LQG/LTR CONTROLLER DESIGN FOR GROUND ALIGNMENT OF INERTIAL PLATFORM

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Abstracts The LQG/LTR controller design procedure for ground alignment of inertial platform is accomplished. Due to the alignment system dynamics, LQG/LTR controller is proposed to overcome both singular problem and nonsquare problem. To show the effectiveness of this control system, computer simulation was performed under the assumption of random sway motion.

Keywords LQG/LTR, Ground Alignment, Inertial Platform, Singular System, Nonsquare System

1. INTRODUCTION

The ground alignment of inertial platform has a great deal of importance and INS has accuracy as much as that of the initial ground alignment. Gyro compassing principle has long been applied to this problem.[1,2,3,4] But it is often noted that the conventional gyro compassing method does not give good results in the presence of random sway motions during ground alignment.[5,6] Since the platform alignment accuracy depends on both sensor noise and external acceleration disturbance, it is desirable to consider their statistical characteristics and to construct robust controller.

Since LQG/LTR control scheme has engineeringly well defined procedures and the benefits capable to design multivariable control system systematically, it is being widely used. As you know, LQG/LTR control system is composed of two steps, i.e. , taget filter loop design step and loop transfer recovery step. Generally during the design procedure of the LQG/LTR control system plant must be stabilizable, detectable, nonsingular, minimum phase, and square system to obtain satisfactory loop formation and loop recovery.[7] But this platform alignment system dynamic model is singular system which has 0 eigenvalues and nonsquare system which has 3 inputs, 2 outputs. In this system we can not construct control system with standard LQG/LTR controller design procedure. In this paper to solve the singular problem, we substitute singular eigenvalue, which does not affect to the low frequency characteristics of plant with stable eigenvalue.[8] And, to solve the nonsquare system problem, we introduced input and output compensators using the principles of singular value decomposition.[9,10]

2. ALIGNMENT SYSTEM MODEL

System dynamic model

The mathematical model of ground alignment of inertial platforms is described by eq. (1). It is consisted of platform error dynamics, sensor errors, and control inputs.

$$\dot{x} = Fx + Bu + Gw \tag{1}$$

Measurement equation is represented as following.

$$z = Hx + v \tag{2}$$

We assume process noise and measurement noise are white gaussian and uncorrelated.

$$\begin{aligned} &E[\,w(t)\,] = E[\,v(t)\,] = E[\,w(t)\,v^T(\tau)\,] = 0 \\ &E[\,w(t)\,w^T(\tau)\,] = Q\,\delta\,(t-\tau) \\ &E[\,v(t)\,v^T(\tau)\,] = R\,\delta\,(t-\tau) \end{aligned} \tag{3}$$

Platform error dynamic model

The error sensitivity studies reported by Winter[11] indicate that 13 states are really significant as far as ground alignment of inertial platforms. But since we are interested in the case that sway acceleration is severe, we assumed 3rd order model to simplify the system.[12]

$$\begin{bmatrix} \dot{E}_N \\ \dot{E}_E \\ \dot{E}_D \end{bmatrix} = \begin{bmatrix} 0 & -\Omega\sin\varphi & 0 \\ \Omega\sin\varphi & 0 & \Omega\cos\varphi \\ 0 & -\Omega\cos\varphi & 0 \end{bmatrix} \begin{bmatrix} E_N \\ E_E \\ E_D \end{bmatrix} + \begin{bmatrix} dN \\ dE \\ dD \end{bmatrix} \tag{4}$$

where Ω : *earth rate*
φ : *longitude of alignment*
$[dN\ dE\ dD]^T$: *gyro drift rate*
$[E_N\ E_E\ E_D]^T$: *misalignment*

Measurement equation is as following.

