Tracking a Constant Speed Maneuvering Target Using IMM Method

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Abstracts An interacting multiple model (IMM) approach which merges two hypotheses for the situations of constant speed and constant acceleration model is considered for the tracking of maneuvering target. The inflexibility of uncertainty which lies in the kinematic constraint (KC) represented by pseudomeasurement noise variance is compensated by the mixing of estimates from two model *Kalman* tracker: one with KC and one without KC. The numerically simulated tracking performance is compared for the "great circular like turning" trajectory maneuver by the single model tracker with constant speed KC and two model tracker which is developed in this paper.

Keywords Constant speed maneuver, Interacting multiple model, *Kalman* filter, Kinematic constraint

1. INTRODUCTION

The kinematic constraint (KC) gives effectual additional *a priori* information to the target tracking filter when its motion is delivered within the given constraints specially for the maneuver [1]. And the incorporation of additional constraint as a pseudomeasurement was proposed [1,2] by virtue of the computational rationale to manage the nonlinear structures of most KCs in extended forms. The constraint of constant speed which is one of the most possible cases of maneuver was simulated with various tracking filter structures [2,3,4].

However, most of the actual maneuvering motions can not always satisfy the given specific additional KC during the tracking periods. Thus, despite the noticeable achievements in the state estimations by the kinematically constrained *Kalman* filter (KCKF), the KC sometimes causes mismodeling to the filter formulations and finally results in track loss. Some formulations selecting a large initial pseudomeasurement noise variance and decreasing it with an empirical rates [2,3] gives reasonable management of KC under the satisfaction of KC by the target dynamics. But the abrupt outbreak of target states from the specified KC during the maneuver still makes the tracking filter mismodeled. Therefore, the rigid adherence of KC according to the invariant existence once it is incorporated into the filter structure needs an adaptive adjustment or reasonable reformulation to maintain the benefits of KC for the maneuver.

One of the noble approaches to the maneuvering target tracking problems represented as interacting multiple model (IMM) algorithm is the assumptions of multiple *Gaussian* hypotheses which can be possibly modeled mathematically and merged in fixed depth with certain degree of freedom [5,7]. IMM algorithm enables to construct the other target model to compensate the single KCKF. Thus two different model filters are batched in parallel structure and their estimations are combined according to the corresponding model probability.

The general dynamic model formulation for this target state estimation problem is reviewed in section 2 and the incorporation of kinematic constraint for constant speed maneuver and its new management with IMM are described in section 3. Section 4 shows the simple numerical comparison of tracking performances between single modeled kinematically constrained Kalman filter (KCKF) and two modeled parallel Kalman filters using IMM algorithm (KCKF-IMM). The conclusions and future aspects of this study are discussed in section 5.

2. PROBLEM FORMULATION

2.1 Modeling of Target

The linear dynamic model for the state estimation in discrete time-invariant system can be given by

$$X_{k+1}^i = \Phi^i X_k^i + \Psi^i \omega_k^i \qquad (1)$$

where $X_k^i = R^{9\times 1} (i = 1,2)$ is the state vector with position, velocity and acceleration and $\omega_k^i \in N(0, Q^i)$ is process noise sequence at time k for i-th model. For the computational simplicity, the system is assumed in three dimensional Cartesian coordination and the control input which is not observable directly from the tracker is included in the state of acceleration for convenience. Thus the state vector X_k^i and its transition matrix $\Phi^i \in R^{9\times 9}$ in (1) can be written with sampling period T, respectively as

$$X_k^i = [x_k^i \;\; \dot{x}_k^i \;\; \ddot{x}_k^i \;\; y_k^i \;\; \dot{y}_k^i \;\; \ddot{y}_k^i \;\; z_k^i \;\; \dot{z}_k^i \;\; \ddot{z}_k^i]^T \qquad (2)$$

$$\Phi^i = \begin{bmatrix} F^i & 0 & 0 \\ 0 & F^i & 0 \\ 0 & 0 & F^i \end{bmatrix}, \text{ where } F^i = \begin{bmatrix} 1 & T & 0.5T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

and the process noise gain matrix $\Psi^i = R^{9\times 3}$ is given as

$$\Psi^i=\begin{bmatrix} G^i & 0 & 0 \\ 0 & G^i & 0 \\ 0 & 0 & G^i \end{bmatrix}, \text{where } G^i=\begin{bmatrix} 0.5T^2 \\ T \\ 1 \end{bmatrix} \quad (4)$$

