A GA BASED ON-LINE TUNING OF ROBUST MINIMAX I-PD CONTROLLER WITH PENALTY ON MANIPULATED VARIABLE

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Abstract. In this paper we propose an on-line tuning method by using genetic algorithm for robust minimax I-PD controller based on new criterion. The new criterion is the Integral of Squared Error (ISE) with a penalty of the derivative of manipulated variable. The work focuses on robust tuning of I-PD controller's parameters in the presence of plant parameter uncertainty. The result of several simulation studies are provided to illustrate the performance of this robust tunig method.

Keywords. Robustness, I-PD controller, Minimax optimization, Integral-Squared-Error, Genetic Algorithm

1. Introduction

PID and I-PD controllers are effective enough to have desirable control performance for practical use in most industrial processes. Although many methods of the design of I-PD controller have been developed [1],[2], these methods of tuning parameters of PID and I-PD controllers are experimental and heuristic. One of the main reason for this is that industrial processes are too complex to obtain precise dynamics of a plant. Therefore, a controller is required to have robustness property in that it can attain acceptable control performance and closed-loop stability in the presence of plant prarameter uncertainty.

More recently, Lu [3],[4] has developed a novel idea of designing a PID controller based on minimax criterion. Since no solution algorithms are provided therein, we have developed a method for designing robust I-PD controllers [5],[6], in which the design problem is formulated as a minimax optimization problem, due to Lu's novel idea[3],[4]. Genetic Algorithms (GAs)[7]~[9] are known as one of most effective methods to solve the complex, large-scale and dynamic optimization problems.

GAs are search algorithm based on the mechanics of natural selection and natural genetics. They provide robust yet efficient procedure in finding near-optimal solutions in complex and large-scale problem spaces. Many optimization methods require much auxiliary information or computational time in order to work properly. For example, gradient techniques need derivatives in order to be able to climb the current peak, all search techniques require huge computational time and they can not be applied to the dynamic optimization problems, and other local search procedures like the greedy techniques of combinatorial optimization require access to most if not all tabular parameters. By contrast, GAs have no need for all this auxliary information.

In this paper, we use the Integral of Squared Error (ISE) with a penalty of the derivative of manipulated variable[11] as the performance criterion and proposed the GA based on-line tuning method of robust minimax I-PD contoller. And we investigate the effectiveness of this method in the presence of plant parameteres uncertainty in siveral simulation studies.

2. Problem formulation

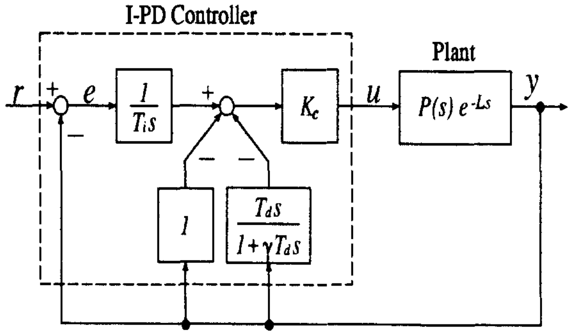

Fig.1 Block Diagram

Consider the control system shown in **Fig.1**, where

$G(s) = P(s)e^{-Ls}$ is a plant and L is a delay. We use the second order Padé approximation to the delay, so that the plant is approximated as

$$G(s) \;\cong\; P(s) \cdot \frac{1 - \frac{L}{2}s + \frac{L^2}{12}s^2}{1 + \frac{L}{2}s + \frac{L^2}{12}s^2} \qquad \textbf{(2.1)}$$

where $P(s)$ is a rational transfer function. Let θ be the parameters in $P(s)$. Taking account of the plant parameter uncertainty, we assume that θ belongs to a bounded set Θ.

