'95 KACC (1995. 10. 23 ~ 25)

ESTIMATION ERROR BOUNDS OF DISCRETE-TIME OPTIMAL FIR FILTER UNDER MODEL UNCERTAINTY

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Abstract In this paper, estimation error bounds of the optimal FIR (Finite Impulse Response) filter, which is proposed by Kwon *et al.*[1, 2], are presented in discrete-time systems with the model uncertainty. Performance bounds are here represented by the upper bounds on the difference of the estimation error covariances between the nominal and real values in case of the systems with the noise or model parameter uncertainty. The estimation error bounds of the discrete-time optimal FIR filter is compared with those of the Kalman filter via a numerical example applied to the simulation problem by Toda and Patel[3]. Simulation results show that the former has robuster performance than the latter.

Keywords Estimation error bounds, Model uncertainty, Discrete-time optimal FIR filter, Kalman filter.

1. INTRODUCTION

Since the mathematical model of real systems gives only an approximate description of them, there always exist uncertainties in the system model. Therefore, the robustness to the model uncertainty has been one of hot issues in several areas including control, estimation, and system identification. It has been shown that, when the precise knowledge about the system configuration as well as *a priori* statistics of the noise models are not known, the Kalman filters may show poor performance and even divergence phenomenon[1, 2, 8, 9]. Also it is known that Kalman filter has a poor performance on systems with model uncertainty so that the estimation error bounds increases. Previous research to assess the effect of modeling error on Kalman filter performance has been reported in [3]-[6].

In order to overcome these problems in Kalman filter, lots of methods are proposed by many researchers. As one method among them, Kwon *et al.* [1, 2] have introduced the optimal FIR filter, and Yoo and Kwon [5] have analyzed the estimation error bounds of the optimal FIR filter for continuous-time systems. The bounds are calculated from the estimation error covariances of the continuous-time optimal FIR filter. Performance error bounds are here represented by the upper bounds on the difference of the estimation error covariance between the nominal and real values in case of the systems with noise or parameter uncertainty.

In the current paper, the estimation error bounds of the discrete-time optimal FIR filter due to model uncertainties are analyzed. The performance bounds of the filter are derived under the assumption that the system parameter and noise statistics are imperfectly known *a priori*. The estimation error bounds of the discrete-time optimal FIR filter is compared with those of the discrete-time Kalman filter via a numerical example applied to the system model in Toda and Patel[3].

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2. DISCRETE-TIME OPTIMAL FIR FILTER

2.1 Notation

The notation $\|\cdot\|$ is denotes the Euclidean 2 norm of an arbitrary vector, and $\Delta(\cdot)$ is a model uncertainty in any function. The notation $W \otimes Z$ is used for the Kronecker product of matrices W and Z. The column string of an $n \times m$ matrix W, denoted by $cs(W)$, is defined as the following nm-dimensional column vector:

$$cs(W) = [w_{11}, \ldots, w_{n1}, \ldots, w_{1m} \ldots w_{nm}]^T, \quad (1)$$

where w_{jk} is the (j,k)th element of W. It can be easily shown that the trace of an $n \times n$ matrix W can be written as

$$tr(W) = [cs(I_n)]^T cs(W), \quad (2)$$

where I_n is the $n \times n$ unit matrix. Also for any matrix Z

$$\|cs(Z)\| = \|Z\|, \quad (3)$$

and for conformable matrices W, Y and Z [7]

$$cs(WYZ) = (Z^T \otimes W)cs(Y). \quad (4)$$

From the relations (3), (4) and the triangular inequality of the norm, it can be shown that any conformable matrices W, Y and Z satisfy the following inequality[3, 4]):

$$|tr[WYZ]| \leq \|ZW\|\|Y\|. \quad (5)$$

The above inequality (5) will be used in Section 3 to derive the bounds of the estimation error covariance of the discrete-time optimal FIR filter.