$$\begin{bmatrix} Z_E \\ Z_N \end{bmatrix} = \begin{bmatrix} -g & 0 & 0 \\ 0 & g & 0 \end{bmatrix} \begin{bmatrix} E_N \\ E_E \\ E_D \end{bmatrix} + \begin{bmatrix} b_E \\ b_N \end{bmatrix} \tag{5}$$

where sensor noise covariance matrices are

$$Q = \begin{bmatrix} 4.25\times 10 & 0 & 0 \\ 0 & 4.25\times 10^{-15} & 0 \\ 0 & 0 & 4.25\times 10^{-15} \end{bmatrix} \tag{6}$$

$$R = \begin{bmatrix} 9.6\times 10^{-8} & 0 \\ 0 & 9.6\times 10^{-8} \end{bmatrix} \tag{7}$$

Modeling of sway motion

From Fig. 1, it is clear that the random sway disturbance exhibits periodic behavior. The autocorelation function model of random variable with periodic behavior is given as

$$\phi_{x_1,x_1} = \sigma^2 e^{-\beta|\tau|}\cos w|\tau| \tag{8}$$

where σ^2, β, and w are obtained by experiments. Two state variables are necessary to represent random variables with above autocorelation function.[13] A pair of state equation to represent this relation is as followings.

$$\begin{aligned} \dot{N}_1 &= N_2 + w_{s1} \\ \dot{N}_2 &= -\alpha_1^2 N_1 - 2\beta_1 N_2 + (\alpha_1 - 2\beta_1)\,w_{s1} \end{aligned} \tag{9}$$

$\alpha = (\beta^2 + w^2)^{\frac{1}{2}}$, $\phi(0) = \sigma^2$, $\beta = \xi w_n$,
where $w_n = 2\pi f$ (*f* : *center frequency*) , $w = w_n(1-\xi^2)^{1/2}$,
the spectral density of white noise w_{s1} *is* $2\beta\sigma^2$

Thus, two level axes random sway disturbances are equated as state space equation.

$$\begin{bmatrix}\dot{N}_1\\ \dot{N}_2\\ \dot{N}_3\\ \dot{N}_4\end{bmatrix} = A_s \begin{bmatrix}N_1\\ N_2\\ N_3\\ N_4\end{bmatrix} + \begin{bmatrix}w_{s_1}\\ (\alpha_1-2\beta_1)\,w_{s_1}\\ w_{s_2}\\ (\alpha_2-2\beta_2)\,w_{s_2}\end{bmatrix} \tag{10}$$

where
$$A_s = \begin{bmatrix}0 & 1 & 0 & 0\\ -\alpha_1^2 & -2\beta_1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -\alpha_2^2 & -2\beta_2\end{bmatrix} \tag{11}$$

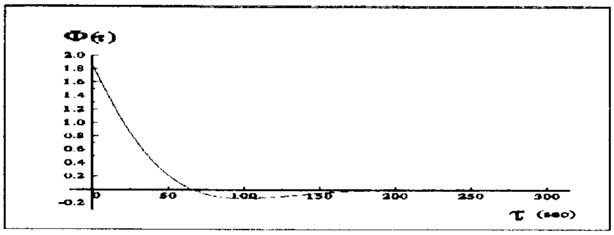

Fig. 1 Autocorelation Function of Random Sway Motion

3. LQG/LTR CONTROLLER DESIGN

3.1 Standard LQG/LTR Controller Design

Generally plant model equations are expressed as followings.

$$\begin{aligned}\dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t)\\ y_p(t) &= C_p x_p(t)\end{aligned} \tag{12}$$

Also we assume nonsingular system, that is, A_p^{-1} exists and want to design control system with no steady state error under certain fixed reference input and disturbance. The design plant model, including free integrator without feedback at each control channel, is to be created as followings.

