac{d^3x}{dt^3} + \omega_1^2\omega_2^2\frac{dx}{dt} = 0. \tag{10}$$

Naturally $\{a_i\}_{i=1}^2$, $\{\varphi_i\}_{i=1}^2$ and b are different between the two signals. Introducing the phase variables similarly as before, we get a similar dynamic model as (7).

Only with the models, however, an accurate estimation of the heaving, rolling and pitching cannot be necessarily expected, because the three signals usually include non-dominant sinusoidal waves other than the two dominant ones. This fact suggests that our models must be slightly modified. We thus adopt the type of dynamics

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bw \tag{11}$$

where $B=(0,0,1,0)^T$ for the heaving, $B=(0,0,1,0,0)^T$ for the rolling and pitching and $w(t)$ is a white Gaussian noise with zero mean and adequate variance σ^2. Contribution of non-dominant sinusoidal waves to each signal being much less than that of the dominant ones, the paper takes, from the viewpoint of on-line measurement, the approach of giving flexibility to the waveforms rather than increasing the dimensions of the models.

3.2 On-Line Sensing of Rolling and Pitching

Data are acquired each sampling time and the observation equations (1) and (2) are written using the phase variable representation as

$$y_k = H\boldsymbol{x}_k + v_k \tag{12}$$

where $y_k = z_1(k\Delta T)$ and $v_k = v_1(k\Delta T)$ for the rolling, $y_k = z_2(k\Delta T)$ and $v_k = v_2(k\Delta T)$ for the pitching, and the observation matrix $H = [1, 0, -L/g, 0, 0]$ for both signals. Furthermore, $\boldsymbol{x}_k = \boldsymbol{x}(k\Delta T)$, where ΔT represents the sampling time. On the other hand, for ease of information processing, the dynamics (11) is discretized as

$$\boldsymbol{x}_{k+1} = F\boldsymbol{x}_k + \boldsymbol{w}_k \tag{13}$$

where $F \triangleq \Phi(t)\,|_{t=\Delta T}$ and $\Phi(t) \triangleq \mathcal{L}^{-1}\left\{(sI - A)^{-1}\right\}$. Here, $\mathcal{L}^{-1}$ denotes the *Laplace Inverse Transformation.* The discretized transition noise $\boldsymbol{w}_k$ becomes a white Gaussian noise with zero mean and covariance

$$W \triangleq \sigma^2\int_0^{\Delta T}\Phi(\Delta T - \tau)BB^T\Phi^T(\Delta T - \tau)\,d\tau. \tag{14}$$

Consequently, we find that the measurement of the rolling and pitching can be reduced to the state estimation of the linear discrete dynamic systems (12), (13). If the angular frequencies ω_1 and ω_2 (hereafter the periodic lengths $\tau_1 \triangleq 2\pi/\omega_1$ and $\tau_2 \triangleq 2\pi/\omega_2$ are used) are known and v_k is assumed to have a white Gaussian property, the estimation of the rolling and pitching can be respectively achieved by the Kalman filter[3]. However, a difficulty in implementing the filter is that we do not know a priori the exact values of the two periodic lengths τ_1, τ_2 for each signal. In addition, they are time-variant. We thus introduce adequate candidates $\left\{(\tau_1^i, \tau_2^i);\ (1 \le i \le M)\right\}$ for the pair of periodic lengths (τ_1, τ_2) and use a bank of Kalman filters. Methods for setting values of the periodic lengths of the candidates will be discussed in the section 3.4. Consequently, the final estimate of the sensing system is obtained as the conditional expectation of the state estimate for given candidates

$$\hat{\boldsymbol{x}}^o_{k/k} \triangleq \sum_{i=1}^{M} p_k^i \hat{\boldsymbol{x}}^i_{k/k} \tag{15}$$

where $\hat{x}^i_{k/k}$ denotes the state estimate $\hat{x}_{k/k}$ obtained by the i-th candidate $T_i=(\tau_1^i, \tau_2^i)$, and p_k^i is the posteriori probability calculated based on the Bayesian theorem

$$p_k^i = \frac{p\left(y_k/T_i, Y^{k-1}\right) p_{k-1}^i}{\sum_{j=1}^{M} p_{k-1}^j\, p\left(y_k/T_j, Y^{k-1}\right)} \tag{16}$$

