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Robust L2 Optimization For Uncertain Systems

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Abstracts This note proposes a robust LQR method for systems with structured real parameter uncertainty based on Riccati equation approach. Emphasis is on the reduction of design conservatism in the sense of quadratic performance by utilizing the uncertainty structure. The class of uncertainty treated includes all the form of additive real parameter uncertainty, which has the multiple rank structure. To handle the structure of uncertainty, the scaling matrix with block diagonal structure is introduced. By changing the scaling matrix, all the possible set of uncertainty structures can be represented. Modified algebraic Riccati equation (MARE) is newly proposed to obtain a robust feedback control law, which makes the quadratic cost finite for an arbitrary scaling matrix. The remaining design freedom, that is, the scaling matrix is used for minimizing the upper bound of the quadratic cost for all possible set of uncertainties within the given bounds. A design example is shown to demonstrate the simplicity and the effectiveness of proposed method.

Keywords Robust, Structured real parameter uncertainty, Input-output decomposition, Riccati approach

1. Introduction

One of the most interesting robust control methodologies in time domain is the quadratic stabilization (QS) technique whose essence is to manipulate the quadratic Lyapunov functions[1]-[4]. QS methods that are the same kind of Petersen's Riccati approach[2] have been extensively applied to systems with structured or unstructured uncertainty in the same manner. It is surprising that there has been little consideration, in QS methods, for the conservatism of resulting control laws such as the high level of control effort and unnecessarily high bandwidth of the closed loop. Besides systems with unstructured uncertainty, it seems to be natural that the reduction of design conservatism is possible for those with structured uncertainty, remembering μ-analysis including D-scaling technique in frequency domain analysis. In this note, emphasis is on the generalization of previous works based on QS methods for structured real parameter uncertainty, and the reduction of the design conservatism, in the sense of quadratic performance, by using the uncertainty structure. An example shows that the proposed method effectively reduces the design conservatism and generates a practical control law compared to the conventional approach.

2. Characterization of uncertain linear systems

Consider the following linear systems with time varying structured uncertainty:

$$\frac{d\mathbf{x}}{dt} = \{\mathbf{A} + \Delta\mathbf{A}(t)\}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \tag{2.1}$$

$$\Delta\mathbf{A}(t) = \sum_{i=1}^{r} \delta_i(t)\,\mathbf{E}_i \tag{2.2}$$

where $\mathbf{x}\in R^n, \mathbf{u}\in R^m$ and $\mathbf{E}_i$ has the rank of q_i in $R^{n\times n}$. Assume that $\delta_i(\cdot)$ are real-valued Lebesgue measurable functions. In this description, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}_i$ are assumed to be known. The assumed form of uncertainty has frequently appeared in literature for robust control. Any type of $\Delta\mathbf{A}(\cdot)$ in (2.2) can be expressed by the input-output(I/O) structure[11] as follows:

$$\Delta\mathbf{A}(t) = \mathbf{M}_o\,\Delta(t)\,\mathbf{N}_o \tag{2.3}$$

where $\Delta(t) = diag[\delta_1(t)\mathbf{I}_1, \cdots, \delta_r(t)\mathbf{I}_r]$ and $\mathbf{I}_i$ denotes identity matrix in $R^{q_i\times q_i}$. In delta block, $\delta_i(\cdot)$ is repeated as many as the rank of $\mathbf{E}_i$. If all the $\mathbf{E}_i$'s are of rank '1', i.e., uncertainties are so called 'rank one uncertainty', then the delta block consists of only r - different uncertain parameters. It is noted that (2.3) is not unique because of the scaling matrix defined by the set

$$\mathbf{S} := \{\Gamma \mid \Gamma\,\Delta(t)\,\Gamma^{-1} = \Delta(t) \text{ for all } t > 0\} \tag{2.4}$$

Note that Γ is a block diagonal matrix with r - submatrices. With the help of (2.3) and (2.4), we can represent all the possible I/O decomposition of given uncertainty as follows:

$$\Delta\mathbf{A}(t) = \mathbf{M}\,\Delta(t)\,\mathbf{N} \qquad (2.5)$$

where $\mathbf{M} = \mathbf{M}_o\Gamma$ and $\mathbf{N} = \Gamma^{-1}\mathbf{N}_o$ for some $\Gamma \in \mathbf{S}$. Note that Γ should be a scalar if $\Delta(\cdot)$ does not have the diagonal structure. Due to the diagonal representation of parameter uncertainties, we can define the block diagonal set **S**.