From **Fig.1**, the I-PD controller is described by

$$u(s) \;=\; K_c\left[\frac{1}{T_i s}e(s) - (1 + T_d s)y(s)\right] \qquad \textbf{(2.2)}$$

so that for the step input $r(s) = 1/s$ and disturbance $d(s) = 0$, the error is expressed as

$$e(s) \;=\; \frac{1 + K_c(1 + T_d s)G(s)}{1 + K_c(1 + \frac{1}{T_i s} + T_d s)G(s)} \cdot \frac{1}{s} \qquad \textbf{(2.3)}$$

and the derivative of manipulated variable is

$$s{\cdot}u(s) = \frac{\frac{K_c}{T_i}}{1 + K_c\left[\frac{1}{T_i s} + \left(1 + \frac{T_d s}{1 + \gamma T_d s}\right)\right]G(s)} \cdot \frac{1}{s} \textbf{(2.4)}$$

We use the performance mesure given by ISE (Integral of Squared Error) with a penalty of the derivative of manipulated variable[11]. Thus the performance measure is described by

$$J(q,\theta) = \int_0^{\infty} \{e^2(t) + \rho \dot{u}^2(t)\}\, dt$$

$$= \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} [e(s)e(-s) + \rho\{su(s)\}\{-su(-s)\}]\, ds \textbf{(2.5)}$$

where $q := (K_c\ T_i\ T_d)$ is the parameter vector of the controller and $\theta \in \Theta$ is the parameter of the plant and ρ is a nonnegative weighting constant.

Assume that the denominator of the transfer function of a plant with Padé approximation for the time delay is the polynomial of degree n. Then, from (2.3) and (2.4), $e(s)$ and $su(s)$ are expressed as

$$e(s) = \frac{B(s)}{A(s)}\ , \quad s \cdot u(s) = \frac{C(s)}{A(s)} \qquad \textbf{(2.6)}$$

where $A(s)$, $B(s)$, $C(s)$ are polynomials of degrees $n+3$, $n+2$, $n+2$, respectively. Thus both $e(s)$ and $su(s)$ are necessarily strictly proper.

We see from (2.5) that

$$J = J_1 + \rho \cdot J_2 \qquad \textbf{(2.7)}$$

where

$$J_1 = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} e(s)e(-s)ds \qquad \textbf{(2.8)}$$

$$J_2 = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} \{su(s)\}\{-su(-s)\}ds \qquad \textbf{(2.9)}$$

Hence, we can see from (2.6) that both J_1 and J_2 are convergent. Therefore, the performance measure J is computed by

$$J_1 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{B(s)B(-s)}{A(s)A(-s)}ds = I_1^k \qquad \textbf{(2.10)}$$

and

$$J_2 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{C(s)C(-s)}{A(s)A(-s)}ds = I_2^k \qquad \textbf{(2.11)}$$

where k is the degree of $A(s)$ and where $I_{1,2}^k$ for $k = 1$ to 10 are listed in Newton *et al.*[10] .

3. On-line tuning algorithm

The problem to be considered is the tuning of I-PD controller that guarantees certain level of performance in the presence of plant parameter uncertainty. The problem is conveniently considered as a minimax optimization problem [3]~[5],[11] as follows.

Tuning problem :

$$\min_{q}\ \max_{\theta \in \Theta} J(q,\theta) \qquad \textbf{(3.1)}$$

s.t. closed loop system is stable of $\forall\ \theta \in \Theta$

We assume that $q := (K_c\ T_i\ T_d) \in Q = \{q \mid q_l \leq q \leq q_u\}$, a subset of $\mathbf{R}^3$. For computational purpos e, let $Q_d := \{q_i, i = 1, \ldots, N\}$ be a discrete approximatio n to the set Q. And we can check the robust stability by the Kharitonov's Theorem.

A constraint condition of this problem is considered the stability of closed-loop system. The auto-tuning algorithm of I-PD controller consists of the following steps:

1. Set time $t = 0$.
2. Observe the plant parameters θ_t at time t.
3. For the plant parameters θ_t, the parameters of I-PD controller, K_c, T_i and T_d, are tuned by GA during the sampling interval. (We test the robust stability by using Kharitonov's Theorem.)
   If time is up, return the best parameters to the controller.
4. If terminal condition is satisfied, stop. Otherwise go to **2.**

What has to be noticed is that an interval of sampling time t is fixed and determined beforehand. Assume that we wish to construct a genetic algorithm to optimize the problem.

