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FLIGHT TRAJECTORY CONTROLLER FOR NONLINEAR MANEUVER (GENERATION OF A DESIRED TRAJECTORY BY SPLINE THEORY)

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Abstracts: To force an aircraft to track the specified path, the generation of the smooth desired trajectory is essential. In this paper, the cubic spline function is used to generate the trajectory which passes through the specified intercept points. The simulation results show that the desired trajectory generated by the spline interpolation is very smooth and the aircraft tracks it with small position errors.

Keywords: Flight trajectory control, Cubic spline, Intercept points, Minimum acceleration trajectory,

NOMENCLATURE

$a_{()}, b_{()}, c_{()}$	: coefficients of a spline function
$C_{L0}, C_{L\alpha}, C_{Y\beta}$	: aerodynamic coefficients
D	: drag
F_x, F_y, F_z	: components of the guidance force
$K_{()}$	: gain for the guidance error correction
L	: lift
m	: aircraft mass
q_t	: dynamic pressure
P_i	: given intercept point
S	: wing area
$S(t), S_{()}(t)$	: cubic spline functions
t	: time
t_i	: discrete time ($t_0 < t_1 < \cdots < t_N$)
T	: thrust
V	: velocity
V_{dx}, V_{dy}, V_{dz}	: components of the desired velocity ($V_{dx}=\dot{x}_d$; $V_{dy}=\dot{y}_d$; $V_{dz}=\dot{z}_d$)
x, y, z	: components of aircraft trajectory
Y	: side force
α	: angle of attack
β	: sideslip angle
γ	: vertical flight path angle
λ	: horizontal flight path angle
ξ_i, η_i, ζ_i	: components of the intercept point
ρ	: air density
ϕ	: bank angle
$(\ )_c$	: commanded value of ( )
$(\ )_d$	: desired value of ( )

1. INTRODUCTION

Recently, many papers on nonlinear flight control for a desired trajectory have been presented.[1)-6)] This is because a recent airborne digital computer enables the implementation of the control system. Flight trajectory control may be useful in case of precision trajectory control during landing at an airport or especially on a carrier at sea, terrain following with a fighter or a helicopter, etc. Also unmanned aircrafts and cruise missiles require complete automatic flight control.

Figure 1 represents the simplified block diagram of aircraft guidance and control concept for a desired trajectory. It consists of four blocks. The feed-forward control is shown in solid lines and the feed-back control in dotted line. The first block generates the smooth desired trajectory in response to corresponding rough trajectory commands which are supplied from various sources. For example, in case of landing at an airport, they are given from an air traffic control system and in case of terrain following, they may be given from the pilot or an AWACS (Airborne Warning And Control System) or a radar base. The second block generates the guidance forces which force the aircraft to track the desired trajectory. The third block generates the commanded values of angle of attack, sideslip angle, bank angle and thrust in response to the guidance forces from the guidance force generator. The forth block represents the flight control system and the aircraft dynamics. Though many design techniques for nonlinear flight control have been proposed, dynamic inversion approach is the most popular and useful one.[2)-4)] This approach cancels the nonlinearities by state feedback and replaces them with desired linear dynamics. In this approach, it is required that the governing equations are known precisely, the aircraft states are measured or estimated accurately and the dynamic inversion can be almost realized. In this paper, dynamic inversion approach is used for the flight control and it is assumed that the requirements just mentioned are all met except that the actuator dynamics cannot be neglected.[4)]