The uncertain parameters are assumed to be bounded in time domain such that

$$|\delta_i(t)| \leq \lambda_i, \qquad i = 1, \cdots, r. \qquad (2.6)$$

If $\delta_i(\cdot)$ represents a normalized variation from a nominal value, an example such that $\lambda = 0.1$ implies that a designer wants the parameter to be allowed upto 10 (%) variation from its nominal value. For the r - uncertain parameters, it is convenient to define

$$\Lambda = diag[\lambda_1\mathbf{I}_1, \cdots, \lambda_r\mathbf{I}_r]. \qquad (2.7)$$

Λ is called tolerance matrix for convenience. The allowable variations are represented by the set

$$\Omega(\Lambda) := \{\,\Delta(t)\,|\;|\Delta(t)| \leq \Lambda \text{ for all } t\,\} \qquad (2.8)$$

3. Robust LQR Control

3.1. Modified Algebraic Riccati Equation

It is well known fact that the classical LQR control cannot guarantee the closed loop stability and designed performance under parameter uncertainties. In this section, emphasis is on developing robust control laws which make the quadratic cost finite. Quadratic performance index is defined as follows:

$$\mathrm{I}(\Delta, \mathbf{G}) = \int_0^\infty (\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt, \qquad (3.1)$$

where $\mathbf{Q} \geq 0$, $\mathbf{R} > 0$ and arguments mean that performance index is evaluated when state feedback gain **G** is used for the uncertain system with $\Delta(\cdot)$. For an example, $\mathrm{I}(0, \mathbf{G}_{LQR})$ implies the quadratic cost obtained by LQR control for a nominal system. It is evident that a state feedback gain which makes $\mathrm{I}(\Delta, \mathbf{G})$ finite also stabilizes the uncertain system. To establish the required results, the following assumtion is necessary.

Assumption: The pair of $(\mathbf{A} + \Delta\mathbf{A}, \mathbf{B}, \sqrt{\mathbf{Q}})$ is controllable and detectable.

The following theorem presents robust feedback laws.

Theorem 3.1: (MARE)

For some $\Gamma \in \mathbf{S}$, if there exists $\mathbf{P} > 0$ such that

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{Q} + \mathbf{N}^T\Lambda\mathbf{N} - \mathbf{P}(\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T - \mathbf{M}\Lambda\mathbf{M}^T)\mathbf{P} = 0, \qquad (3.2)$$

uncertain systems with any $\Delta \in \Omega(\Lambda)$ are stabilized by using the state feedback with $\mathbf{G}_{RLQ} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}$. Moreover, if $\mathbf{Q} + \mathbf{G}_{RLQ}^T\mathbf{R}\mathbf{G}_{RLQ}$ is positive definite, the compensated systems are asymptotically stable and the performance index is bounded as follows:

$$\mathrm{I}(\Delta, \mathbf{G}_{RLQ}) \leq \mathbf{x}_o^T\mathbf{P}\,\mathbf{x}_o \qquad (3.3)$$

for any $\Delta \in \Omega(\Lambda)$, where $\mathbf{x}_o$ denotes the initial condition of system states.

(Proof) For $\mathrm{V}(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\,\mathbf{x}$, by using the fact

$$\begin{aligned}\mathbf{N}^T\Delta\mathbf{M}^T\mathbf{P} + \mathbf{P}\mathbf{M}\Delta\mathbf{N} &= \mathbf{N}^T|\Delta|^{1/2}\mathbf{S}(\Delta)|\Delta|^{1/2}\mathbf{M}^T\mathbf{P} + \mathbf{P}\mathbf{M}|\Delta|^{1/2}\mathbf{S}(\Delta)|\Delta|^{1/2}\mathbf{N}\\ &\leq \mathbf{N}^T|\Delta|^{1/2}|\Delta|^{1/2}\mathbf{N} + \mathbf{P}\mathbf{M}|\Delta|^{1/2}\mathbf{S}(\Delta)^2|\Delta|^{1/2}\mathbf{M}^T\mathbf{P}\\ &= \mathbf{N}^T|\Delta|\mathbf{N} + \mathbf{P}\mathbf{M}|\Delta|\mathbf{M}^T\mathbf{P}\\ &\leq \mathbf{N}^T\Lambda\mathbf{N} + \mathbf{P}\mathbf{M}\Lambda\mathbf{M}^T\mathbf{P}\end{aligned}$$

for $\Delta \in \Omega(\Lambda)$, where $\mathbf{S}(\cdot)$ denotes the signum function, it can be shown that

$$\dot{\mathrm{V}}(\mathbf{x}) \leq -\mathbf{x}^T(\mathbf{Q} + \mathbf{G}_{RLQ}^T\mathbf{R}\,\mathbf{G}_{RLQ})\,\mathbf{x} \leq 0.$$

