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CONTROL INPUT RECONSTRUCTION USING REDUNDANCY UNDER TORQUE LIMIT

Jonghoon Park*, Wan-Kyun Chung**, Youngil Youm**

* Graduate Student in Dept. of Mech. Eng, Pohang University of Science and Technology (POSTECH)
** Professors in Dept. of Mech. Eng, Pohang University of Science and Technology (POSTECH)
Tel:+82-562-279-2844; Fax: +82-562-279-5899; E-mail: jhpark@risbot.rist.re.kr

Abstracts Various physical limitations which intrinsically exist in the manipulator control system, for example kinematic limits and torque limit, cause some undesirable effects. Specifically, when one or more actuators are saturated the expected control performance can not be anticipated and in some cases it induces instability of the system. The effect of torque limit, especially for redundant manipulators, is studied in this article, and an analytic method to reconstruct the control input using the redundancy is proposed based on the kinematically decomposed modeling of redundant manipulators. It results to no degradation of the output motion closed-loop dynamics at the cost of the least degradation of the null motion closed-loop dynamics. Numerical simulations help to verify the advantages of the proposed scheme.

Keywords Kinematic redundancy, Torque limit, Kinematically decomposed dynamic controller

INTRODUCTION

Application of a manipulator to a certain task is often made difficult due to the kinematic and/or hardware limitations which intrinsically exist in a manipulator control system. For example, the kinematic singularity and work space disconnectivity belongs to the kinematic limitations, and the hardware limitations include the joint travel, velocity, and torque limits. To overcome the various kinematic limitations kinematically redundant manipulators, which are given more degrees of freedom than required to specify a task position, were proposed and the control methods were developed, for example [7]. Some recent works addressed the problem of the joint travel and velocity limits in redundant manipulators [1, 2, 4].

In this article, we are to propose a dynamic control algorithm for redundant manipulators which can guarantee the linear decoupled closed loop dynamics in the null motion as well as in the task motion. Next, an analytic method will be proposed to reconstruct the control torque using redundancy which guarantees the expected task motion dynamics in spite of some torque saturation. Some numerical simulations will verify the efficiency and validity of the proposed schemes.

KINEMATICALLY-DECOMPOSED DYNAMIC CONTROLLER

Assume that a manipulator has n degrees of freedom and are to execute a task parametrized with m-vector. A task position is denoted by $\boldsymbol{p} \in \Re^n$ and a pose of the manipulator is specified by $\boldsymbol{q} \in \Re^n$. The redundant degrees of freedom is $r = n - m$. The homogeneous velocity is reparametrized with the minimal number, r, of parameters and denoted by $\dot{\boldsymbol{q}}_{null}$ called the null velocity. The control torque is $\boldsymbol{\tau} \in \Re^n$. On this setting, the dynamics of a redundant manipulators is assumed to be of the form

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}), \quad (1)$$

where $\boldsymbol{M}(\boldsymbol{q}) \in \Re^{n\times n}$ is the inertia matrix, and $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \Re^n$ includes all torque but the inertial torque. Then, by the help of the kinematically decomposed modeling, the standard joint space dynamics can be decomposed into the following (open-loop) output space and null space dynamics [5]

$$\boldsymbol{J}\boldsymbol{M}^{-1}\boldsymbol{\tau} = \ddot{\boldsymbol{p}} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} + \boldsymbol{J}\boldsymbol{M}^{-1}\boldsymbol{h} \quad (2)$$

$$\boldsymbol{N}^T\boldsymbol{M}^{-1}\boldsymbol{\tau} = \ddot{\boldsymbol{q}}_{null} - \dot{\boldsymbol{N}}^T\dot{\boldsymbol{q}} + \boldsymbol{N}^T\boldsymbol{M}^{-1}\boldsymbol{h}. \quad (3)$$

The matrices $\boldsymbol{R} \in \Re^{n\times m}$ and $\boldsymbol{N} \in \Re^{n\times r}$ constitutes the right singular matrices of the manipulator Jacobian $\boldsymbol{J} \in \Re^{m\times n}$, that is

$$\boldsymbol{J} = \boldsymbol{U}\left[\ \tilde{\boldsymbol{\Sigma}} \quad \boldsymbol{0}\ \right]\left[\ \boldsymbol{R} \quad \boldsymbol{N}\ \right]^T, \quad (4)$$

where $\boldsymbol{U} \in \Re^{m\times m}$ is the orthogonal matrix, and $\boldsymbol{\Sigma} \in \Re^{m\times m}$ is a diagonal matrix of the singular values of $\boldsymbol{J}$.