2.2 Discrete-Time Optimal FIR Filter

It is assumed that the true system is described by the discrete time-varying state-space model

$$\begin{aligned} x(i+1) &= A_i x(i) + B_i w(i) \\ z(i) &= C_i x(i) + v(i), \end{aligned} \quad (6)$$

where $x(\cdot)$ and $z(\cdot)$ are the state vector and the observation vector, respectively. The initial state vector $x(0)$ is a random variable with $E[x(0)] = m_0$ and $Cov[x(0)] = P_0$ and the system noise $w(\cdot)$ and the observation noise $v(\cdot)$ are zero-mean white with covariances $E[w(i)w^T(j)] = Q_i\delta_{ij}$ and $E[v(i)v^T(j)] = V_i\delta_{ij}$, respectively. It is also assumed that $x(0)$, $w(\cdot)$, and $v(\cdot)$ are uncorrelated each other. Though all matrices A_i, B_i, C_i, Q_i and V_i are time-varying, the subscript i which denotes time-dependence will be deleted hereafter for the notational convenience.

The discrete-time optimal FIR filter $\hat{x}(i \mid N)$ for the state $x(\cdot)$ of the true system (6) is presented in [1] as follows:

$$\hat{x}(i \mid N) = \sum_{k=-1}^{N-1} H(i,k;N)z(k) \tag{7}$$

$$J \equiv E\left\|x(i) - \hat{x}(i \mid N)\right\|^2, \tag{8}$$

where J is the cost function of the filter, and the impulse responses $H(i,\cdot;N)$ is calculated by

$$H(i,j;n+1) = [I - R(i,n+1)C^TV^{-1}C]AH(i,j;n), \tag{9}$$
$$0 \le N - i + j < n \le N-1$$

$$H(i,j;N-i+j) = R(i,N-i+j)C^TV^{-1}$$

$$R(i,n+1) = \bar{R}(i,n) - \bar{R}(i,n)C^T[I + C\bar{R}(i,n)C^T]^{-1}C\bar{R}(i,n)$$

$$-1 < n \le N-1 \tag{10}$$

$$R(i,-1) = P(i-N-1, i-N-1) = Cov[x(i-N-1)].$$

$$\bar{R}(i,n) = AR(i,n)A^T + BQB^T.$$

$$P(i+1,i+1) = AP(i,i)A^T + BQB^T \tag{11}$$
$$P(0,0) = P_0.$$

If the system has the model uncertainty, the discrete-time FIR filter $\hat{x}_m(i \mid N)$ is given for the system represented by the incorrect model $\{A_m, B_m, C_m, Q_m, V_m\}$ as follows :

$$\hat{x}_m(i \mid N) = \sum_{k=-1}^{N-1} H_m(i,k;N)z(k), \tag{12}$$

where the impulse response $H_m(i,k;N)$ is calculated by the same algorithm (9)-(11) using A_m, B_m, C_m, Q_m, V_m instead of A, B, C, Q, V, respectively. If the model uncertainty is described by an additive parametric one, *i.e.*, $A_m \equiv A + \Delta A$, $C_m \equiv C + \Delta C$, $Q_m \equiv Q + \Delta Q$, and $V_m \equiv V + \Delta V$, the relationship between $H_m(i,k;N)$ and $H(i,k;N)$ is also represented by

$$H_m(i,k;N) = H(i,k;N) + \Delta H(i,k;N). \tag{13}$$

The estimation error covariances of the discrete-time FIR filters (7) and (12), which are defined by

$$R(i,N) := E[x(i) - \hat{x}(i \mid N)][x(i) - \hat{x}(i \mid N)]^T \tag{14}$$

$$R_r(i,N) := E[x(i) - \hat{x}_m(i \mid N)][x(i) - \hat{x}_m(i \mid N)]^T, \tag{15}$$

have the following relationship:

