'95 KACC (1995. 10. 23~25)

THE ESTIMATION OF THE ROBUSTNESS BOUNDS OF THE SYSTEMS HAVING STRUCTURED PERTURBATIONS

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Abstract The stability of system is one of the important aspects and to judge the system's stability is another complicated problem. Previously, new technique derived from relaxing Lyapunov conditions has been already introduced and in this paper, this proposed technique applies to the practical dynamic systems. This utility of numerical procedures prove the comparable improvements of the estimation of robustness for dynamic systems having structured (bounded) perturbations.

Keywords Lyapunov Direct Method, Robustness Bounds, Structured Perturbation

1. INTRODUCTION

The stability concept based on Lyapunov direct method is mainly used in this paper and this proposed method warrants that estimates of robustness will be extended and improved. This new approach is derived from modified consideration of the sign properties of the time derivative of Lyapunov function along finite time interval instead of the conventional method of the sign property of the time derivative itself. Systems which have disturbances from their surroundings, it is so difficult to obtain the analytical solutions. But from the practical point of view, it is very important to obtain the allowable perturbation bounds so that the stability of the original system may be maintained. The control of dynamic systems which contain uncertain elements and are with uncertain inputs is treated by the application of stochastic control theory. The construction of measured state feedback controls that provide a guarantee that system responses enter and remain within a particular nighborhood of the zero state after a finite interval of times was considered by Leitmann [9], as well as controller design for uncertain systems. The main objective of the current investigation is to analyze the results from the perspective of computational programming. This main purpose of this paper starts with the new Lyapunov based technique for the robust design of control systems subject to structured perturbations. In concluding remarks, results from computational program can show the further researches directions as well as the improvement of robustness bounds in nonlinear system having structured perturbations.

2. THEORETICAL BACKGROUNDS

The current investigation is to improve the robustness estimate of dynamic systems with structured uncertainties using Lyapunov stability conditions to weaken the stability condition formulated in conventional Lyapunov theorems. For analyses of the sign properties of the Lyapunov function derivative integrated along the finite interval of time are adapted, rather than the sign properties of the derivative itself, which has been the conventional method of deciding the system's stability. The system investigated in this paper is selected as form of nominally linear, with time variant, nonlinear bounded perturbations. For system analysis of the robust design of control systems, stability must be considered in view of the uncertainties of the system equations. This is particular true of the current investigation since stability is the principal area of research interest. Specific explanations are considered in next section, in which vertical take-off and landing aircraft system, with several parameters varying over time, result in substantial changes in dynamics. The system equations require an adequate controller to achieve satisfactory and stable performance with different flight conditions. Parameters with certain condition stated in the system equations can be determined if the aircraft conditions to be stable over a large parametric space. The proposed technique serve to improve the parametric range of the structured perturbations considered to be the robustness bounds.

Following corollary is utilized for the estimation of robustness bounds : Consider a system

$$\dot{x} = f(x) \ , \ f(0) = 0 \qquad (1)$$

where $f \in C^{(1)}(R^n)$.
Let all solution of this equation be difined in the future. If there exist:

1) a continuously differentiable positive definite function V(x),
2) a bounded function T(x) defined for $x \in R^n$ and having a positive lower bound, and
3) a continuous, positive-definite function W(x),

such that the function

$$V^*(x) = \int_{-T(x)}^{0} \dot{V}(x(\tau, 0, x))d\tau \qquad (2)$$

fulfills the condition

$-V^*(x) \geq W(x)$ and $V^* = W(0)$ then the trivial solution of (1) is globally asymptotically stable. In this proposed technique, the perturbations are considered as structured, nonlinear time-variant, and systems are nominally linear. This class of systems is particularly suited to the utilization of the corollary for the estimation of robustness bounds.

The procedure based on corollary is then a natural extension of the Lyapunov direct method procedure. Firstly, the selection of the Lyapunov candidate function V(x) is to check the sign of $\dot{V}$. Observe that for each $x \in \Gamma_1$, there exists an $\varepsilon > 0$ such that $\int_{-\varepsilon}^{0} \dot{V}(x(\tau, 0, x))d\tau < 0$. Therefore, further investigation of (2) is required only for $x \in \Gamma_1$. Given the class of systems under discussion, it is enough to consider only the points x inside the unit sphere S_1. Secondly, the approach utilizes the fact that the class of systems under discussion allows for the derivative of the analytical expression of the difference

$$V(x(t, 0, x_0)) - V(x_0). \qquad (3)$$

When V(x) is selected as the quadratic form, this leads to the analysis of the properties of matrices, which are used to describe how the difference in (3) behaves for different perturbation. It will be possible to obtain the robustness bounds analytically and that this approach will be a simple technique to the multi-dimensional cases.