2.2 Measurement and Estimation of Target States

The measurement by discrete single sensor in polar coordinate frame can be written as

$$z_{k+1}=H_k(X^i_{k+1})+v_{k+1} \quad (5)$$

where z_{k+1} is the measurement vector with range, azimuth and elevation. Despite the correlation between the *nongaussian* measurement noise and the measurement vector components, $v_{k+1}\in N(0,\Theta)$ is usually assumed to be Gaussian and uncorrelated with measurement components for the convenience in computing probabilities in later derivations. And the process and measurement noises are assumed to be uncorrelated so that

$$E[\omega^i_k(v_j)^T]=0, \quad \text{for } \forall\ k,j \quad (6)$$

3. KC MANAGEMENT WITH IMM

3.1 KC for Constant Speed Motion.

To introduce the kinematic constraint (KC) in the filtering problem, the selected set of state variables for the constraint is treated as an additional pseudomeasurement [1,2,3]. The rationale for this method is due to the fact that the pseudomeasurement can be structured linearized form algebraically. The noise variance of pseudomeasurement interpreted as an uncertainty of additional KC relaxes the error from the linearization of nonlinear KCs. Therefore, the additional KC can be easily incorporated to the cost function of the least square filter as an additional *a priori* information. The improvement in target tracking problem with this idea is mainly attributed to the reduced errors of the estimated state variables which are relevant to the given KC.

The nonlinear KC is defined as

$$C(X_k)=0 \quad (7)$$

And (7) can be linearized using the predicted estimation of optimal filter which is numerically reasonably close to the true state. So by using the first two terms of *Taylor* series, (7) can be written as

$$C(X_k)=C(X_{k|k-1})+\left.\frac{\partial C}{\partial X}\right|_{X=X_{k|k-1}}\cdot(X_k-X_{k|k-1}) \quad (8)$$

By defining the uncertainty factor of KC as κ_k, we obtain

$$\kappa_k\equiv\left.\frac{\partial C}{\partial X}\right|_{X=X_{k|k-1}}\cdot X_k-[\left.\frac{\partial C}{\partial X}\right|_{X=X_{k|k-1}}\cdot X_{k|k-1}-C(X_{k|k-1})] \quad (9)$$

κ_k is zero-mean white Gaussian with variances as

$$E[\kappa_k(\kappa_k)^T]=\Omega\delta_{kj} \quad (10)$$

Defining the vectors for the velocities and accelerations in Cartesian coordination, respectively like

$$\vec{V}_k\equiv[\dot{x}_k\ \ \dot{y}_k\ \ \dot{z}_k]^T \quad (11)$$

$$\vec{A}_k\equiv[\ddot{x}_k\ \ \ddot{y}_k\ \ \ddot{z}_k]^T \quad (12)$$

We can get the KC of constant speed as

$$C(X_k)=\vec{V}_k\cdot\vec{A}_k=\dot{x}_k\cdot\ddot{x}_k+\dot{y}_k\cdot\ddot{y}_k+\dot{z}_k\cdot\ddot{z}_k=0 \quad (13)$$

By extending (13) as in (9) and calculating the *Jacobian* of KC about the predicted estimates it leads to

$$\bar{\bar{C}}_{k|k-1}=\bar{C}_{k|k-1}\cdot X_k+\kappa_k \quad (14)$$

where
$$\bar{C}_{k|k-1}\equiv\left.\frac{\partial C}{\partial X_k}\right|_{X_k=X_{k|k-1}} \quad (15)$$

and
$$\bar{\bar{C}}_{k|k-1}\equiv\bar{C}_{k|k-1}\cdot X_{k|k-1}-\vec{V}_{k|k-1}\cdot\vec{A}_{k|k-1} \quad (16)$$