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t)\end{aligned} \tag{13}$$

where $x(t) = [u_p(t)\ x_p(t)]^T$, $u(t) = \dot{u}_p(t)$,
$$A = \begin{bmatrix}0 & 0\\ B_p & A_p\end{bmatrix},\quad B = \begin{bmatrix}I\\ 0\end{bmatrix},\quad C = [0\ \ C_p] \tag{14}$$

Transfer function matrix of LQG/LTR compensator $K(s)$ is

$$K(s) = G(sI-A+BG+HC)^{-1}H \tag{15}$$

Taget filter loop design (TFL)

The designed state space models of the plant are

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + L\xi(t)\\ y(t) &= Cx(t) + \theta(t)\end{aligned} \tag{16}$$

where $\xi(t)$ and $\theta(t)$ are zero mean, ideal white noise with intensities I and μI. i.e.,

$$\begin{aligned}&E[\ \xi(t)\ ] = E[\ \theta(t)\ ] = 0\\ &E[\ \xi(t)\ \xi^T(\tau)\ ] = I\delta(t-\tau)\\ &E[\ \theta(t)\ \theta^T(\tau)\ ] = \mu I\delta(t-\tau)\end{aligned} \tag{17}$$

At first, to design taget filter loop under above imaginary sensor noise, we must select filter gain matrix H by

$$H = \frac{1}{\mu}PC^T \tag{18}$$

where P is obtained by following FARE.

$$AP + PA^T + LL^T - \frac{1}{\mu}PC^TCP = 0 \tag{19}$$

For selecting design parameters μ and L, we utilize the result of Kalman filter frequency domain equality and obtain the transfer function matrix of TFL $G_F(s) = C(sI-A)^{-1}H$ as

$$G_F(s) \cong \frac{1}{\sqrt{\mu}}C(sI-A)^{-1}L \tag{20}$$

To obtain desired loop shape, we select a design parameter L by coincidence of singular value at high and low frequency simultaneously.

$$L = \begin{bmatrix}-(C_p A_p^{-1} B_p)^{-1}\\ C_p^T(C_p C_p^T)^{-1}\end{bmatrix} \tag{21}$$

In these manners if a parameter L is selected, we can choose design parameter μ that satisfying desired cutoff frequency and bandwidth by moving loop shape up and down. After forming desired TFL, we can select design parameter H.

Loop transfer recovery (LTR)

LTR is possible by solving the cheap control LQR problem. To accomplish LTR, we must compute the solution of following CARE, K as $\rho \to 0$.

$$KA + A^TK + C^TC - \frac{1}{\rho}KBB^TK = 0 \tag{22}$$

and control gain matrix G is computed by

$$G = \frac{1}{\rho}B^TK \tag{23}$$

To derive the basic concept of LTR, we will survey the limit behaviour of CARE as ρ closes to 0. Under the assumption that system (A, B) is stabilizable, (A, C) is detectable and design plant is minimum phase. The limit behaviour of eq. (22), as $\rho \to 0$, is as following eg. (24).

$$C^TC - (\frac{1}{\sqrt{\rho}}KB)(\frac{1}{\rho}B^TK) \to 0 \tag{24}$$

Combining eq. (23) and eqn. (24),

$$(\sqrt{\rho}\,G)^T(\sqrt{\rho}\,G) \to C^TC \tag{25}$$

Therefore, as ρ closes to 0, the limit behavior of control gain matrix G is as following.

$$\lim_{\rho\to 0}\sqrt{\rho}\,G = UC \tag{26}$$

where U is unitary matrix satisfing $U^TU = I$.

Now, the limit behaviour of model based compensator $K(s)$ and loop transfer function matrix $T(s)\ (= G(s)K(s))$ by using eq. (26) are as followings.

$$\begin{aligned}\lim_{\rho\to 0}K(s) &= [\,C(sI-A)^{-1}B]^{-1}[\,C(sI-A)^{-1}H]\\ &= G^{-1}(s)\ G_F(s)\end{aligned} \tag{27}$$

$$\begin{aligned}\lim_{\rho\to 0}T(s) &= \lim_{\rho\to 0}G(s)D(s)\\ &= G(s)\,G^{-1}(s)\ G_F(s)\\ &= G_F(s)\end{aligned} \tag{28}$$

From above equations as ρ approaches to 0, we can know that the limit behaviour is the product of the inverse of plant transfer function matrix $G(s)$ and the desired target filter loop transfer function $G_F(s)$.

