ON ORDER DETERMINATION IN IDENTIFICATION OF CLOSED-LOOP SYSTEMS

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Abstract Identification of a process in closed-loop control system is an important problem in practice. This paper deals with parameter estimation using input-output data of the process operating in a closed-loop system. It is necessary to determine orders and delay-time to get consistent estimators by least square method for input-output data collected from the process. The authors considered a problem to determine delay-time in the condition that orders were known, in last KACC. So we extend the range to determine orders and delay-time in this paper.
Keywords Order Determination, Closed-Loop System, Identification, Model Selection

1. INTRODUCTION

It is an important problem to determine orders and delay-time of time-series models in order to get available parameter estimates of the process in identification. There are some useful methods for determining them. AIC, pole-zero placement and data matrix are available to determine the orders. For delay-time determination, there is a method by checking estimates of high order ARX models. These methods have been known to be useful for the data collected in open-loop experiments. But most of the processes are controlled through feedback loop and it is often necessary in practice to identify the processes operating in closed-loop control systems. It is suspicious that the above methods are useful for the data collected in closed-loop experiments, because the input has correlation with the output in these cases. Therefore, determination method of orders and delay-time should be firstly considered in identification of the process operating in closed-loop system. In the last KACC, we suggested a method to determine delay-time of the process in closed-loop system with known orders. In this paper, we extend the range to determine orders and delay-time in closed-loop identification.

First, the authors investigate the difference between estimated properties by ARX and DARX model with various orders and delay-times, and second we consider a method to determine orders of the process in closed-loop identification, and last we practically apply the method to the data collected from the real plant operating in closed-loop system.

2. PROBLEM STATEMENT

Consider a closed-loop control system shown in figure1.

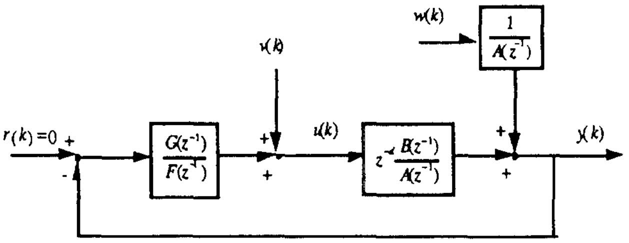

Fig.1. Closed-loop system

$$y(k) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u(k) + \frac{1}{A(z^{-1})}w(k) \quad (1)$$

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b}$$

$$u(k) = \frac{G(z^{-1})}{F(z^{-1})}\{r(k) - y(k)\} + v(k) \quad (2)$$

$$F(z^{-1}) = 1 + f_1 z^{-1} + f_2 z^{-2} + \cdots + f_{n_f} z^{-n_f}$$

$$G(z^{-1}) = g_0 + g_1 z^{-1} + g_2 z^{-2} + \cdots + g_{n_g} z^{-n_g}$$

where $u(k)$ is input to the process and $y(k)$ is output.

$w(k)$ is additive noise to the process output and $v(k)$ is additive noise to process input. Both of them are Gaussian white noises which have the following properties.

$$E\{w(k)\} = 0, \quad E\{w(j)w(k)\} = \sigma_w^2 \delta_{jk}$$

$$E\{v(k)\} = 0, \quad E\{v(j)v(k)\} = \sigma_v^2 \delta_{jk}$$

$$E\{w(j)v(k)\} = 0$$

The input $u(k)$ to the process is determined from output feedback by a regulator. That is $r(k)=0$. The output $y(k)$ and the input $u(k)$ are available to estimate process parameters $\{a_1,a_2,\cdots,a_{n_a},b_0,b_1,\cdots,b_{n_b}\}$.

It is assumed that the structure of regulator is given, but orders and delay-time of the process aren't given in advance. Least square method using ARX model is adopted as identification method. We will consider how to determine orders and delay-time of the process suitably among estimated models of various orders and delay-time, and will show the procedure to determine orders and delay-time.

3. SELECTION OF MODEL STRUCTURES

There are two relations between the input and output data of the process in closed-loop. One is defined by the process dynamic and the other is by the controller. We encounter the difficulties in selection of the identification model due to this fact.