The successful design of the generators or controll-

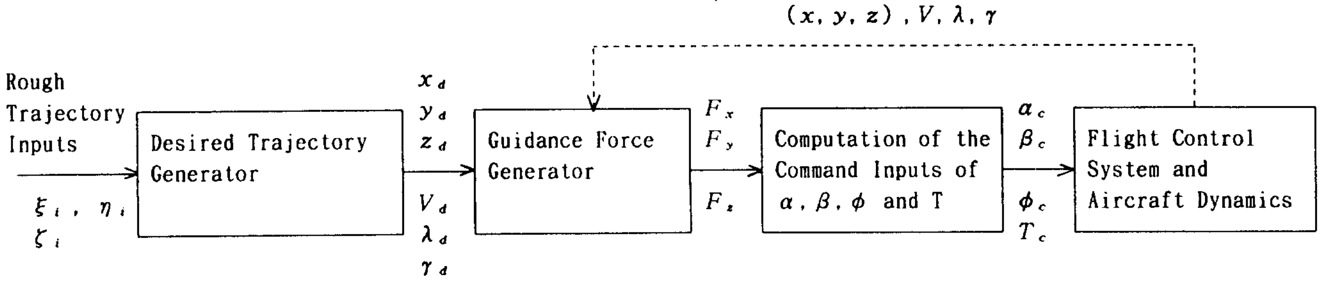

Fig. 1 Aircraft Guidance and Control Concept for a Desired Trajectory

ers in each block is essential in order to guide an aircraft precisely on the desired trajectory. Though many problems are left unsettled in each block, this paper deals primarily with the generation of the desired trajectory using spline theory.

2. GENERATION OF A DESIRED TRAJECTORY BY SPLINE THEORY

If desired trajectories are predetermined flight paths such as a horizontal turn or a barrel roll, they can be represented with analytical continuous functions of time. However, in case of landing at an airport or terrain following, the trajectory command is usually given by not a predetermined function of time, but a set of intercept points, that is, waypoints. And it is easier to force an aircraft to follow a trajectory specified in terms of a set of intercept points than an exact trajectory. When the trajectory command is given by a set of intercept points, it is necessary to generate the desired trajectory from the points. Though there are many methods to determine the trajectory which passes through the specified points, the cubic spline is chosen in this paper for the following reason.[5),6)] The cubic spline $S(t)$ has a continuous second derivative and gives the smoothest curve in the sense that it minimizes the integral[7)]

$$\int_a^b |\ddot{S}(t)|^2 \tag{1}$$

Since $S(t)$ represents one of the coordinates of the desired trajectory, $\ddot{S}(t)$ turns out to be one of the components of the desired acceleration. Thus, the cubic spline generates the minimum acceleration trajectory which passes through the given intercept points.

Let us assume that the desired flight path on the time interval $[t_0, t_N]$ is given by (N+1) intercept points $P_i(\xi_i, \eta_i, \zeta_i; t_i)$ (i=0, 1, 2, ..., N) as functions of discrete time. Figure 2 shows the typical desired trajectory specified in terms of intercept points. According to the spline theory, the desired trajectory on the time subinterval $[t_i, t_{i+1}]$ can be expressed by the following cubic spline functions.

$$S_{xi}(t)=\xi_i+a_{1i}(t-t_i)+a_{2i}(t-t_i)^2+a_{3i}(t-t_i)^3 \tag{2}$$

$$S_{yi}(t)=\eta_i+b_{1i}(t-t_i)+b_{2i}(t-t_i)^2+b_{3i}(t-t_i)^3 \tag{3}$$

$$S_{zi}(t)=\zeta_i+c_{1i}(t-t_i)+c_{2i}(t-t_i)^2+c_{3i}(t-t_i)^3 \tag{4}$$

Connecting $S_{xi}(t)$, $S_{yi}(t)$ and $S_{zi}(t)$ for i=0, 1, ..., N-1, one by one, respectively, the elements of the desired trajectory on the interval $[t_0, t_N]$, x_d, y_d, z_d, can be generated. Figure 3 explains x_d and $S_{xi}(t)$ as a function of time. Differentiating Eqs.(2), (3) and (4) with respect to time, we obtain the desired velocity components on the subinterval $[t_i, t_{i+1}]$ as follows.