If $\mathbf{Q} + \mathbf{G}_{RLQ}^T\mathbf{R}\,\mathbf{G}_{RLQ}$ is positive definite, then the closed loop system is asymptotically stable. In such case, one can obtain (3.3) by integrating both sides of the above equation. (QED)

Remarks: When the uncertainty is time-invariant, the asymptotic stability is guaranteed by the detectablility condition without the positive definiteness of $\mathbf{Q} + \mathbf{G}_{RLQ}^T\mathbf{R}\,\mathbf{G}_{RLQ}$. For the existence of $\mathbf{P} > 0$, quadratic stabilizabilty condition is necessary[2]. To remove the positive definiteness of $\mathbf{Q} + \mathbf{G}_{RLQ}^T\mathbf{R}\,\mathbf{G}_{RLQ}$, **Q** may be simply assumed to be positive definite as in lots of literature[9].

Note that if there is no consideration for system variations, that is, tolerance matrix Λ is set to be zero, (3.2) becomes the classical LQR solution. Hence, the term 'Robust LQR' is used, which is appeared in [5].

When $\Lambda = \mathbf{I}$ and $\Gamma_i = \varepsilon\,\mathbf{I}_i$ for $i = 1, \cdots, r$, MARE is identical or similar to results of the classical QS technique [2]. $\Lambda = \mathbf{I}$ implies that the tolerance matrix is set by the upper bounds of normalized uncertainties. One should note that the structural information of uncertainty is mostly ignored by setting Γ to only a scalar. The inequality in (3.3) results from using the following extended Petersen's bounding technique:

$$\mathbf{X}^T\mathbf{Y} + \mathbf{Y}^T\mathbf{X} \leq \mathbf{X}^T\mathbf{W}\mathbf{W}^T\mathbf{X} + \mathbf{Y}^T\mathbf{W}^{-T}\mathbf{W}^{-1}\mathbf{Y} \qquad (3.4)$$

where **X**, **Y** and **W** are matrices with proper sizes. When it is applied to the unstructured cases, **W** would be a scalar. However, in the structured cases like the representation (2.5), **W** can be generalized upto a block diagonal matrix. Note that Γ has the same role with the scaling matrix **W**. It is evident that the bounding conservatism may be reduced by selecting a proper **W** with higher dimension. Diagonal scaling matrix, which is the subset of block diagonal matrix, is introduced in [10], however, proper selection method is not addressed except for recommending the trial and error selection. Because $\mathbf{G}_{RLQ}$ is a function of Γ for a given Λ, taking Γ as a scalar narrows the search space for the required $\mathbf{G}_{RLQ}$ unnecessarily. This is the major difference between the proposed method and the previous works by QS methods.

3.2. Optimization For Uncertainty Structure

In developing Theorem 3.1, a scaling matrix representing uncertainty structure is assumed to be given. Note that the control laws given by Theorem 3.1 are the function of $\Gamma \in S$. It is important to note that controller has one more design freedom, that is, scaling matrix for uncertainty structure. In the sense of quadratic performance, arbitrary choice of $\Gamma \in S$ may produce the high upper bound of performance index in (3.3). In general, high performance index implies the high level of control effort or the sluggish convergence behavior. Therefore, it is necessary to find $\Gamma \in S$, which minimizes the upper bound, $\mathbf{x}_o^T \mathbf{P} \mathbf{x}_o$ or $\mathrm{trace}(\mathbf{P}\,\mathbf{x}_o\mathbf{x}_o^T)$ to obtain the guaranteed best performance in the sense of quadratic performance. However, the resulting solution should depend on the initial states if the objective function is chosen by $\mathrm{trace}(\mathbf{P}\,\mathbf{x}_o\mathbf{x}_o^T)$. To avoid this kind of dependency on initial conditions, trace(**P**) instead of $\mathrm{trace}(\mathbf{P}\,\mathbf{x}_o\mathbf{x}_o^T)$ has been used in [8]. In this note, we adopt more reasonable cost index. Generally, initial states represent the possible perturbation of states due to disturbances in regulating systems. Because disturbances are unknown in general, they can be assumed to be a random process so that initial states can be. Hence, the following assumption is made.