Lemma 1 If the incorrect model is represented with A_m, B_m, C_m, Q_m, V_m, and P_m, the real estimation error covariance $R_r(i,N)$ can be represented as follows:

$$R_r(i,N) = R(i,N) + U(i,N), \tag{16}$$

where

$$U(i,N) := \sum_{j=i-N}^{i} \sum_{k=i-N}^{i} \Delta H(i,j;N)R_z(j,k)\Delta H(i,k;N) \tag{17}$$

and $R_z(j,k) := E[z(j)z^T(k)]$. $U(i,N)$ satisfies the following recursion:

$$\begin{aligned}
U(i,n+1) &= [I - K(i,n+1)C]AU(i,n)A^T[I - K(i,n+1)C]^T \\
&+ \Delta K(i,n+1)R_z(i-N+n+1, i-N+n+1)\Delta K^T(i,n+1) \\
&+ \sum_{j=i-N}^{i-N+n} \sum_{k=i-N}^{i-N+n} \Big[[I - K(i,n+1)C]AH_m^T(i,k;n)F^T(i,n) \\
&+ F(i,n)H_m(i,j;n)\Delta H^T(i,k;n)A^T[I - K(i,n+1)C]^T \\
&+ F(i,n)H_m(i,j;n)H_m^T(i,k;n)F^T(i,n)\Big]
\end{aligned} \tag{18}$$
$$U(i,-1) = 0, \quad -1 \le n \le N-1,$$

$$\begin{aligned}
F(i,n) &:= [I - K_m(i,n+1)C_m]\Delta A \\
&\quad -[K(i,n+1)\Delta C + \Delta K(i,n+1)C_m]A. \\
K(i,n) &= R(i,n+1)C^T \\
K_m(i,n) &= R_m(i,n+1)C_m^T \\
\Delta K(i,n) &= K_m(i,n) - K(i,n).
\end{aligned}$$

Proof : The definition of $R_r(i,N)$ gives

$$\begin{aligned}
&R_r(i,N) \\
&= E[x(i) - \hat{x}(i \mid N) + \hat{x}(i \mid N) - \hat{x}_m(i \mid N)] \\
&\quad \times[x(i) - \hat{x}(i \mid N) + \hat{x}(i \mid N) - \hat{x}_m(i \mid N)]^T \\
&= E[x(i) - \hat{x}(i \mid N)][x(i) - \hat{x}(i \mid N)]^T \\
&\quad + E[\hat{x}(i \mid N) - \hat{x}_m(i \mid N)][\hat{x}(i \mid N) - \hat{x}_m(i \mid N)]^T.
\end{aligned} \tag{19}$$

The second equality of (19) comes from the orthogonality of the optimal FIR filter $\hat{x}(i \mid N)$. The first and second terms of (19) are equal to $R(i,N)$ and $U(i,N)$, respectively. Hence (16) is derived. The equations (9) and (13) yield

$$\begin{aligned}
\Delta H(i,j;n+1) &= [I - K(i,n+1)C]A\Delta H(i,j;n) \\
&+ \Big\{[I - K_m(i,n+1)C_m]\Delta A - [K(i,n+1)\Delta C \\
&+ \Delta K(i,n+1)C_m]A\Big\}H_m(i,j;n),
\end{aligned} \tag{20}$$
$$\Delta H(i,j;N-i+j) = \Delta K(i,N-i+j).$$

Substituting (20) into (17) gives (18). Hence the proof is completed. □□□

Although (16)-(18) describe the true estimation error covariance $R_r(i,N)$, which can be taken as the performance measure of the discrete-time FIR filter, it cannot be readily calculated. That is because the modeling error matrices are generally not known exactly. Therefore, it is desired to obtain the upper and lower bounds of $R_r(i,N)$ under the modeling errors such as uncertainties in model parameters and noise statistics. These bounds will provide a measure of the worst performance to be expected from the filter based on imperfect knowledge of the system configuration, noise statistics and initial covariance.

3. ESTIMATION ERROR BOUNDS

3.1 Under Noise Uncertainty

In this section, the upper and lower bounds of the estimation error covariance $R_r(i,N)$ will be derived in case the noise statistics Q_m and V_m are imperfect, but the system matrices A and C are known exactly.