$$\dot{S}_{xi}(t)=a_{1i}+2a_{2i}(t-t_i)+3a_{3i}(t-t_i)^2 \tag{5}$$

$$\dot{S}_{yi}(t)=b_{1i}+2b_{2i}(t-t_i)+3b_{3i}(t-t_i)^2 \tag{6}$$

$$\dot{S}_{zi}(t)=c_{1i}+2c_{2i}(t-t_i)+3c_{3i}(t-t_i)^2 \tag{7}$$

The desired velocity elements on $[t_0, t_N]$, V_{dx}, V_{dy}, V_{dz}, can be generated by connecting $\dot{S}_{xi}(t)$, $\dot{S}_{yi}(t)$, $\dot{S}_{zi}(t)$ for i=0, 1, ..., N-1 one by one, respectively. Then, V_d, λ_d and γ_d can be computed from the following equations:

$$V_d = \sqrt{V_{dx}^2 + V_{dy}^2 + V_{dz}^2} \tag{8}$$

$$\lambda_d = \tan^{-1}(V_{dy}/V_{dx}) \tag{9}$$

$$\gamma_d = -\sin^{-1}(V_{dz}/V_d) \tag{10}$$

Differentiating Eqs.(5), (6) and (7) again, we have

$$\ddot{S}_{xi}(t) = 2a_{2i} + 6a_{3i}(t-t_i) \tag{11}$$

$$\ddot{S}_{yi}(t) = 2b_{2i} + 6b_{3i}(t-t_i) \tag{12}$$

$$\ddot{S}_{zi}(t) = 2c_{2i} + 6c_{3i}(t-t_i) \tag{13}$$

As mentioned above, connecting $\ddot{S}_{xi}(t)$, $\ddot{S}_{yi}(t)$, $\ddot{S}_{zi}(t)$ for i=0, 1, ..., N-1, one by one, respectively, the components of the desired acceleration on $[t_0, t_N]$, $\dot{V}_{dx}$, $\dot{V}_{dy}$, $\dot{V}_{dz}$, can be generated. Finally, we obtain $\dot{V}_d$, $\dot{\lambda}_d$ and $\dot{\gamma}_d$ as follows.

$$\dot{V}_d = \frac{V_{dx}\dot{V}_{dx} + V_{dy}\dot{V}_{dy} + V_{dz}\dot{V}_{dz}}{V_d} \tag{14}$$

$$\dot{\lambda}_d = \frac{V_{dx}\dot{V}_{dy} - \dot{V}_{dx}V_{dy}}{V_{dx}^2 + V_{dy}^2} \tag{15}$$

$$\dot{\gamma}_d = \frac{\dot{V}_{dz}V_d - V_{dz}\dot{V}_d}{V_d\sqrt{V_{dx}^2 + V_{dy}^2}} \tag{16}$$