3.2 Two Model Tracker Using IMM

Considering the 2 models for the maximum *a posteriori* estimates with the Gaussian conditional *pdf* of X_k.and using *Chapmann-Kolmogorov* equation for the discrete *pdf*, the conditional *pdf* of the state X_k can be written as

$$p[X_k|Z^k]=\sum_{i=1}^{2}p[X_k|M^i_k,Z^k]p[M^i_k|Z^k] \quad (17)$$

where M^1_k is the model parameter of KCKF and M^2_k is that of KF. Z^k is the set of cumulated measurement vectors up to time *k* so that $p[M^i_k|Z_k]$ can be a probability of model *i*(=1,2) during the time $0\le t\le k$. For the maximum *a posteriori* estimation, the given model conditioned *pdf* in (17) can be written as

$$p[X_k|M^i_k,Z^k]=p[X_k|M^i_k,z_k,Z^{k-1}] \quad (18)$$

Using the *Bayes* theorem the *pdf* of (18) can be rewritten as

$$p[X_k|M^i_k,z_k,Z^k]=p[X_k|M^i_k,Z^{k-1}]\frac{p[z_k|M^i_k,X_k]}{p[z_k|M^i_k,Z^{k-1}]} \quad (19)$$

The first term of right side in (19) is the conditional Gaussian *pdf* for the one-step prediction in *i*-th model. And for the model transition from time *t=k*-1 to *t=k* and interacting (mixing) with the rest of the models including itself, it can be decomposed using the *Champmann-Kolmogorov* equation as follows

$$p[X_k|M^i_k,Z^{k-1}]$$
$$=p[|X_k|M^i_k,\{\sum_{j=1}^{2}p^j[X_{k-1}|Z^{k-1}]p[M^j_{k-1}|M^i_k,Z^{k-1}]\} \quad (20)$$

The mixing of models is approximated by the probabilitically weighted summations of two model conditioned estimates and their covariances at past through time. and Gaussian *pdf* is approximated by matching the moment of two *Gaussians* to single Gaussian *pdf*.

The second term of the summation in (20) can be decomposed by using the *Markovian* chain like

$$p[M^j_{k-1}|M^i_k,Z^{k-1}]=\frac{\Lambda_{ij}p[M^j_{k-1}|Z^{k-1}]}{p[M^i_k|Z^{k-1}]} \quad (21)$$

And the term Λ_{ij} stands for the probability of the model transition from model *j* to *i* like $p[M^i_k|M^j_{k-1},Z^{k-1}]$ and

$$p[M^i_k|Z^{k-1}]=\sum_{j=1}^{N}\Lambda_{ij}p[M^j_{k-1}|Z^{k-1}] \quad (22)$$

Therefore, using these results (20) leads to

$$p[X_k|M^i_k,Z^{k-1}]$$

$$= p[X_k|M_k^i,(\sum_{j=1}^{N} p^j[X_{k-1}|Z^{k-1}]\Lambda_{ij}\frac{p[M_{k-1}^j|Z^{k-1}]}{p[M_k^i|Z^{k-1}]} \quad (23)$$

$p[M_{k-1}^i|Z^{k-1}]$ of (30) is updated by the likelihood function using the residuals and its covariances as

$$p[M_k^i|Z^{k-1}]=\frac{1}{p[Z^k]}\lambda_k^i\sum_{j=1}^{2}\Lambda_{ij}p[M_{k-1}^j|Z^{k-1}] \quad (24)$$

The term λ_k^i which is $p[z_k|M_k^i,Z^{k-1}]$ can be interpreted as the *likelihood* function for the i-th model.