Theorem 1 If there is no parameter uncertainty in the system (6), i.e., *$\Delta A = 0$ and $\Delta C = 0$, then the estimation error bound of the discrete-time optimal FIR filter (12) due to the* *uncertainty in the noise statistics is given by*

$$\left|tr[R_r(i,N)-R(i,N)]\right| \le \Delta\pi_1(i), \quad (21)$$

where

$$\Delta\pi_1(i)=\sum_{k=-1}^{N-1}\left\|\Phi(N,k+1)M(i,k)\Phi^T(N,k+1)\right\|, \quad (22)$$

and $\Phi(N,\cdot)$ is the transition matrix of $[I-K(i,n+1)C]A$ satisfying

$$\Phi(N,k)=\begin{cases}\prod_{n=k+1}^{N}[I-K(i,n+1)C]A & ,N>k\\ I_n & ,N=k\end{cases} \quad (23)$$

where

$$\begin{aligned}M(i,n):=&\\ &\Delta K(i,n+1)R_z(i-N+n+1,i-N+n+1)\Delta K^T(i,n+1)\\ &+\sum_{j=i-N}^{i-N+n}\sum_{k=i-N}^{i-N+n}\Big[[I-K(i,n+1)C]AH_m^T(i,k;n)F_1^T(i,n)\\ &+F_1(i,n)H_m(i,j;n)\Delta H^T(i,k;n)A^T[I-K(i,n+1)C]^T\\ &+F_1(i,n)H_m(i,j;n)H_m^T(i,k;n)F_1^T(i,n)\Big] \quad (24)\end{aligned}$$

$$F_1(i,n):=-\Delta K(i,n+1)CA$$

Proof: In the case where the parameter uncertainty $\Delta A=0$ and $\Delta C=0$, (18) gives

$$U(i,N)=\sum_{k=-1}^{N-1}\Phi(N,k+1)M(i,k)\Phi^T(N,k+1).$$

Taking traces of both sides of (16), we get

$$tr[R_r(i,N)]\le tr[R(i,N)]+\sum_{k=-1}^{N-1}\left|tr[\Phi(N,k+1)M(i,k)\Phi^T(N,k+1)]\right|.$$

Applying the property (5) to the above inequality, the following bound of the estimation error covariance is obtained:

$$tr[R_r(i,N)]-tr[R(i,N)]\le\Delta\pi_1(i),$$

which completes the proof. □□□

3.2 Under Parameter Uncertainty

Now, in this section, it is assumed that the noise statistics are exactly known, *i.e.*, $\Delta Q=0$ and $\Delta V=0$, but that there exists the modeling uncertainty in the system parameter matrices. Here, the upper and lower bounds of the estimation error covariance $R_r(i,N)$ is derived as follows:

Theorem 2 If there is no noise uncertainty in the system (6), i.e., $\Delta Q=0$ and $\Delta V=0$, then the estimation error bound of the discrete-time optimal FIR filter (12) due to the parameter uncertainty is given by

$$\left|tr[R_r(i,N)-R(i,N)]\right|\le\Delta\pi_2(i), \quad (25)$$

where

$$\Delta\pi_2(i)=\sum_{k=-1}^{N-1}\|\Phi(N,k+1)\|^2\|\alpha(i,n)\|\|\beta(i,n)\|\|\gamma(i,n)\| \quad (26)$$

$$\alpha(i,n):=\sum_{j=i-N}^{i-N+n}\sum_{k=i-N}^{i-N+n}\left\|[I-K(i,n+1)C]A\right\|\times\|H_m(i,k;n)\|\|F(i,n)\|$$

$$\beta(i,n):=\sum_{j=i-N}^{i-N+n}\sum_{k=i-N}^{i-N+n}\|F(i,n)\|\|H_m(i,j;n)\|\times\|\Delta H(i,k;n)\|\|[I-K(i,n+1)C]A\|$$

$$\gamma(i,n):=\sum_{j=i-N}^{i-N+n}\sum_{k=i-N}^{i-N+n}\|F(i,n)\|^2\|H_m(i,j;n)\|^2.$$