Here, $p(y_k/T_i, Y^{k-1})$ represents the conditional probability density function of y_k under the i-th candidate $T_i = (\tau_1^i, \tau_2^i)$ and $Y^{k-1} \triangleq \{y_j; j \leq k-1\}$, i.e.,

$$p(y_k/T_i, Y^{k-1}) = \frac{1}{\sqrt{2\pi\Lambda_k^i}} \exp\left[-\frac{\left(y_k - H\hat{x}^i_{k/k-1}\right)^2}{2\Lambda_k^i}\right] \tag{17}$$

where $\hat{x}^i_{k/k-1}$ and Λ_k^i are the estimate and the variance corresponding to the i-th Kalman filter. Consequently, the proposed sensing system adaptively and automatically selects the most appropriate candidate from the candidates given each sampling time, according to the changed periodic lengths of the two dominant periodic waves.

3.3 On-Line Sensing of Heaving

A discretized observation equation for the heaving is obtained as

$$\begin{aligned} y_k &= \left[z_3(t) - g\cos\hat{\theta}(t)\cos\hat{p}(t)\right]\big|_{t=k\Delta T} \\ &= H_k x_k + v_k \end{aligned} \tag{18}$$

where H_k is a time-variant observation matrix, $H_k=[0, 0, \cos\hat{\theta}(t)\cos\hat{p}(t), 0]\,|_{t=k\Delta T}$, and v_k is an observation noise on S_3, $v_k = v_3(k\Delta T)$. The dynamics of the heaving given by (11) similar to that of the rolling and pitching, and on-line measurement of the heaving is realized by executing the same procedure as described before. The first component of the final estimate $\hat{x}^o_{k/k}$ gives the measurement of the heaving displacement. Furthermore, the second and third components of the estimate $\hat{x}^o_{k/k}$ offer, respectively, the velocity and acceleration of the heaving. The situation is of course the same to the rolling and pitching. Thus the proposed sensing system has an advantage that it measures not only the displacement but also the velocity and acceleration of the three signals.

3.4 Arrangement of Candidates

3.4.1 Method I (Time-Invariant Arrangement). Although the periodic-lengths of the two dominant waves to each signal are known to be in the ranges of 6 s to 10 s and 2 s to 3 s, respectively, their exact values are unknown. It is therefore important to consider the arrangement of the candidates for the Kalman filters which minimize the deterioration of the measurement accuracy for the periodic-lengths in the above ranges. Here we propose, from the viewpoint of the minimax criterion, the optimal arrangement of candidates by use of mumerical simulations (method I). Let $X \triangleq \{T_i\}_{i=1}^M = \{(\tau_1^i, \tau_2^i)\}_{i=1}^M$ be an arrangement of the candidates, the optimal arrangement is given by $\max_{\tau_1,\tau_2} \delta \to \min_X$, where δ denotes the measurement error-rate of the sensing system defined by [effective value of measurement error] /[effective value of simulated signal]×100 (%).

Figure 2 shows the measurement error-rates of the proposed sensing system by use of the method I and the previously proposed one for the short-periodic length τ_2. Here the rolling angle is given by $\theta(t) = 0.45\sin(2\pi t/\tau_1+\pi/2) + 1.5\sin(2\pi t/\tau_2+8\pi/9)$ +0.8(deg) (τ_1=8s, 2s$\leq\tau_2\leq$3s). Considering the conditions of the real seas, the amplitude of the short-periodic wave is set to be larger than that of the long-periodic one. On the other hand, from the viewpoint of the on-line measurement, totally twenty-seven candidates are considered; nine short-periodic lengths and three long-periodic ones are arranged in the ranges of 2 s to 3 s and 6 s to 10 s, respectively. For the method I, the short periodic-lengths of the candidates are arranged as $\{\tau_2^{*j}\}_{j=1}^9$={2.0, 2.03, 2.07, 2.11, 2.15, 2.21, 2.28, 2.36, 2.50} (s), while for the previously proposed system they are set at equally-spaced points, $\{\tau_2^j\}_{j=1}^9$ ={2.0, 2.13, 2,25, 2.38, 2.50, 2.63, 2.75, 2.88, 3.0} (s). For the long-periodic lengths τ_2 the same arrangement $\{\tau_1^k\}_{k=1}^3$={6.0s, 8.0s, 10.0s} is used for both measurement systems because rearrangement of the long-periodic lengths does not remarkably improve the measurement accuracy. In Fig.2, it is clearly seen that the worst value of the measurement error-rate of the method I is improved by 50% in comparison with that of the previously proposed system. Similar arrangements are obtained when the amplitudes and/or periodic-lengths of the simulation data are rather changed. For the pitching, the same arrangement as for the rolling can be used because short-priodic waves are observed apparently in the pitching as well as the rolling one. On the other hand, for the heaving long-periodic waves are seen more conspicuously than short-periodic ones. Thus nine long- and three short-periodic lengths are arranged as $\{\tau_1^{*j}\}_{j=1}^9$={6.0, 6.41, 6.83, 7,27, 7.73, 8.21, 8.72, 9.26, 9.84} (s) and $\{\tau_2^{*k}\}_{k=1}^3$={2.0, 2.30, 2.72} (s).