3. RESULTS FROM AIRCRAFT SYSTEM

Concerning the dynamic systems, we select the linearized model of the VTOL(Vertical Take-Off and Landing) aircraft in the vertical plane and describe:

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u.$$

The state vector $x \in R^4$, x_1:horizontal velocity, x_2: vertical velocity, x_3:pitch rate, x_4:pitch angle. For typical load and flight condition for this system at an airspeed of 135 knots, the matrices A and B are :

$$A = \begin{bmatrix} -0.0336 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1001 & 0.3681 & -0.707 & 1.42 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \end{bmatrix}$$

The most significant events take place in the elements a_{32}, a_{34} and b_{21}. And all the other elements are considered as constants. Thus, in the matrices $\Delta A(t)$ and $\Delta B(t)$, these are the only non-zero terms. To obtain appropriate hadling characteristics at the nominal airspeed of 135 knots, the feedback gain, as provided by Sundararajan [15], is

$$K = \begin{bmatrix} -0.8143 & -1.2207 & 0.266 & 0.826 \\ -0.2582 & 1.178 & 0.0623 & -0.212 \end{bmatrix} .$$

Input u=Kx and, the system is three-degree of freedom dynamic case with the structured perturbations Δa_{32}, Δa_{34} and Δb_{21}. The system equation is therefore

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u \ , \quad u = Kx$$

and

$$\begin{aligned} \dot{x} &= \underline{(A + BK)}x + (\Delta A + \Delta BK)x \\ &= \bar{A}x + \Delta \bar{A}x \ . \end{aligned}$$

Using the Lyapunov direct method, numerical method is used for $V=x^TPx$ and $\dot{V}=x^T(\overline{A}^TP+P\overline{A})x$ to solve the equation $\overline{A}^TP+P\overline{A}=-I$. Finally, the matrix P is

$$P=\begin{bmatrix} 2.3651 & 0.1903 & 0.2075 & -1.2156 \\ 0.1903 & 0.3487 & 0.3661 & 0.0797 \\ 0.2075 & 0.3661 & 0.4609 & 0.1702 \\ -1.2156 & 0.0797 & 0.1702 & 2.1871 \end{bmatrix}$$

and

calculate the $\overline{A}^TP+P\overline{A}$.

Therefore $\dot{V}=x^T(\overline{A}^TP+P\overline{A})x$ is

$$\begin{aligned}\dot{V} = & (-0.3099\Delta b_{21}-1.0)x_1^2 \\ & +(-1.0325\Delta b_{21}+0.4149\Delta a_{32})x_1x_2 \\ & +(-0.8514\Delta b_{21}+0.7322\Delta a_{32}-1.0)x_2^2 \\ & +(-0.4950)x_1x_3 \\ & +(-0.7083\Delta b_{21}+0.9218\Delta a_{32})x_2x_3 \\ & +(0.1948\Delta b_{21}-1.0)x_3^2 \\ & +(0.1845\Delta b_{21}+0.4149\Delta a_{34})x_1x_4 \\ & +(0.3815\Delta b_{21}+0.7322\Delta a_{34}+0.3404\Delta a_{32})x_2x_4 \\ & +(0.6472\Delta b_{21}+0.9218\Delta a_{34})x_3x_4 \\ & +(0.1317\Delta b_{21}+0.3404\Delta a_{34}-1.0)x_4^2 \quad .\end{aligned}$$

If $\dot{V}$ is always negative, the matrix $\overline{A}^TP+P\overline{A}$ is always negative-definite, the system is asymptotically stable. To satisfy this condition, the parametric spaces Δa_{32}, Δa_{34} and Δb_{21} have certain limits. In the case of $|\Delta a_{32}|<0.43$, $|\Delta a_{34}|<0.24$ are $|\Delta b_{21}|<0.44$ system is stable in the sense of conventional Lyapunov stability concept.

Applying the new technique, the space which fulfills the stability conditions are extended. Thus, the final results for regions are, respectively, $|\Delta a_{32}|<0.47$, $|\Delta a_{32}|<0.26$ and $|\Delta b_{21}|<0.48$.