Proof: In order to obtain the estimation error bound due to the model parameter uncertainty with $\Delta Q=0$ and $\Delta V=0$, the following equation can be derived from (18):

$$tr[R_r(i,N)]=tr[R(i,N)]+\sum_{k=-1}^{N-1}tr[\Phi(N,k+1)E(i,k)\Phi^T(N,k+1)] \quad (27)$$

where

$$\begin{aligned}E(i,n):=&\\ &\Delta K(i,n+1)R_z(i-N+n+1,i-N+n+1)\Delta K^T(i,n+1)\\ &+\sum_{j=i-N}^{i-N+n}\sum_{k=i-N}^{i-N+n}\Big[[I-K(i,n+1)C]AH_m^T(i,k;n)F^T(i,n)\\ &+F(i,n)H_m(i,j;n)\Delta H^T(i,k;n)A^T[I-K(i,n+1)C]^T\\ &+F(i,n)H_m(i,j;n)H_m^T(i,k;n)F^T(i,n)\Big]. \quad (28)\end{aligned}$$

The upper bound on (27) is obtained as follows:

$$\begin{aligned}tr[R_r(i,N)]=&\ tr[R(i,N)]\\ &+\sum_{k=-1}^{N-1}tr\Big[\Phi(N,k+1)\Big\{\Delta K(i,n+1)\\ &\cdot R_z(i-N+n+1,i-N+n+1)\Delta K^T(i,n+1)\\ &+\sum_{j=i-N}^{i-N+n}\sum_{k=i-N}^{i-N+n}[I-K(i,n+1)C]AH_m^T(i,k;n)F^T(i,n)\\ &+F(i,n)H_m(i,j;n)\Delta H^T(i,k;n)A^T[I-K(i,n+1)C]^T\\ &+F(i,n)H_m(i,j;n)H_m^T(i,k;n)F^T(i,n)\Big\}\\ &\cdot\Phi^T(N,k+1)\Big] \quad (29)\end{aligned}$$

Next we bound each of the terms on the right-hand side of (29) from above:

$$\begin{aligned}&\Big|tr\Big[\Phi(N,k+1)\Delta K(i,n+1)R_z(i-N+n+1,i-N+n+1)\\ &\quad\cdot\Delta K^T(i,n+1)\Phi^T(N,k+1)\Big]\Big|\\ &\le\ \|\Phi(N,k+1)\Delta K(i,n+1)\Delta K^T(i,n+1)\Phi^T(N,k+1)\|\\ &\quad\cdot\|R_z(i-N+n+1,i-N+n+1)\| \quad (30)\end{aligned}$$

where we have used relation (5) to obtain the inequality. Hence, it can be obtained

$$\begin{aligned}&\|\Phi(N,k+1)\Delta K(i,n+1)\Delta K^T(i,n+1)\Phi^T(N,k+1)\|\\ &\quad\cdot\|R_z(i-N+n+1,i-N+n+1)\|\\ &\le\ \|\Phi(N,k+1)\|^2\|\Delta K(i,n+1)\|^2\\ &\quad\cdot\|R_z(i-N+n+1,i-N+n+1)\| \quad (31)\end{aligned}$$

The bounds on the remaining terms are obtained in a similar manner using (5) as follows:

$$\begin{aligned}&\sum_{j=i-N}^{i-N+n}\sum_{k=i-N}^{i-N+n}\Big|tr\Big[\Phi(N,k+1)[I-K(i,n+1)C]A\\ &\quad\cdot H_m^T(i,k;n)F^T(i,n)\\ &\quad+F(i,n)H_m(i,j;n)\Delta H^T(i,k;n)A^T[I-K(i,n+1)C]^T\\ &\quad+F(i,n)H_m(i,j;n)H_m^T(i,k;n)F^T(i,n)\Phi^T(N,k+1)\Big]\Big|\\ &\le\sum_{j=i-N}^{i-N+n}\sum_{k=i-N}^{i-N+n}\Big[\|\Phi(N,k+1)[I-K(i,n+1)C]A\\ &\quad\cdot H_m^T(i,k;n)F^T(i,n)\Phi^T(N,k+1)\|\\ &\quad+\|\Phi(N,k+1)F(i,n)H_m(i,j;n)\Delta H^T(i,k;n)\end{aligned}$$

Table 1 Allowable bounds of discrete time optimal FIR filter and Kalman filter.