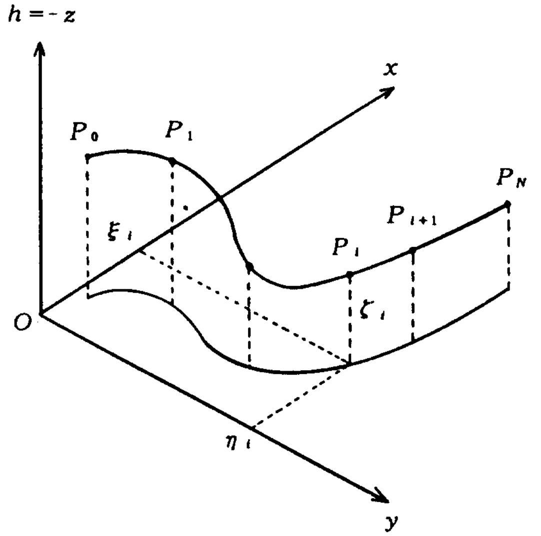

Fig. 2 The trajectory specified by intercept points

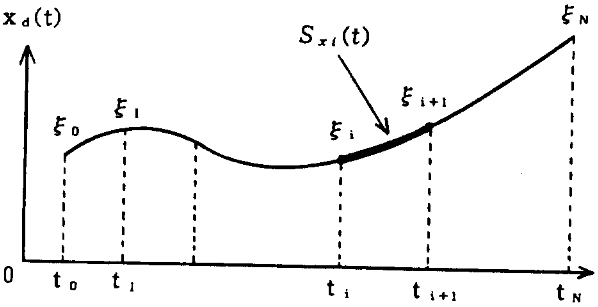

Fig. 3 x_d and $S_{xd}(t)$

3. GUIDANCE FORCE GENERATOR

The guidance forces which guide the aircraft on the desired trajectory are basically constructed of two parts. One is the force necessary for the program motion and the other is the force to eliminate the guidance errors, which consist of the position error, velocity error, horizontal flight path angle error and vertical flight path angle error. Thus, taking these terms into accounts, the guidance forces can be generated from the following equations:[4]

$$F_x = m\{\dot{V}_d + K_v(V_d - V) + g\sin\gamma + K_x(x_d - x)\cos\lambda\cos\gamma + K_y(y_d - y)\sin\lambda\cos\gamma - K_z(z_d - z)\sin\gamma\} \quad (17)$$

$$F_y = m\{V_d\dot{\lambda}_d\cos\gamma + K_\lambda V_d(\lambda_d - \lambda)\cos\gamma - K_x(x_d - x)\sin\lambda + K_y(y_d - y)\cos\lambda\} \quad (18)$$

$$F_z = m\{-V_d\dot{\gamma}_d - V_d K_\gamma(\gamma_d - \gamma) - g\cos\gamma + K_x(x_d - x)\cos\lambda\sin\gamma + K_y(y_d - y)\sin\lambda\sin\gamma + K_z(z_d - z)\cos\gamma\} \quad (19)$$

From Eqs.(17), (18) and (19), we see that the guidance forces depend significantly on the magnitude of the gains for the guidance error correction. Many simulation results showed that when some guidance errors become large, the aircraft could not be forced to track the desired trajectory with the fixed gains. Thus, we used the fuzzy control theory to determine the gains.

4. COMPUTATION OF α_c, β_c, ϕ_c and T_c

At least three control variables are required to guide an aircraft on the desired trajectory in space. On the other hand, a conventional aircraft has four control variables, which are an elevator angle, an aileron angle, a rudder angle and thrust. Thus, we can give one more constraint to the guidance problem for an aircraft. A bank angle is usually specified. For example, if a coordinated flight is specified, the bank angle command ϕ_c is given by

$$\phi_c = \sin^{-1}\left(\frac{F_y}{\sqrt{F_y^2 + F_z^2}}\right) \quad (20)$$

Assuming that the command ϕ_c is specified, the thrust command T_c and the desired values of side force Y_d and lift L_d are obtained from F_x, F_y and F_z .

$$T_c = F_x + D \quad (21)$$

$$Y_d = F_y\cos\phi_c + F_z\sin\phi_c \quad (22)$$

$$L_d = F_y\sin\phi_c - F_z\cos\phi_c \quad (23)$$

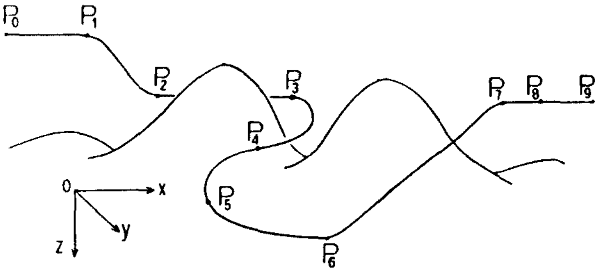

Fig. 4 Terrain following with an aircraft

Thus, the commanded values of angle of attack α_c and sideslip angle β_c are computed from the following equations:

$$\alpha_c = \frac{1}{C_{L\alpha}}\left(\frac{L_d}{q_t S} - C_{L0}\right) \quad (24)$$

$$\beta_c = Y_d/(q_t S C_{Y\beta}) \quad (25)$$

where $q_t = \rho V^2/2$

The flight control system for the commands α_c, β_c, ϕ_c and T_c is designed using the dynamic inversion approach in Ref.(4).