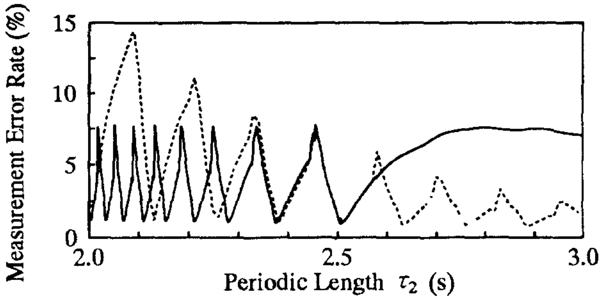

Fig. 2. Measurement error-rate. Solid: method I, dotted: previously proposed system.

3.4.2 Method II (Adaptive Arrangement). In the real seas, the periodic-lengths of the two dominant waves naturally vary with the time. A new type of candidate, whose long- and short- periodic lengths adaptively vary with those of the dominant waves, is thus introduced to further improve the measurement accuracy. The time-invariant candidates are also needed to arrange the new, called adaptive, candidates. For the rolling and pitching, the total number of the adaptive and time-invariant candidates are set to be the same as the method I to compare the measurement accuracy. Twelve adaptive candidates, composed of four short-periodic lengths and three long-periodic ones, are given, while fifteen time-invariant candidates of five short- and three long-periodic lengths are set. For the heaving, six adaptive candidates composed of three long- and two short-periodic lengths and twenty-one time-invariant candidates of seven long- and three short-periodic lengths are considered.

The time-invariant candidates can be arranged optimally by the minimax criterion similarly as the method I. On the other hand, the adaptive candidates are set based on information given by the state estimation for the time-invariant candidates. Namely, the long- and short-periodic lengths of the adaptive candidates are closely arranged near the long- and short-periodic length of the time-invariant candidate which gives the maximum posteriori probability in (16). The final estimate of the method II is given by the conditional expectation of the estimates using the adaptive candidates. The adaptive candidates can be thus automatically rearranged according to the change of the most appropriate candidate in the time-invariant candidates and very accurate on-line sensing is expected.

4. SIMULATION AND EXPERIMENT

Numerical simulations are performed to show the validity of the proposed sensing system. The rolling angle is assumed to be $\theta(t)=\sum_{i=1}^{10} a_{\theta i}\sin(2/T_{\theta i}+\varphi_{\theta i})+0.8$ (deg) where $\{a_{\theta i}\}_{i=1}^{10}=\{0.45, 0.2, 0.1125, 0.0675, 0.1125, 1.5, 0.5, 0.375, 0.225, 0.375\}$ (deg), $\{T_{\theta i}\}_{i=1}^{10}=\{8.0, 8.5, 9.0, 9.5, 7.5, 2.1, 2.15, 2.2, 2.25\ 2.15\}$ (s) and $\{\varphi_{\theta i}\}_{i=1}^{10}=\{\pi/2, \pi/18, 11\pi/18, 2\pi/9, 5\pi/9, 8\pi/9, 5\pi/6, \pi/6, \pi, \pi/9\}$ (rad). Displacements of the pitching are similarly given as the rolling angle, although for the heaving the amplitudes of the long-periodic waves are set to be larger than those of the short-periodic ones. The vertical distance from the intersection of the rolling and pitching axes to the three sensors S_i $(1\leq i\leq 3)$ is set as L=0.5 m. The means of the noises of the sensors are all set to be zero. Considering the specification of the sensors, the variances of the noises are set as 2.5×10^{-9} for the inclinometers and 4.0×10^{-4} for the accelerometer. The sampling time ΔT taken here is 0.2 s.