	Kalman filter	Optimal FIR filter N=5	N=10	N=15	N=20
$\Delta\pi_1(i)$	11.3586	7.3713	7.8259	8.6731	9.1984
$\Delta\pi_2(i)$	12.6989	8.2657	9.2165	9.8536	10.2478

$$\begin{aligned}
&\cdot A^T[I - K(i,n+1)C]^T\Phi^T(N,k+1)\| \\
&+\|\Phi(N,k+1)F(i,n)H_m(i,j;n) \\
&\cdot H_m^T(i,k;n)F^T(i,n)\Phi^T(N,k+1)\|\Big] \\
\leq & \sum_{j=i-N}^{i-N+n}\sum_{k=i-N}^{i-N+n}\Big[\|\Phi(N,k+1)\|^2\|[I-K(i,n+1)C]A\| \\
&\cdot\|H_m(i,k;n)\|\|F(i,n)\| \\
&+\|\Phi(N,k+1)\|^2\|F(i,n)\|\|H_m(i,j;n)\|\|\Delta H(i,k;n)\| \\
&\cdot\|[I-K(i,n+1)C]A\| \\
&+\|\Phi(N,k+1)\|^2\|F(i,n)\|^2\|H_m(i,j;n)\|^2\Big] \qquad (32)
\end{aligned}$$

The required allowable bound is then derived

$$tr[R_r(i,N)] - tr[R(i,N)] \leq \Delta\pi_2(i),$$

which is equivalent to eq(25). This completes the proof of the theorem. □□□

It in noted that bounds of $\|\Delta A\|$, $\|\Delta C\|$, $\|\Delta Q\|$, $\|\Delta V\|$, and $\|\Delta K\|$ should be given in order to compute performance bounds by Theorem 1 and Theorem 2. The upper bound of $\|\Delta K\|$ can be calculated by the method proposed in [3] or assumed to be given.

4. SIMULATION

The estimation error bound of the discrete-time optimal FIR filter is here analyzed via a numerical example, which applies the filter to the estimation problem for a time-invariant process given in Toda and Patel [3]. The result will be compared to that of the Kalman filter, which is presented in [3]. The time-invariant process is modeled by

$$\begin{aligned}
x(i+1) &= \begin{bmatrix} A_{m1} & 0 \\ A_{m2} & A_{m3} \end{bmatrix} x(i) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(i) \\
z(i) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(i) + v(i).
\end{aligned}$$

The numerical value for the model are specified as

$$A_{m1} = -1/3, \quad A_{m2} = 1/10, \quad and \quad A_{m3} = -1/4.$$

The model variances of the zero-mean white noises are

$$q_m = E[w(k)^2] = 10 \ ; \ v_m = E[v(k)^2] = 5.$$

The parameter A_{m2} and A_{m3} are assumed to be correct and the modeling error bounds for the parameter A_{m1}, q_m and v_m are given by

$$|\Delta A_{m1}| \leq 0.1, \ |\Delta q| \leq 1, \ |\Delta v| \leq 1,$$

Bounds of estimation error covariance for the discrete time optimal FIR filter and Kalman filter are summarized in Table 1. This table shows that bounds of the discrete-time optimal FIR filter are smaller than those of Kalman filter and hence, in both the case, the former has better robust performance than the latter when applied to systems with incorrect noise statistics and model parameter uncertainty.

5. CONCLUSIONS

In this paper, the performance bounds of the discrete-time optimal FIR filter for the discrete-time system with model uncertainty have been analyzed. The trace of the difference between the estimation error covariances of the true and nominal systems is here taken as the performance measure. Two types of uncertainty have been considered here; one is the uncertainty in the noise statistics, and another is the parameter uncertainty. The FIR filter is applied to the estimation problem in time-invariant processes given in [3]. The simulation result has shown that the discrete-time optimal FIR filter has better robust performance than Kalman filter.

Acknowledgement

This work was support in part by Inha University Research Grant, Inchon 402-751, Korea in 1994.

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