5. NUMERICAL SIMULATION

To illustrate the effectiveness of the generation of a desired trajectory by spline functions, we performed the numerical simulation for a hypothetical terrain following with an aircraft, as shown in Fig. 4. The aircraft is a jet fighter such as F-4 and its geometrical and aerodynamic data are given in Ref.(8). Other data are as follows: the actuator lags are 0.2 seconds ; the maximum thrust is about 151 kN; and the constraints for α_c and β_c are given by

$$-4° \le \alpha_c \le 17°, \quad -4° \le \beta_c \le 4°$$

The intercept points in x-y plane are shown in Fig. 5 and those in x-z plane in Fig. 6. The solid lines connect the intercept points one by one with straight lines and circular arcs. The dotted lines indicate the expected desired trajectory, which were generated using draftsman's curves. The coordinates of the intercept points and the intercept time are shown in Table 1. The intercept time is calculated under the assumption that the aircraft flies along the trajectory indicated by the solid lines in Figs. 5 and 6 and the velocity is constant at 250 m/s. The simulation results are shown in Figs. 7-10. From Figs. 7, 8 and 9, we see that the desired trajectory generated by spline interpolation is very smooth and the aircraft tracks the trajectory with small position errors. The time histories of the velocity is depicted in Fig. 10. Though the

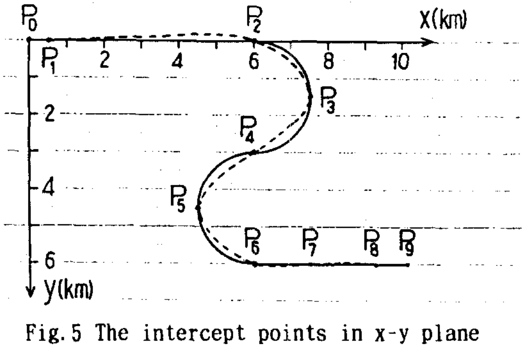

Fig. 5 The intercept points in x-y plane

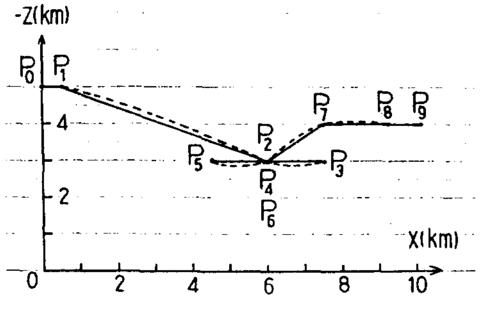

Fig. 6 The intercept points in x-z plane

Table 1. Terrain following intercept points

Point	x(km)	y(km)	-z(km)	time(sec)
P_0	0.0	0.0	5.0	0.0
P_1	0.5	0.0	5.0	2.0
P_2	6.0	0.0	3.0	25.4
P_3	7.5	1.5	3.0	34.8
P_4	6.0	3.0	3.0	44.2
P_5	4.5	4.5	3.0	53.7
P_6	6.0	6.0	3.0	63.1
P_7	7.5	6.0	4.0	74.8
P_8	9.24	6.0	4.0	76.8
P_9	10.1	6.0	4.0	80.0