In the case of minimum phase system, the closer ρ is to 0, the better recovery is achieved. But it has the effects of enlargement of control inputs.

3.2 LQG/LTR Control of Singular and Nonsquare System

Control of Singular System

We must design TFL by augmenting free integrators to each control input channel without touching plant free integrators. However, plant is singular system, A_p^{-1} does not exist, we can not select design parameter L by using this method directly. Thus to solve such problems, we use stable pole replacement methods, which exists A_p^{-1} and does not affect much the low freqency characteristics of system. By substituting the eigenvalue 0, which makes A matrix singular, with stable eigenvalue $\varepsilon(\to 0)$, we can make new A_p' matrix that exists inverse and can set up augumented design plant model with free control integrating element at each control input channel. In the manner refered to previous section 3.1, LQG/LTR control system is designed for modified design plant model, but the performance test of designed LQG/LTR control system must be performed with the actual plant model. i.e. , we must restore substituted poles to original 0, when we perform simulation.

Control of Nonsquare System

Nonsquare system is one of the most difficult problems in design of linear multivariable system. When the system is nonsquare, we can not calculate solution by direct modern control algebra equation technique. However, since the physical relationship exists among system inputs and outputs, we can construct control system. In the case that system has not the same input and output numbers, the general control design methods are giving or taking a few inputs and outputs. However, in actual cases if the input is added, then the actuator is also added, and if the output is subtracted, the proficiency may be reduced. The system will be called "Overcontrolled" if there are more inputs than outputs. Alignment system is belong to this. In this paper we designed square compensator using the methods of singular value decomposition and applied the multivariable LQG/LTR control technique. By this squaring compensator design technique, we can make square system. Fig. 2 is block diagram of nonsquare system which have square system as design plant model and K(s) is designed for the 2-input 2-output system.

Fig. 3 is singular value diagram of nonsingularized and squared system by above method. The same shape are represented comparing with original plant at low frequency but some detaching shape is viewed at high frequency singular values.

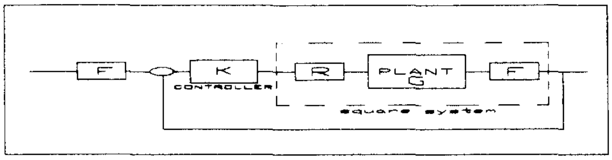

Fig. 2 Block Diagram of Squaring Alignment System

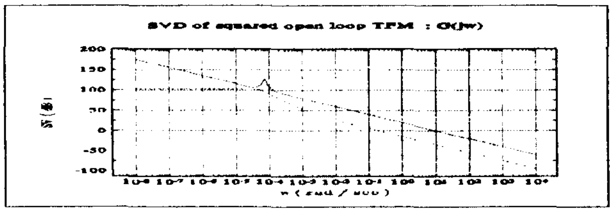

Fig. 3 Singular Value Diagram of nonsingularized and Squared Plant

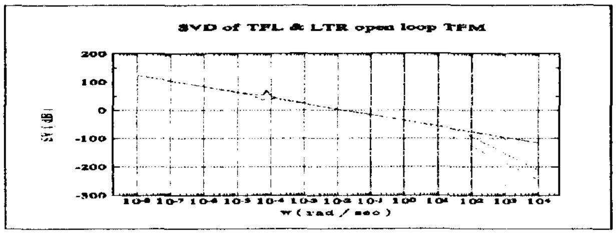

Fig. 4 Singular Value Diagram of Target Filter Loop and Recovered Loop Transfer Function

From the base of design plant model augmented with free integrators, we set up TFL and perfomed LTR which results are represented favorably by Fig. 4. System original characteristics exhibits some resonance behaviour near 10^{-4} rad/sec, and may affect bad effects to the performance of the system.

4. SIMULATION AND RESULTS

Simulation of 3rd order alignment system was performed under the assumption that process noise and measurement noise are white gaussian. The initial misalignment angles are 2 mrad, 2 mrad, 100 mrad, N,E,D-axes in order.