Now, consider this in view of the spectrum. We have equation (3) about power spectrum of the input-output data

$$\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)}=\frac{\left|z^{-d}B(z^{-1})F(z^{-1})\right|^2_{z=e^{j\omega}}\sigma_v^2+\left|F(z^{-1})\right|^2_{z=e^{j\omega}}\sigma_w^2}{\left|A(z^{-1})F(z^{-1})\right|^2_{z=e^{j\omega}}\sigma_v^2+\left|G(z^{-1})\right|^2_{z=e^{j\omega}}\sigma_w^2}\quad(3)$$

If the noise $v(k)$ is intense in comparison with the noise $w(k)$, namely $\sigma_v^2>>\sigma_w^2$, equation (3) tends to equation (4).

$$\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)}\approx\left|z^{-d}\frac{B(z^{-1})}{A(z^{-1})}\right|^2_{z=e^{j\omega}}\quad(4)$$

If the noise $v(k)$ does not exist or it has less influence on the process than the noise $w(k)$, namely $\sigma_v^2<<\sigma_w^2$, equation (3) tends to equation (5).

$$\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)}\approx\left|\frac{F(z^{-1})}{G(z^{-1})}\right|^2_{z=e^{j\omega}}\quad(5)$$

We notice that spectral analysis cannot distinguish between the properties of the process and the controller from equations (3) and that the results of spectral analysis depends on intensities of external noises from equations (4) and (5). Therefore we must evaluate intensities of noises before spectral analysis.

In time-series model representation of the input-output data, we have equation (6) from equation (1) and (2).

$$\begin{aligned}\{L(z^{-1})A(z^{-1})+M(z^{-1})G(z^{-1})\}y(k)=\\ \{z^{-d}L(z^{-1})B(z^{-1})-M(z^{-1})F(z^{-1})\}u(k)\\ +L(z^{-1})w(k)+M(z^{-1})F(z^{-1})v(k)\quad(6)\end{aligned}$$

where $L(z^{-1})$ and $M(z^{-1})$ are arbitrary real-coefficient polynomials. In identification of time-series model, this equation shows that the estimated relation between input-output data depends on the structure of used models.

The process dynamic provides the relation between input sequence $\{u(k-d)\cdots u(k-d-n_b)\}$ and output sequence $\{y(k),\cdots,y(k-n_a)\}$, and the controller constitutes $u(k)$ based on $\{u(k-1)\cdots u(k-n_f)\}$ and $\{y(k)\cdots y(k-n_g)\}$. As $y(k)$ is represented by $\{y(k-1),\cdots,y(k-\hat{n}_a)\}$ and $\{u(k)\cdots u(k-\hat{n}_b)\}$ in ARX model given in (7), high order ARX model can extract both properties of the process and the controller.

$$\hat{A}(z^{-1})y(k)=\hat{B}(z^{-1})u(k)+\tilde{w}(k)\quad(7)$$

$$\hat{A}(z^{-1})=1+\hat{a}_1z^{-1}+\cdots+\hat{a}_{\hat{n}_a}z^{-\hat{n}_a}$$

$$\hat{B}(z^{-1})=\hat{b}_0+\hat{b}_1z^{-1}+\cdots+\hat{b}_{\hat{n}_b}z^{-\hat{n}_b}$$

Therefore it will be significant to compare the property of estimated ARX model with the results of spectrum analysis.

$$\hat{A}(z^{-1})y(k)=z^{-\hat{d}}\hat{B}(z^{-1})u(k)+\tilde{w}(k)\quad(8)$$

DARX model in (8) represents the relation between $\{u(k-\hat{d})\cdots u(k-\hat{d}-\hat{n}_b)\}$ and $\{y(k),\cdots,y(k-\hat{n}_a)\}$. It is possible to estimate only the property of the process if we can select the same delay-time and orders in models as true process's ones. These are shown in figure2.

As mentioned above, we must pay special attention to estimates in closed-loop identification by ARX and DARX models. To determine delay-time and orders, different interpretations for the estimated results are needed in closed-loop identification. In general, low order part of identification model represents the property of the controller, and high order part represents the process in identification of process control system whose delay-time is comparatively long.

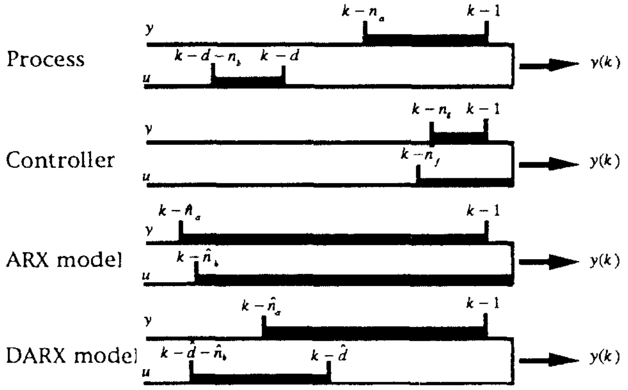

Fig.2. Relations between input and output data

4. DETERMINATION OF ORDERS AND DELAY-TIME

In this section, a determination method of orders and delay-time in closed-loop experiment will be discussed considering the above. Before it, we shall review the way to determine them in the open-loop experiment.