The kinematically decomposed dynamic controller has as its control law

$$\begin{aligned}\boldsymbol{\tau} &= \boldsymbol{M}\left[\boldsymbol{R}(\boldsymbol{J}\boldsymbol{R})^{-1}|\boldsymbol{N}\right]\begin{pmatrix}\boldsymbol{u}_p\\ \boldsymbol{u}_{null}\end{pmatrix}\\ &+ \boldsymbol{M}\frac{\mathrm{d}}{\mathrm{d}t}\left[\boldsymbol{R}(\boldsymbol{J}\boldsymbol{R})^{-1}|\boldsymbol{N}\right]\begin{pmatrix}\dot{\boldsymbol{p}}\\ \dot{\boldsymbol{q}}_{null}\end{pmatrix}\\ &+ \boldsymbol{h}.\end{aligned} \quad (5)$$

When there is no parametric uncertainties, it realizes the following decoupled and linear closed-loop dynamics:

$$\ddot{\boldsymbol{p}} = \boldsymbol{u}_p \quad (6)$$

$$\ddot{\boldsymbol{q}}_{null} = \boldsymbol{u}_{null}. \quad (7)$$

It remains to generate those two auxiliary control inputs $\boldsymbol{u}_p$ and $\boldsymbol{u}_{null}$ to stabilize each motion. The detailed description of the kinematically decomposed controller can be found in [5].

EFFECT OF TORQUE LIMITS ON CONTROL PERFORMANCE

However, when the calculated control torque by Eq. (5) can not be achieved within the torque limits, the expected performance by Eqs. (6) and (7) cannot be anticipated. In this section, the effect of torque limits, or torque saturations, on the control performance is to be analyzed.

Assume that s joints among n joints are saturated in torque, that is

$$\tau_{k_i} < \underline{\tau}_{k_i} \quad \text{or} \quad \tau_{k_i} > \overline{\tau}_{k_i} \tag{8}$$

for $i = 1, \ldots, s$ and k_i is an integer between 1 and n. Then the actual control input would be

$$\hat{\tau} = \begin{pmatrix} \tau_1 \\ \vdots \\ \underline{\tau}_{k_1} \\ \vdots \\ \overline{\tau}_{k_s} \\ \tau_n \end{pmatrix} \quad \begin{matrix} \\ \\ \leftarrow k_1\text{-th element} \\ \\ \leftarrow k_s\text{-th element} \\ \\ \end{matrix} \tag{9}$$

When the saturated control torque is input, we have the following closed-loop output space dynamics:

$$\ddot{p} = u_p - \overline{(JM^{-1})}\Delta\tau \tag{10}$$

$$= u_p - J\overline{M^{-1}}\Delta\tau \tag{11}$$

where $\overline{(JM^{-1})}$ is the $m \times s$ matrix which consists of k_i-th column of JM^{-1} associated to each limited joint, and $\Delta\tau$ is an $s-$vector whose $k-$th element is defined as $\tau_k - \underline{\tau}_k$(or $\overline{\tau}_k$) for k ranging over $k_1, \ldots, k_s$. Note that the matrix $\overline{(JM^{-1})}$ is equal to the matrix $J\overline{M^{-1}}$ where $\overline{M^{-1}}$ consists of k_i-th column of M^{-1} associated to each limited joint. The above output space dynamics follows because

$$\begin{aligned} JM^{-1}\hat{\tau} &= JM^{-1}(\tau - \tau + \hat{\tau}) \\ &= JM^{-1}\tau - JM^{-1}(\tau - \hat{\tau}) \\ &= u_p - \dot{J}\dot{q} + JM^{-1}h - J\overline{M^{-1}}\Delta\tau. \end{aligned} \tag{12}$$

As for the closed-loop null space dynamics, a similar result is obtained as follows

$$\ddot{q}_{null} = u_{null} - N^T\overline{M^{-1}}\Delta\tau. \tag{13}$$

Roughly speaking, the torque limits introduce errors in the output motion as well as in the null motion. Moreover, two motions are no more linear and decoupled.