We considered nonlinearity of actuators by introducing saturation elements between controller and actuators. i.e.,

$$u_{ai} = u_i \quad \text{if } u_i \le u_{imax} \qquad (29)$$
$$= u_{imax} \quad \text{if } u_i > u_{imax}$$

where U_{imax} is set 0.003. The variance of gyro drift rates and acceleration biases are $4.25e^{-15}\,rad/sec^2$ and $9.6e^{-8}\,m^2/sec^4$.

Two cases of random acceleration disturbance are considered.

(i) Single frequency disturbance, which has random varying amplitude with variance $1.86\times10^{-6}\,m^2/sec^4$.

(ii) Random sway disturbance having a center frequency and narrow band as stated section 2. $\phi(0)=\sigma^2=1.86\times10^{-6}\,m^2/sec^4$.

Simulation block disagram is represented as Fig. 5. First, we consider case(i), assumed 0.05 Hz as single frequency. Fig. 6 show that N-axis is aligned below 0.1 mrad, E-axis below 0.3 mrad and azimuth axis(D-axis) below 1 mrad within 300 sec. Next, we performed simulation to the case(ii). We consider center frequency as 0.5 Hz, 0.1 Hz, 0.05 Hz and ζ as 0.1, 0.5, 0.7. The smaller ζ and the larger center frequency, the smaller misalignment is acquired. The range of misalignment is from 0.02 mrad(N-axis), 0.25 mrad(E-axis), 0.8 mrad (D-axis) at center frequency of 0.5 Hz to 0.1 mrad(N-axis), 0.3 mrad(E-axis), 2 mrad(D-axis) at 0.05 Hz.

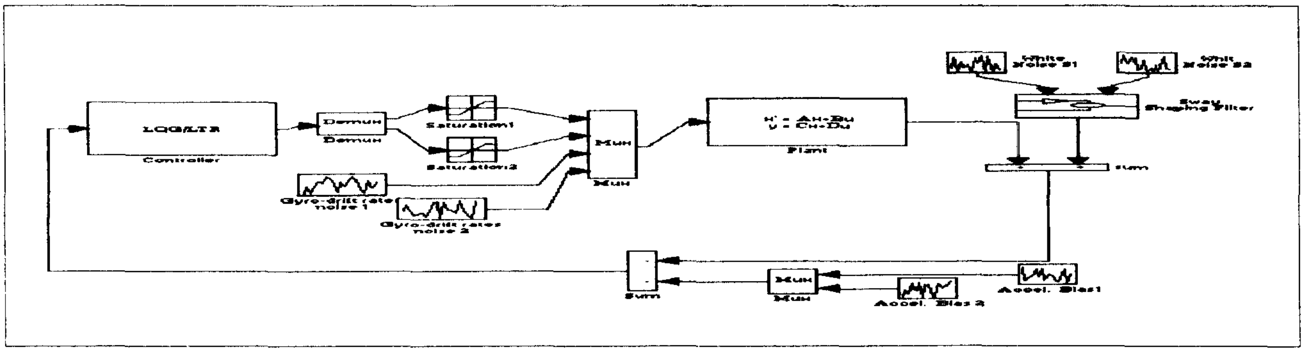

Fig. 5 Simulation Block Diagram of Ground Alignment Control System

Fig. 7 shows that the shape of random sway. From Fig. 8 with the center frequency of 0.05 Hz and ξ of 0.7, we can see that the misalignment of case(ii) are slightly worse than case(i). But, especially for azimuth axis, this results show that the alignment was performed well within the allowable accuracy, even under further severe conditions.

5. CONCLUSION

In spite of two beseting problems, singular system problem and nonsquare system problem, we showed that LQG/LTR control technique is successfully constructed for ground alignment of inertial platforms.

Under assumed random acceleration disturbance, the control system exhibit good performance. Therefore our proposed control system shows less sensitive to the random acceleration disturbance, which may be applied in the case of actual alignment mission. So, we expect that initial misalignment angle can be maintained at small values when initial platform is exposed to the extent of vibration or sway motions.