Figure 3 shows the simulated rolling angle $\theta(t)$, the estimate by the method II and that by the previous arrangement[1]. An accurate and stable measurement is realized by using the method II. It is also clearly seen that the estimate by the method II becomes more accurate than that by the previous arrangement. Similar results are obtained for the pitching and heaving. Moreover, for the two periodic-lengths varied in the ranges of 2 s to 3 s and 6 s to 10 s, respectively, the worst values of the averages of the measurement error-rates for fifty different noise sequences are evaluated. For the previous arrangement, the worst values are 19.3%, 23.1% and 12.9% for the rolling, pitching and heaving, respectively. On the other hand, for the method I and method II, the values become 11.3%, 5.9% and 12.7% and 7.4%, 5.3% and 11.6%, respectively. The measurement accuracy is thus remarkably improved through the method I by 40% and 70% for the rolling and pitching, and further through the method II by 35% and 10%.

Furthermore, experimental data taken in the Seto inland sea of Japan are used instead of the simulation data. The sensors S_i $(1\leq i\leq 3)$ are located at the point 0.66 m distant just over the intersection of the rolling and pitching axes of the ship. For the measurement of the rolling, pitching and heaving, the same candidates as are used in the simulation are adopted again. Due to the engine vibration of the ship, the variances of the observation noises are set to be larger than those used in the simulation; 2.5×10^{-7} for the inclinometers and 4.0×10^{-2} for the accelerometer.

Figure 4 illustrates the actual output of the inclinometer S_1 and the computed output based on the estimate of the rolling (method II). The computed output almost completely agrees with the actual one. In the estimate of the rolling and pitching short-periodic waves are mainly observed as we predicted (data are not shown here). On the other hand, for the estimate of the heaving short-periodic waves are seen more apparently than we expected, because

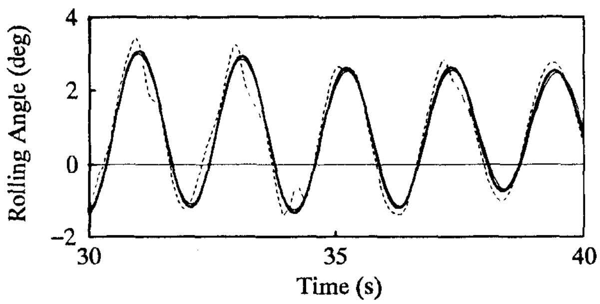

Fig. 3. Measurement of rolling angle. thick: simulated, thin:estimated by the method II, dotted:estimated by the previous arrangement.

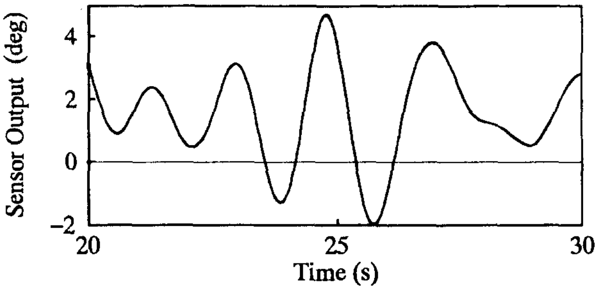

Fig. 4. Sensor output of the inclinometer S_1. thick; actual, thin;method II

the ship used for the experiment was small. These results are considered to be reasonable as the heaving, rolling and pitching in the Seto inland sea. Combining the fact with the simulation results, we can conclude that the proposed sensing system offers a stable and on-line accurate measurement of the ship's attitude.

The computational time of the proposed system was about 30 ms with TOSHIBA AS4080 from the acquisition of the sensor outputs to the measurement of the ship's attitude. If advanced computer technology will be used or the sampling time is a little enlarged, a personal computer will be available for the on-line measurement.

5. CONCLUSION

The paper proposed an on-line dynamic sensing system which accurately measured the heaving, rolling and pitching of the ship by adequately processing the outputs of two inclinometers and one accelerometer appropriately located on the ship. By modeling the three signals as outputs of adequate linear dynamic systems and using a bank of Kalman filters, on-line accurate measurement of the ship's attitude was realized.

REFERENCES

[1] S. Tanaka,"Development of an Automatic Measurement Systen for Ship's Attitude," *T.IEE Japan*, vol. 113-C, No.2, pp. 122-130, 1993.

[2] J. Fukuda,"Statistical Prediction of Ship's Motion," *Proc. the Symposium on Seaworthiness, the Society of Naval Architects of Japan*, Kobe, pp. 99-119, 1969.

[3] R.E.Kalman,"A New Approach to Linear Filtering and Prediction Problems," *Trans. ASME*, vol. 82, pp. 35-45, 1960.