RECONSTRUCTION ALGORITHM OF A SATURATED TORQUE

Since we have redundancy, we want to compensate the output space dynamic error, i.e. the second term in the right hand side of Eq. (10), by distributing it to the unlimited joints. That is,

$$J\hat{M}^{-1}\tilde{\tau} = J\overline{M^{-1}}\Delta\tau, \tag{14}$$

where $\hat{M}^{-1}$ is the $n \times (n-s)$ matrix consisting of $n-s$ columns of M^{-1} associated with the unlimited joints. By reconstructing the command torque by

$$\tau^* = \hat{\tau} + \tilde{\tau}_0 \tag{15}$$

where the $n-$ vector $\tilde{\tau}_0$ is the vector $\tilde{\tau}$ with its k_i-th($i = 1, \ldots, s$) element filled with 0, we can retain the original linear and decoupled closed-loop output space dynamics. That is,

$$\begin{aligned} JM^{-1}\tau^* &= JM^{-1}\hat{\tau} + JM^{-1}\tilde{\tau}_0 \\ &= u_p - \dot{J}\dot{q} + JM^{-1}h, \end{aligned} \tag{16}$$

from Eqs. (12) and (14).

The effect of the reconstructed torque on the null motion is

$$N^TM^{-1}\tau - N^TM^{-1}\tau^* = f - N^T\hat{M}^{-1}\tilde{\tau}, \tag{17}$$

where $f \triangleq N^T\overline{M^{-1}}\Delta\tau$. Note that the second term in the right hand side is due to the reconstructed torque.

In the meantime, $J\hat{M}^{-1}$ is a $m \times (n-s)$ matrix and it is assumed that $n - s \geq m$. Eq. (14) leads to a unique solution $\tilde{\tau} \in \Re^{n-s}$ if $n = m+s$. However, when $n > m+s$, there exist many $\tilde{\tau}$'s to satisfy the equation. A scheme to choose a unique solution is required for practical application. One obvious method is to choose one *nonsingular* square minor out of ${}_{n-s}C_m$ possible square minors of $J\hat{M}^{-1}$. But this may be blind choice not to consider optimality of a certain choice. Instead, we are to choose the unique solution which minimizes a possible null motion degradation due to the solution reconstruction. That is, one solution which minimizes

$$\left\| N^T\hat{M}^{-1}\tilde{\tau} - f \right\|$$

is attained as the unique solution.

Thus the problem to resolve a unique solution out of many solutions of Eq. (14) is stated as:

find a solution $\tilde{\tau}^* \in \Re^{n-s}$
which minimizes

$$\left\| N^T\hat{M}^{-1}\tilde{\tau} - f \right\| \tag{18}$$

subject to

$$J\hat{M}^{-1}\tilde{\tau} = d \tag{19}$$

where $d \triangleq J\overline{M^{-1}}\Delta\tau$.

Then the problem is of linear equality constrained least square problem type, and the solution is analytically found by (See Appendix.)

$$\begin{aligned} \tilde{\tau}^* &= \left(J\hat{M}^{-1}\right)^+ d + \left(N^T\hat{M}^{-1}Z\right)^+ \\ &\times \left(f - N^T\hat{M}^{-1}\left(J\hat{M}^{-1}\right)^+ d\right) \end{aligned} \tag{20}$$

where

$$Z = I - \left(J\hat{M}^{-1}\right)^+ J\hat{M}^{-1}. \tag{21}$$

Note that it is the unique minimal length solution if and only if Eq. (14) is consistent and the rank $\begin{bmatrix} J\hat{M}^{-1} \\ N^T\hat{M}^{-1} \end{bmatrix}$ is $n - s$.

Table 1. Parameters of the manipulator

	length $l(m)$	c.o.m. $r(m)$	mass $m(kg)$	inertia $I(kg \times m^2)$
1	0.3	0.15	20.0	0.15
2	0.25	0.125	10.0	0.0521
3	0.2	0.1	10.0	0.0333

NUMERICAL SIMULATIONS

A planar 3-dof manipulator is employed to show the effectiveness of the proposed algorithms. The parameters of the manipulator are shown in Table 1.