3.1 Outline of GA

A general procedure of GA consists of the following steps:

1. Initialize the genes of each individual in the population $G(k = 0)$.
2. Generate $G(k+1)$ from $G(k)$ as follows:
   evaluate fitness of each individual in $G(k)$;
   select individuals from $G(k)$ using fitness;
   recombine them using genetic operators;
3. If terminal condition is satisfied, stop and return the best individual. Otherwise set $k = k + 1$ and go to **2.**

The index k indicates the number of generations.

GA comprises a set of individual elements and a set of biologically inspired operators defined over the population itself. According to evolutionary theories, only the most suited elements in a population are likely to survive and generate offspring, thus transmitting their biological heredity to new generations. In computing terms, a genetic algorithm maps a problem onto a set of strings, each string representing a potential solution of problem. The GA then manipulates the most promising strings in its search for improved solutions.

3.2 GA formulation

To design a genetic algorithm for the tuning of I-PD controller, certain problem-dependent algorithm elements need to be defined. They influence both the efficiency of the algorithm and the quality of its results.

Representation. Gray-code string representation is employed for candidate solution. Each of the parameters K_c, T_i and T_d is subject to interval constraints, for example, an interval $[a_l, a_u]$ is the constraints of the parameter K_c, and Δk_c is the corresponding discretization step. Values of K_c from the interval $[a_l, a_u]$ is represented as binary strings of $\lceil \log_2(\frac{a_u - a_l}{\Delta k_c}) \rceil$ bits. Three such strings are concatenated into a binary, representing a point in the parameter space to be searched by the algorithm.

Fitness function. The linear scaling method is employed as a fitness function. The fitness f_i of the individual X_i at generation k is computed through two steps. First, the order O_i $(i = 1, \cdots, p)$ of each individual X_i in the population is calculated by using the value of Equation (6). Then, the fitness f_i of the individual X_i is defined as follows:

$$\begin{aligned} f_i &= (F_{\max} - F_s \times (O_i - 1)) \\ F_s &= \frac{1}{p-1}(F_{\max} - F_{\min})) \end{aligned} \tag{3.2}$$

where p denotes the population size, $F_{\max}, F_{\min}$ denote the maximum and the minimum with the fitness values.

Genetic operators. The genetic operators applied in algorithm are uniform crossover, bit mutation and roulette wheel selection. Uniform crossover generates two offspring by exchanging a predefined number of alternate subsections between two parent strings. The recombination operator, mutation, is implemented as altering bit values at randomly selected string positions.

4. Experimental results

We consider a plant with transfer function.

$$G_2(s) = \frac{K_p}{1 + T_p s} \cdot e^{-Ls} \tag{4.1}$$

Let the uncertainty of the plant parameters be given by

$$\Theta = \{\theta \mid \underbrace{\begin{bmatrix} 0.8 \\ 5.6 \end{bmatrix}}_{\theta_l} \le \underbrace{\begin{bmatrix} K_p \\ T_p \end{bmatrix}}_{\theta} \le \underbrace{\begin{bmatrix} 1.2 \\ 8.4 \end{bmatrix}}_{\theta_u} \} \tag{4.2}$$

and the delay is $L = 1.0$. We assume that the set Q for controller is given by

$$Q = \{q \mid \underbrace{\begin{bmatrix} 0.1 \\ 1.0 \\ 0.25 \end{bmatrix}}_{q_l} \le \underbrace{\begin{bmatrix} K_c \\ T_i \\ T_d \end{bmatrix}}_{q} \le \underbrace{\begin{bmatrix} 20.0 \\ 100.0 \\ 100.0 \end{bmatrix}}_{q_u} \} \tag{4.3}$$

4.1 Off-line tuning

The result of off-line tuning is shown in **Table 1** for weighting constant $\rho = 0.5$, where J equals guaranteed robust performance.

Table 1 Result of off-line tuning

K_c	T_i	T_d	J
4.354	3.851	0.421	4.9663972

Fig.2 shows the step responses of closed loop system.