There are some procedures to determine orders and delay-time of the process operating in open-loop experiment. AIC method is basic one of them. In this method, orders and delay-time are selected by evaluating maximum likelihood for various sets of orders and delay-time. In identification using primary model (7), delay-time and orders will be determined as following. Provided that orders of $\hat{A}(z^{-1})$ and $\hat{B}(z^{-1})$ are large, appropriate orders are selected by AIC and delay-time is determined comparing with each estimated parameters of $\hat{B}(z^{-1})$, because parameters for the low order parts of $\hat{B}(z^{-1})$ are estimated close to 0. After determining delay-time in this way, we can use DARX model and estimate the parameters of DARX model with selected delay-time. Checking AIC or pole-zero placement of the estimated model (8), we can get appropriate orders. These procedures are available because input-output data shows only property of the process in this case.

However, the above methods are not always useful for the process in closed-loop. When the noise $v(k)$ is large enough compared with $w(k)$, orders and delay-time are determined in the same manner as in open-loop experiment. But in other cases, when the noise $v(k)$ is small, we cannot use the above method straightforward. Therefore, the level of the noise $v(k)$ should be evaluated previously in frequency domain. It can be performed by comparing $\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)}$ with $\left|\frac{F(z^{-1})}{G(z^{-1})}\right|^2_{z=e^{j\omega}}$. After that, taking account of the level of the noise $v(k)$, we should treat the data more carefully than in the open-loop experiment.

In the case that $v(k)$ is small, the procedure of determining them are as follows.

Step1: Estimate ARX model fitting to the input-output data, when $\hat{n}_a = \hat{n}_b$. Calculate prediction errors for various model orders, and select some models which give small prediction errors among them. Those orders are denoted $\hat{n}$.

Step2: Evaluate the estimated models with selected orders $\hat{n}$ in frequency domain. By comparing the frequency characteristics of the estimated models with $\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)}$, we narrow candidates of the orders $\hat{n}$.

Step3: Select delay-time $\hat{d}$ such that $\hat{d} = \hat{n}_b$. Estimate DARX models with delay-time $\hat{d}$ as varying orders $\hat{n}_a = \hat{n}_b$ in (8) and select some DARX models which gives small prediction errors among them in the same manner as step 1. Then we can hold some candidates of DARX models with orders and delay-time $(\hat{n}_a, \hat{n}_b, \hat{d})$.

Step4: Evaluate frequency characteristics of DARX models selected in step 3 comparing with $\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)}$ and narrow candidates of $(\hat{n}_a, \hat{n}_b, \hat{d})$. Difference between estimated DARX model's frequency characteristics and power spectrum of input-output data will give us some information about the level of noises.

Step5: Check pole-zero placements of selected models $(\hat{n}_a, \hat{n}_b, \hat{d})$ in step 4, and reduce orders $\hat{n}_a$ and $\hat{n}_b$.

5. SIMULATION EXAMPLES

We will show an example of the procedure. It is applied to the data collected from the process control system whose delay-time is comparatively long.

The input and output data are shown in figure3.

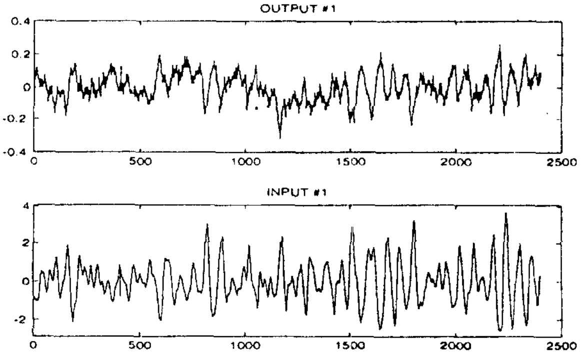

Fig.3: Input-Output data

Step1: First, we use ARX model and evaluate prediction errors of the model $(\hat{n}_a = \hat{n}_b = 1 \cdots 50)$. Loss function defined by equation (9) is calculated.

$$J = \frac{1}{N}\sum_{k=1}^{N}\left(y(k) - \hat{y}(k)\right)^2 \tag{9}$$

where N is the number of data and $\hat{y}(k)$ is defined by the following equation.

$$\hat{y}(k) = -\sum_{i=1}^{\hat{n}_a} \hat{a}_i y(k-i) + \sum_{j=0}^{\hat{n}_b} \hat{b}_j u(k - \hat{d} - j) \tag{10}$$

where $\hat{a}_i$ and $\hat{b}_j$ are estimates. Prediction errors are shown in figure4.