The controller used is the kinematically decomposed dynamic controller which was proposed. It is assumed that the state $(\boldsymbol{q}^T, \dot{\boldsymbol{q}}^T)^T$ is available from perfect sensors. The outer serve loop is the simple linear controller

$$\boldsymbol{u}_p = \ddot{\boldsymbol{p}}_d + \boldsymbol{K}_v(\dot{\boldsymbol{p}}_d - \dot{\boldsymbol{p}}) + \boldsymbol{K}_p(\boldsymbol{p}_d - \boldsymbol{p}) \quad (22)$$

$$\boldsymbol{u}_{null} = \ddot{\boldsymbol{q}}_{null,d} + \boldsymbol{K}_{null}(\dot{\boldsymbol{q}}_{null,d} - \dot{\boldsymbol{q}}_{null}) \quad (23)$$

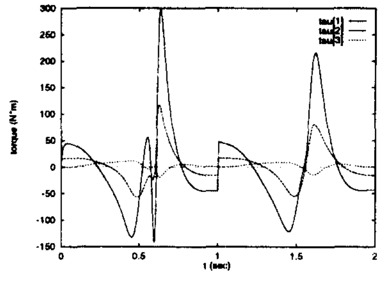
The desired task, $\boldsymbol{p}_d$, is to trace a circle two times in 2(seconds) and the perimetric distance is interpolated using quintic polynomial. The circle is centered at $(0.35, 0.0)^T(m)$ and its radius is $0.15(m)$. The desired null motion is generated in order to maximize the manipulability measure $m(\boldsymbol{q})$ [7] as following

$$\dot{\boldsymbol{q}}_{null,d} = \kappa \boldsymbol{N}^T \nabla m. \quad (24)$$

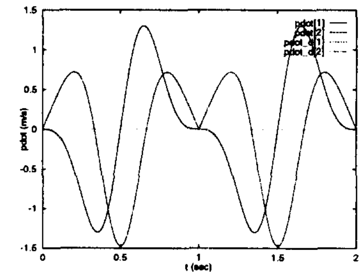
The gains used are: $\boldsymbol{K}_p$ = diag{1000, 1000}, $\boldsymbol{K}_v$ = diag{10, 10}, $\boldsymbol{K}_{null}$ = [100], and $\kappa = 30$. The detailed description of the kinematically decomposed controller can be found in [5].

First simulation shows the control result in Fig. 1 when there is no torque limit. The output motion traced the desired one well. The advantage of the kinematically decomposed controller is that the desired null motion was realized without much error. Almost all the conventional resolved acceleration controllers for redundant manipulators leaves the null motion uncontrolled, which can destabilize the zero dynamics of the manipulator system. Now it is assumed that the feasible torque for the first joint is $[-200.0, 200.0](N \times m)$. Fig. 2 shows control performance degradation because no consideration is taken about the effect of the limit. We can see the output velocity error in Fig. 2(b) as well as the null motion velocity error.

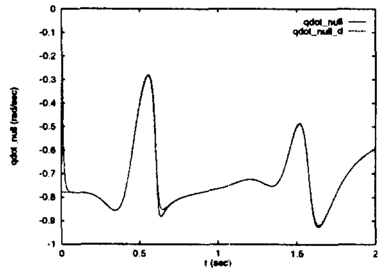
The proposed reconstruction algorithm is applied to compensate the torque saturation. The results is summarized in Fig. 3, where we can see that the reconstructed torque indeed realize the desired output velocity in spite that the first joint is saturated. Moreover, the null motion error was also reduced much, as can be seen in Fig. 3 (c).



(a) The control torque



(b) The desired and actual output velocity



(c) The desired and actual null velocity

Figure 1. The control performance of the kinematically decomposed dynamic controller with no torque limit

CONCLUSION

In this article, the kinematically decomposed dynamic controller was proposed, so that we can control the null motion as well as the output motion. Next, there was proposed the analytic method to reconstruct the control input when some torques are saturated. The expected task motion closed-loop dynamics are recovered under this reconstruction. Moreover, the reconstructed solution is the one which minimizes the error norm of the null motion dynamics. They are very analytic and efficient method as can be seen in the numerical simulations.