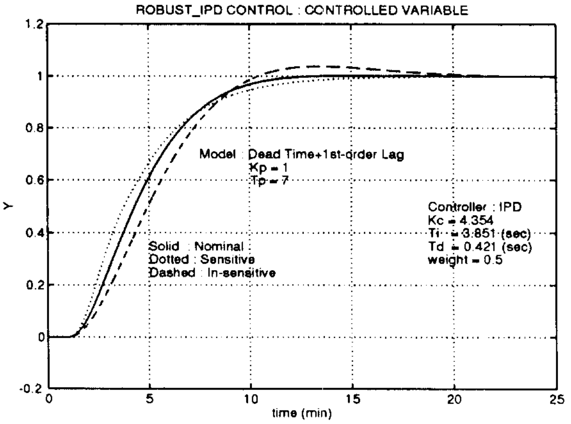

Fig.2 Step responses of closed loop system (off-line tuning)

4.2 On-line tuning using GA

In the simulation, each parameter K_c, T_i and T_d has 16 bits of precision, namely, each population member consists of a 48-bit string. Concerning the parameters of GA, we determine that the number of population is 50, the mutation rate is 0.20 and the control parameters of fitness are $(F_{\max}, F_{\min}) = (10, 1)$. The interval time of observing plant parameters is fixed as 5 sec. The initial values of plant parameters are randomly generated in each constraint condition.

Fig.3 shows the values of the performance measure J during the search. In the on-line tuning method, the robust stability is checked at the interval $\pm 10\%$ of the observed plant parameters.

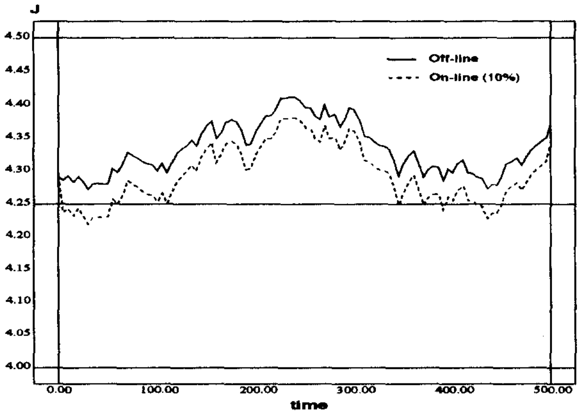

Fig.3 Values of the performance measure J

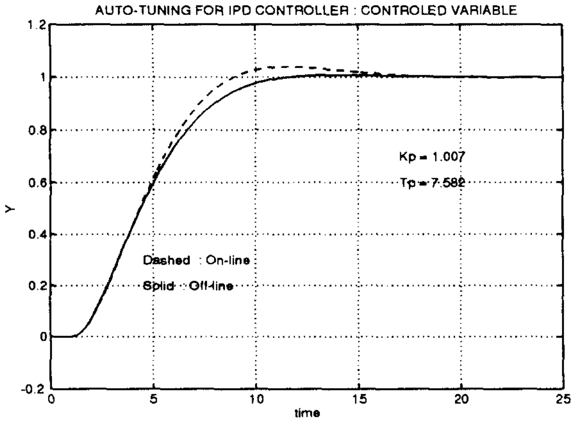

Fig.4 Step responses of closed loop system (contolled variable)

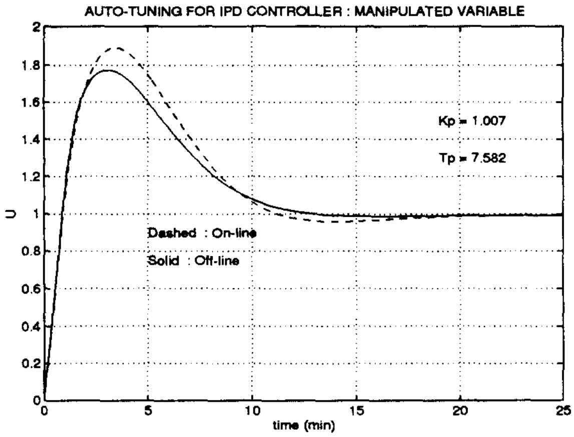

Fig.5 Step responses of closed loop system (manipulated variable)

Figs.4 and **5** show the step responses of closed loop system using the control parameters in 250 sec.

5. Conclusion

In this paper, we have given an on-line tuning method of robust minimax I-PD controller with penalty on manipulated valiable using genetic algorithm. We can recognize from several numerical simulations that the proposed method is effective for the control of time-varying plant. Our next work will be application of the present operation to many problems to investigate its general effectiveness.

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