4. CONCLUSIONS

The proposed technique is applied to VTOL aircraft and the most important parameters for the control and design of the airplane controller were extended in range, assuming the stability of the original system in the context of Lyapunov stability. Robust control design for VTOL aircraft was previously considered by Singh and Coelho [14], who obtained bounds resulting from nonlinear controls of $|\Delta a_{32}|\leq 0.2$, $|\Delta a_{34}|\leq 0.3$ and $|\Delta b_{21}|\leq 0.3$. The results obtained from the new technique, based upon the study of robust stability and ability to stabilize VTOL aircraft systems with parametric uncertainties were $|\Delta a_{32}|<0.47$, $|\Delta a_{32}|<0.26$ and $|\Delta b_{21}|<0.48$. Further research of robustness bounds should directed toward the study of new generations of Lyapunov functions for dynamic systems controlled by various kinds of parameters and forms of perturbations.

REFERENCE

[1] Barmish, B.R., "Stabilization of uncertain systems via linear control," IEEE Trans. Automa. Contr., vol. AC-28, pp. 848-850, 1978.

[2] Barmish, B.R., Coreless, M., and Leitmann, G., "A new class of stabilizing controllers for uncertain dynamical systems," Proc. IEEE Conf. on Decision and Control, Florida, pp. 228-233 1982.

[3] Barrett, M.F., "Conservatism with robustness tests for linear feedback control systems," in Proc. 19th Conf. Decision Contr., pp. 885-890, 1980.

[4] Biernacki, R.M., Hwang, H., and Bhattacharyya, S.P., "Robust stability with structured parameter perturbations," IEEE Trans. Automat. Contr., vol. AC-32, pp. 495-506, 1987.

[5] Brayton, R.K., and Tong, C.H., "Constructive Stability and asymptotic stability of dynamic systems," IEEE Trans. Circ Syst., vol. 27, pp. 1121 -1130, 1980.

[6] Desoer, C.A., and Chan, W.S., "Robustness of stability conditions for linear time-variant feedback systems, "IEEE Trans. Automat. Contr., vol. AC-22, pp. 586-590, Aug. 1977.

[7] Keel, L.H., and Bhattacharyya, S.P., "Robust control with structured perturbations," IEEE Trans. Automat. Contr., vol. 33, pp. 68-78, 1988.

[8] Lasalle, J.F., and Lefschetz, S., "Stability by Lyapunov's direct mothod with applications," Academic Press, New York, 1961.

[9] Leitmann, G., "On the efficacy of nonlinear control in uncertain linear systems," Journal of Dynamic Systems, Measurements, and Control, vol. 102, pp.95-102, Jun. 1981.

[10] Olas, A., "A criterion of stability of the trivial solution of a set of ordinary differential equations," Bull. del' Acad. Polon. des Sci., Ser. des Sci. Tec., vol. XXiX, no. 5/6, 1981.

[11] Patel, R.V., and Toda, M., "Quantitative measures of robustness for multivariable systems," in Proc. Joint Automat. Contr. Conf., San Francisco, CA, TP8-A, 1980.

[12] Patel, R.V., Toda, M, and Sridhar, R., "Robustness of linear quadratic state feedback designs in the presence of system uncertainty," IEEE Trans. Automat. Contr., vol. AC-22, pp. 945-947, Dec. 1977.

[13] Siljak, D.D., "Parameter space methods for robust control design: A guide tour," IEEE Trans. Automat. Contr., vol. 34, no. 7, pp. 674-688, Jul. 1989.

[14] Singh, S.n., and Coelho, Antonio A.R., "Nonlinear control of mismatched uncertain linear systems and application to control of aircraft," Journal of Dynamic Systems, Measurement, and Control, vol. 106, pp.106-210, Sep. 1984.

[15] Sundararajan, N., "Sensitivity reduction in aircraft control systems," IEEE Trans. Aerospace Electr. Syst., vol 14, pp.292-297, Mar. 1978.

[16] Vannelli, A., and Vidyasagar, M., "maximal Lyapunov function and domains of attraction for autonomous nonlinear systems," Automatica, vol. 21, no. 1, pp. 69-80, 1985.

[17] Yedavalli, R.K., "Perturbation bounds for robust stability in linear state space models," Int J. Contr., vol. 42, pp. 1507-1517, Dec. 1985.

[18] Yeadavalli, R.K., and Liang, Z., "Reduced conservatism in stability robustness bounds by state transformation," IEEE Trans. Automat. Contr., vol. AC-31, no.9, pp. 863-866, Sep. 1986.

[19] Yeadavalli, R.K., "Improved measures of stability robustneaa for linear state space models," IEEE Trans. Automat. Contr., vol. AC-30, pp. 577- 579, Jun. 1985.

[20] Zhou, K., and Khargonekar, P.P., "Stability robustness bounds for linear state-space models with structured uncertainty," IEEE Trans. Automat. Contr., vol. AC-32, pp. 621-623, 1987.