Note that the kinematically decomposed controller can be extended to the compliant motion controller [6]. Also, the reconstruction method can be generalized to the inverse kinematic algorithm.

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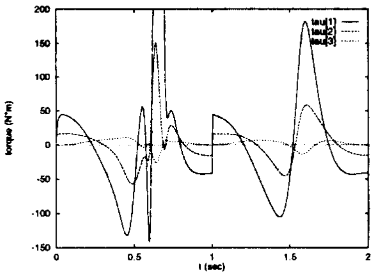
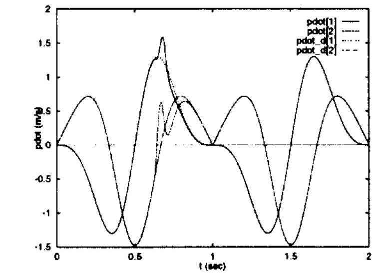
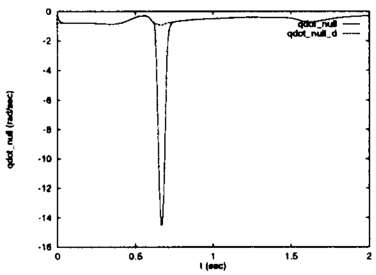


(a) The control torque

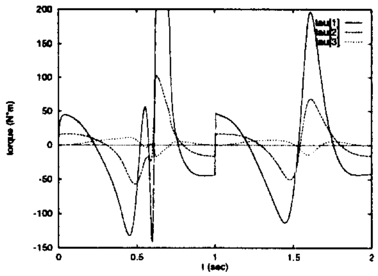


(b) The desired and actual output velocity

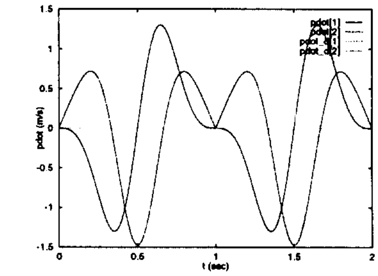


(c) The desired and actual null velocity

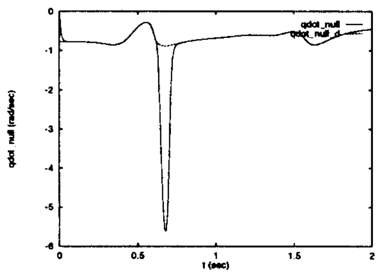
Figure 2. The control performance degradation with torque limit



(a) The control torque



(b) The desired and actual output velocity



(c) The desired and actual null velocity

Figure 3. The reconstructed control performance with torque limit

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APPENDIX A: LEAST SQUARE SOLUTION WITH EQUALITY CONSTRAINTS

The linear equality constrained least squares problem, called LSE hereinafter, addresses:

> Given an $m_1 \times n$ matrix $\boldsymbol{C}$ of rank k_1, an m_1−vector $\boldsymbol{d}$, and $m_2 \times n$ matrix $\boldsymbol{E}$, and an m_2−vector f, among all $n-$ vectors $\boldsymbol{x}$ that satisfy
> $$\boldsymbol{C}\boldsymbol{x} = \boldsymbol{d} \tag{25}$$
> find one that minimizes
> $$\|\boldsymbol{E}\boldsymbol{x} - \boldsymbol{f}\|. \tag{26}$$

The characteristics and a solution of the following problem are well analyzed in Lawson and Hanson [3]. The following theorem summarized all the analytical results.

THEOREM **A.1** *Assuming Eq. (25) is consistent, Problem LSE has a unique minimal length solution given by*

$$\boldsymbol{x}^* = \boldsymbol{C}^+\boldsymbol{d} + (\boldsymbol{E}\boldsymbol{Z})^+ (\boldsymbol{f} - \boldsymbol{E}\boldsymbol{C}^+\boldsymbol{d}) \tag{27}$$

where

$$\boldsymbol{Z} = \boldsymbol{I}_n - \boldsymbol{C}^+\boldsymbol{C}. \tag{28}$$

The vector $\boldsymbol{x}^*$ is the unique solution vector for Problem LSE if and only if Eq. (25) is consistent and the rank of $\begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{E} \end{bmatrix}$ is n.

A full proof of the above theorem and various numerical algorithms can be found in [3].