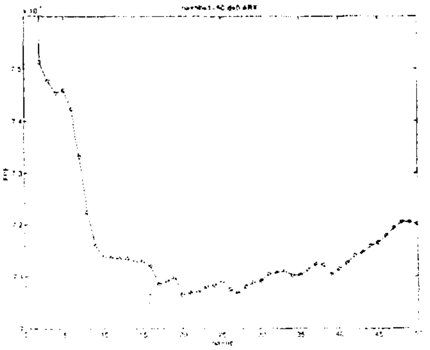

Fig.4: Prediction errors of ARX models

Step2: Figure4 shows that candidates of the order are selected around $\hat{n} = 20$. Then we calculate power spectrum of the models whose orders are around $\hat{n} = 20$. Frequency characteristics of the estimated models $\left|\frac{\hat{B}(z^{-1})}{\hat{A}(z^{-1})}\right|^2_{z=e^{j\omega}}$ and power spectrum of input-output data $\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)}$ are shown in figure5.

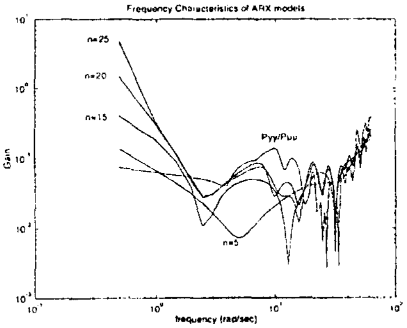

Fig.5: Frequency Characteristics of ARX models

Step3: From figure5, we can confirm that the appropriate candidates are selected. Then we assume $\hat{d} = \hat{n}$ and fit the input-output data to DARX model of the orders around $\hat{n}$. Prediction errors of $\hat{n}_a = \hat{n}_b = 20$, $\hat{d} = 0 \cdots 40$ are shown in figure6.

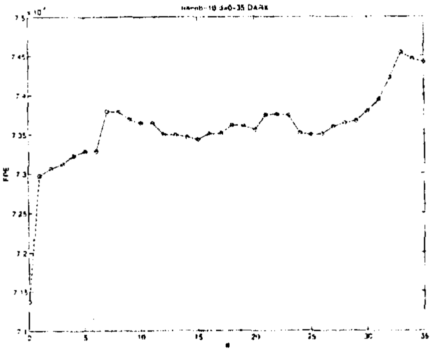

Fig.6:Prediction errors of DARX models

Figure6 shows the availability of estimated delay-time, because prediction errors are small enough around the delay-time $\hat{d} = 20$. By evaluating prediction errors again, candidates of the set $(\hat{n}_a, \hat{n}_b, \hat{d})$ are narrowed.

Step4: Frequency characterisitics of the estimated models $\left|z^{-\hat{d}}\frac{\hat{B}(z^{-1})}{\hat{A}(z^{-1})}\right|^2_{z=e^{j\omega}}$ and power spectrum of the input-output data $\frac{\Phi_{yy}(\omega)}{\Phi_{uu}(\omega)}$ are compared in frequency domain. By considering the level of noise $v(k)$, candidates $(\hat{n}_a, \hat{n}_b, \hat{d})$ are narrowed again . Figure7 shows frequency characteristics of some estimated models and power spectrum of the input-output data.

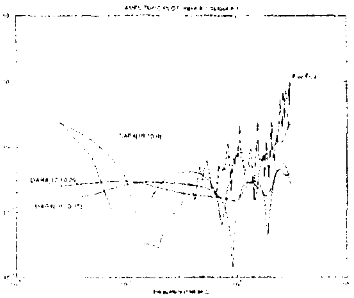

Fig.7: Frequency Characteristics of DARX models

Step5: By checking the pole-zero placement of the candidates $(\hat{n}_a, \hat{n}_b, \hat{d})$, orders are determined. Some examples of pole-zero placement are shown in figure8.

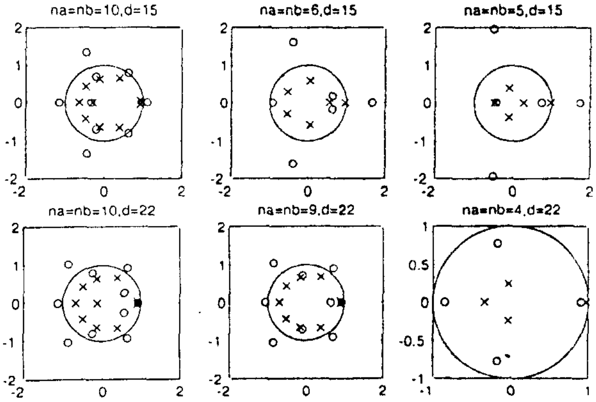

Fig.8: Pole-zero placement of the estimated models

6. CONCLUSION

In this paper, the authors considered the procedure of determining orders and delay-time in identification of the process in closed-loop control systems. By applying the procedure to the real plant, the availability of the procedure were assured.

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