Assumption :

Initial state $\mathbf{x}_o$ is a random process with known covariance such that $E[\mathbf{x}_o\mathbf{x}_o^T] = \mathbf{X}_o$.

The following problem can be understood as an optimization problem of the uncertainty structure.

Problem Definition:

Find $\Gamma \in S$ which minimizes $\mathrm{trace}(\mathbf{P}\,\mathbf{X}_o)$ subject to MARE (3.3).

Remark: $\mathbf{x}_o$ may be considered as the initial energy distribution. For states with large average energy, larger weighting is embedded. When $\mathbf{x}_o$ is unity, the formulation is identical to the case of trace(**P**).

Because the above problem is well defined, it can be easily solved by Lagrange Multiplier method. Resulting equations are as follows:

$$\mathbf{X}_o + (\mathbf{A} - \Pi\mathbf{P})\,\mathbf{F} + \mathbf{F}\,(\mathbf{A} - \Pi\mathbf{P})^T = 0 \tag{3.5}$$

$$\Gamma^{-T}\Lambda\,\Gamma^{-1}\,BD(\mathbf{N}_o\mathbf{F}\,\mathbf{N}_o^T) = BD(\mathbf{M}_o^T\mathbf{P}\,\mathbf{F}\,\mathbf{P}\,\mathbf{M}_o)\,\Gamma\,\Lambda\,\Gamma^T \tag{3.6}$$

where $\Pi = \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T - \mathbf{M}\,\Lambda\,\mathbf{M}^T$ and $BD(\mathbf{Y}) = \mathrm{blockdiag}[\mathbf{Y}_{11}, \cdots, \mathbf{Y}_{rr}]$ for $\mathbf{Y}_{ii} \in R^{q_i \times q_i}$ which is the i-th diagonal block of $\mathbf{Y}$. The following example illustrates the function of $BD(\cdot)$.

Example:

Assume $\Gamma = \begin{bmatrix} \gamma_1 & \gamma_2 & 0 \\ \gamma_3 & \gamma_4 & 0 \\ 0 & 0 & \gamma_5 \end{bmatrix}$. For $\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$, $BD(\mathbf{Y}) = \begin{bmatrix} y_{11} & y_{12} & 0 \\ y_{21} & y_{22} & 0 \\ 0 & 0 & y_{33} \end{bmatrix}$.

Note that equations (3.5), (3.6) and (3.3) should be solved simultaneously for finding Γ, **F** and **P**. Three coupled equations can be solved by an iterative method. Procedures using MATLAB functions are as follows:

(i) Guess an initial Γ_1, which generates **P** > 0 of (3.3). Set k=1.

(ii) Set $\mathbf{M}_k = \mathbf{M}_o\Gamma_k$ and $\mathbf{N}_k = \Gamma_k^{-1}\mathbf{N}_o$ and solve (3.3) by

$$\mathbf{P}_k = \mathrm{are}(\mathbf{A}, \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T - \mathbf{M}_k\Lambda\mathbf{M}_k^T, \mathbf{Q} + \mathbf{N}_k^T\Lambda\mathbf{N}_k).$$

(iii) Solve (3.5) by $\mathbf{F}_k = \mathrm{lyap}(\mathbf{A} - \Pi_k\mathbf{P}_k, \mathbf{X}_o)$.

(iv) Obtain Γ_{k+1} by solving

$$\Gamma_{k+1}^{-T}\Lambda\,\Gamma_{k+1}^{-1}\,BD(\mathbf{N}_o\mathbf{F}_k\mathbf{N}_o^T) = BD(\mathbf{M}_o^T\mathbf{P}_k\mathbf{F}_k\mathbf{P}_k\mathbf{M}_o)\,\Gamma_{k+1}\,\Lambda\,\Gamma_{k+1}^T.$$

(v) If $|\mathrm{trace}(\mathbf{P}_k - \mathbf{P}_{k-1})| \le \varepsilon$, stop iterations. Otherwise, increase $k := k+1$ and go to Step (ii).

At Step (iv), a solving function 'fsolve' in a commercial package 'Optimization toolbox' of MATLAB is used. r-subblock equations can be solved separately because the off-diagonal blocks in Γ consist of zero. If they are solved simultaneously, Γ with nonzero off-diagonal blocks may be obtained. Even though the above procedures seem to be simple, they were successful for every case we tried.