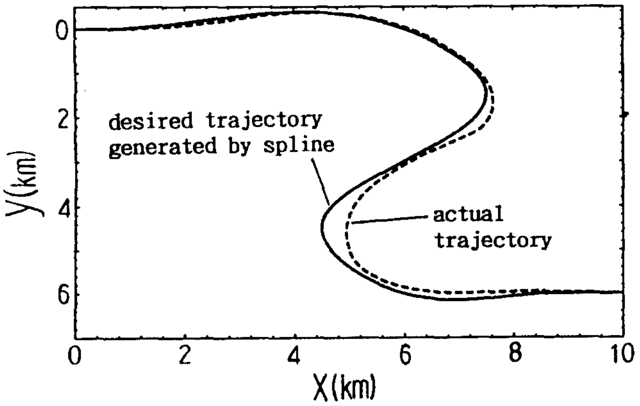

Fig. 7 Trajectories in x-y plane

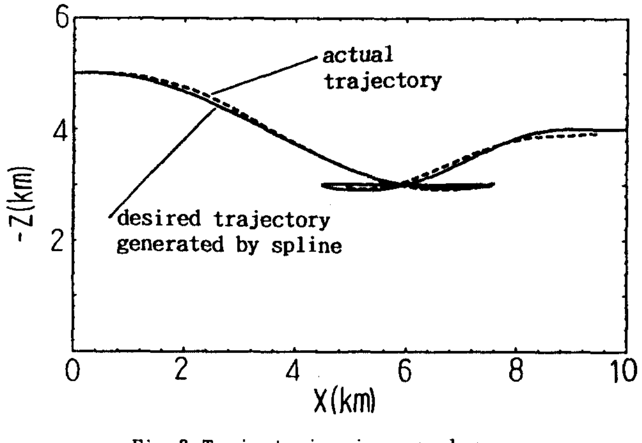

Fig. 8 Trajectories in x-z plane

velocity was assumed to be constant at 250 m/s at first, the generated desired velocity changes sinusoidally. This is because there are differences between the assumed trajectory for computing the intercept time and the desired trajectory generated by the spline functions. Figure 10 shows that the actual velocity cannot follow closely the desired value because of the thrust limitation and the actuator lags. This is one of the causes for the position errors.

6. CONCLUSION

When the intercept points are specified, the aircraft flight trajectory control system must generate the

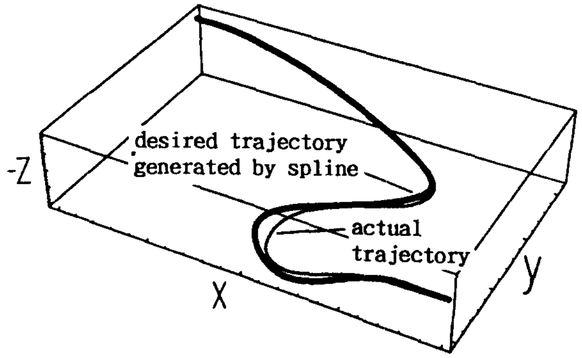

Fig. 9 Trajectories in space

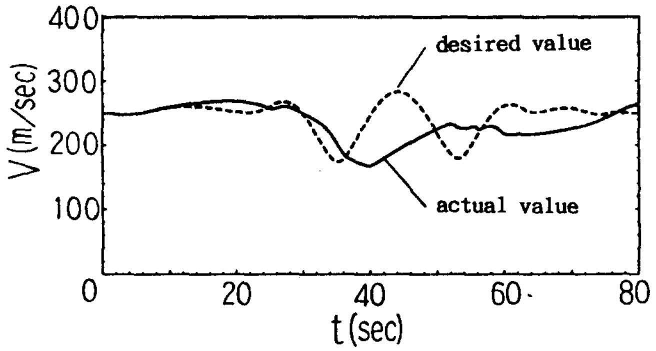

Fig. 10 Time histories of the velocity

desired trajectory which passes through the points. The cubic spline functions were used to generate the trajectory because the cubic spline has a continuous second derivative and gives the smoothest curve, that is, the minimum acceleration trajectory. The numerical simulation for the hypothetical terrain following with a jet fighter was performed and the result showed that the trajectory generated by the cubic spline is very smooth and the aircraft can track it with small position errors.

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