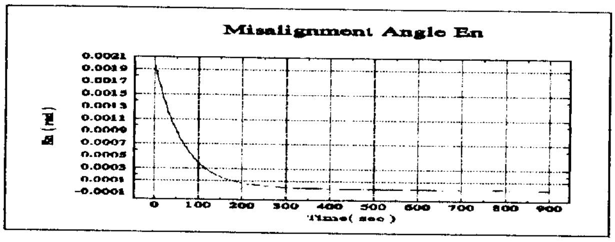

Fig. 6-1 N-Axis Misalignment of Case(i)

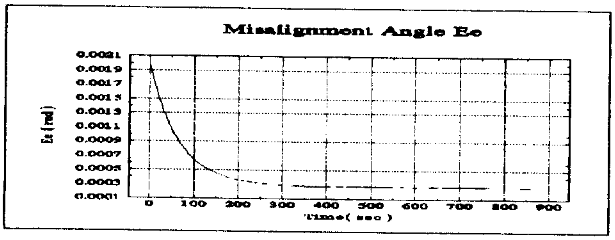

Fig. 6-2 E-Axis Misalignment of Case(i)

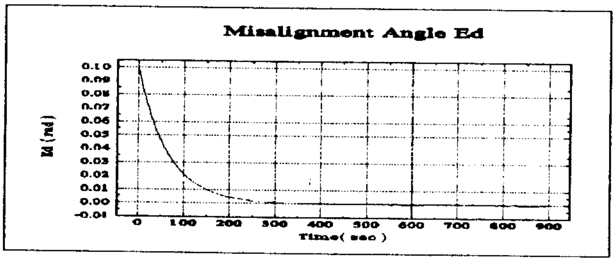

Fig. 6-3 D-Axis Misalignment of Case(i)

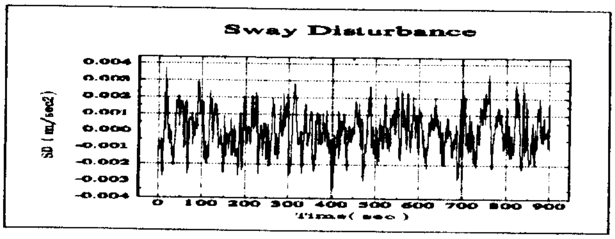

Fig. 7 Acceleration Disturbance of Case(ii) at 0.05 Hz, ξ=0.7

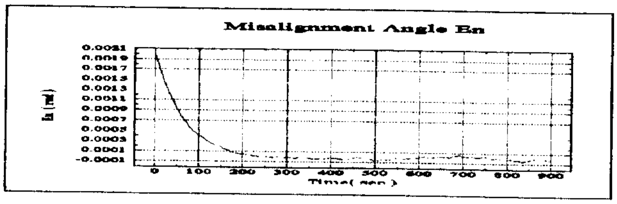

Fig.8-1 N-Axis Misalignment of Case(ii)

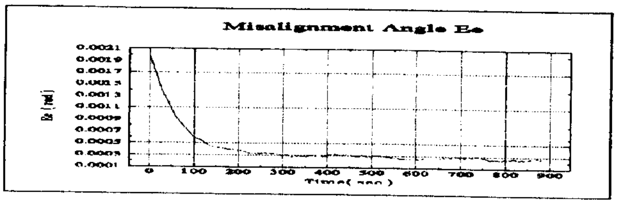

Fig.8-2 E-axis misalignment of case(ii)

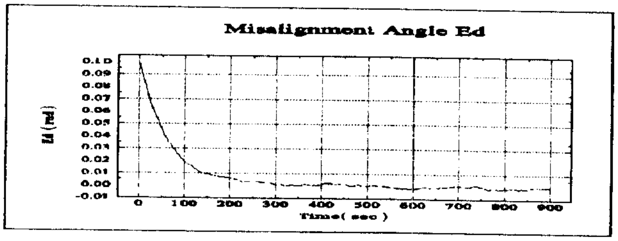

Fig.8-3 D-Axis Misalignment of Case(ii)

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