3.3. Design Example

An uncertain system with variation of two parameters is given as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 + 2.5\delta_1 \\ 1 + 4.5\delta_1 & -1 + 0.5\delta_2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}$$

$$y = [1\ 0]\,\mathbf{x}$$

It is assumed that the changeable range of δ_2 is known as $|\delta_2(t)| \le 1$ and we wish to design a control law such that δ_1 could be tolerated upto 30 (%) variation. **Q** and **R** are set by $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and 0.1, respectively, as in the standard LQR design. Uncertainty factorization is as follows:

$$\mathbf{M}_o = \begin{bmatrix} 0 & 1.58 & 0 \\ 2.18 & 0 & 2.11 \end{bmatrix},\ \mathbf{N}_o^T = \begin{bmatrix} 2.18 & 0 & 0 \\ 0 & 1.58 & 2.11 \end{bmatrix},$$

$$\text{and } \Delta(t) = \begin{bmatrix} \delta_1\mathbf{I}_{2\times 2} & 0 \\ 0 & \delta_2 \end{bmatrix}.$$

Hence, Γ has the same structure with the example in part B. Tolerance matrix is selected such that $\Lambda = \begin{bmatrix} 0.3\mathbf{I}_{2\times 2} & 0 \\ 0 & 1 \end{bmatrix}$. $\mathbf{X}_o$ is chosen to be a unit matrix. To use the iteration method, an initial Γ is chosen by $\Gamma_1 = \mathrm{diag}[0.578, 0.578, 1]$ which generates **P**>0. Fig.1 shows the optimizing results by the presented iteration method. It shows that the fast convergence is achieved. It should be noted that the control law obtained at step k=1 also guarantees the robust stability and finite cost index for any $\Delta \in \Omega(\Lambda)$. However, the resulting feedback gain at step k=1 is much larger than that of step k=7. State feedback gain at step k=1, is $\mathbf{G}_{RLQ}^1 = [25.173\ \ 9.129]$. After 7 iterations when complete convergence is occurred, the obtained scaling matrix and a state feedback gain are as follows:

$$\Gamma^* = \begin{bmatrix} 1.4249 & -0.4676 & 0 \\ -0.3129 & 0.7051 & 0 \\ 0 & 0 & 2.0150 \end{bmatrix} \text{ and } \mathbf{G}_{RLQ}^* = [8.9622\ 6.7711].$$

It is evident that the level of control effort using $\mathbf{G}_{RLQ}^*$ is effectively reduced. It means that the nonoptimal control law design is unnecessarily conservative in the sense of quadratic performance. In other words, the upper bound of the quadratic cost is so high that the quadratic performance has little meaning, remembering that the minimal quadratic cost represents the optimal trade-off between disturbance rejection and control effort. It is noted that $\mathbf{G}_{RLQ}^*$ guarantees the robust performance in the sense of the bounded quadratic criteria as well as the robust stability. It is consistent with 'auxiliary cost minimization problem' in the sense of minimizing the upper limit of cost function[7].

For many examples, the proposed optimization procedure seems to be extremely helpful to solve MARE because the uncertainty structure, which has been tuned by trial and error, is automatically determined.

4. Conclusions

We studied a quadratic stabilization method for linear systems with structured real parameter uncertainty. We investigated the possibility of reducing the design conservatism by using the structure of uncertainty. We adopted the diagonal representation of uncertainty which made it possible to construct a set of block diagonal scaling matrices not a scalar scaling factor as in previous QS methods. Under the general I/O decomposition, we proposed MARE, which was the generalized version for structured real parameter uncertainty. It generates robust feedback laws, which are the function of the scaling matrix and make LQ cost finite, for all the allowable variations of uncertainty. Consequently, among the feasible feedback laws generated by MARE, we can choose the best by optimizing the uncertainty structure, that is, scaling matrix to minimize the quadratic cost. Simulation results show that unnecessary conservatism such as the high level of control effort in the sense of quadratic criteria can be effectively reduced by the proposed method.

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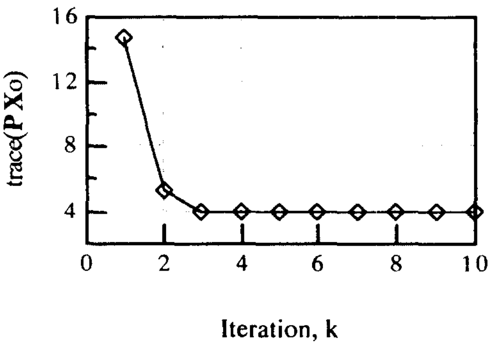

Fig.1 Quadratic cost during iteration. After 7 iterations, complete convergence is obtained.