3.3 Algorithm of Two Model Tracker

Step 1. ***mixing of two different estimates***

$$X_{k-1|k-1}^{oi}=\sum_{j=1}^{2}\frac{1}{c_i}\Lambda_{ij}X_{k-1|k-1}^{j}\eta_{k-1}^{j} \quad (25.1)$$

where η_{k-1}^j is the updated normalized model probability from (24) and $\bar{c}_i$ is the other normalization constant from

$$\bar{c}_i=\sum_{j=1}^{N}\Lambda_{ij}\eta_{k-1}^{i} \quad (25.2)$$

And the error covariances are mixed using the filtered estimates from KC and KCKF and the mixed estimates from (25.1) as follows

$$P_{k-1|k-1}^{oi}=\sum_{j=1}^{2}\frac{1}{\bar{c}_i}\Lambda_{ij}[P_{k-1|k-1}^{j}+(X_{k-1|k-1}^{j}-X_{k-1|k-1}^{oi})(X_{k-1|k-1}^{j}-X_{k-1|k-1}^{oi})^{T}]\eta_{k-1}^{i} \quad (25.3)$$

Step 2. ***Parallel Filtering***

The *Kalman* estimators for KCKF(i=1) and KF(i=2)

$$X_{k|k-1}^{i}=\Phi^{i}X_{k-1|k-1}^{i} \quad (25.1)$$

$$P_{k|k-1}^{i}=\Phi^{i}P_{k-1|k-1}^{i}(\Phi^{i})^{T}+Q^{i} \quad (25.2)$$

$$\bar{H}_k^i=\left.\frac{\partial H_k(X_k^i)}{\partial X_k^i}\right|_{X_k^i=X_{k|k-1}^i} \quad (25.3)$$

$$\Gamma_k^i=\bar{H}_k^iP_{k|k-1}^i(\bar{H}_k^i)^T+\Theta^i \quad (25.4)$$

$$K_k^i=P_{k|k-1}^i(\bar{H}_k^i)^T(\Gamma_k^i)^{-1} \quad (25.5)$$

$$\alpha_k^i=z_k-H_k^i(X_{k|k-1}^i) \quad (25.6)$$

$$X_{k|k}^i=X_{k|k-1}^i+K_k^i\alpha_k^i \quad (25.7)$$

$$P_{k|k}^i=[I-K_k^i\bar{H}_k^i]P_{k|k-1}^i \quad (25.8)$$

where (25.3) is the *Jacobian* matrix of H_k^i for the extended form of measurement to transform Cartesian to polar coordination. And for KCKF

$$z_k^1=[z_k \quad \bar{\bar{C}}(x_{k|k-1})]^T \quad (25.9)$$

$$\bar{H}_k^1=[\left.\frac{\partial H_k(X_k)}{\partial X_k}\right|_{X_k=X_{k|k-1}^1} \quad \bar{C}(X_{k|k-1}^1]^T \quad (25.10)$$

$$\Theta^1=\begin{bmatrix}\Theta & 0\\ 0 & \Omega\end{bmatrix} \quad (25.11)$$

Step 3. ***Model Probability Update***

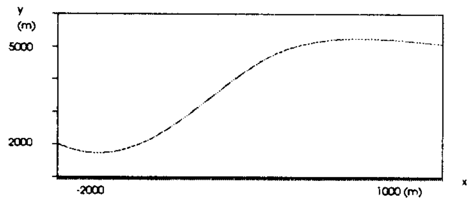


Fig. 1. Trajectory of great circular maneuvering target

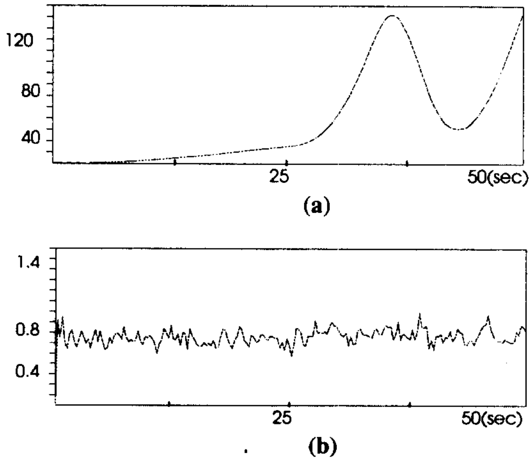


Fig. 2. Position RMSE of (a) KCKF and (b) KCKF-IMM.

$$\lambda_k^i=\frac{1}{\sqrt{2\pi|\Gamma_k^i|}}\exp[-0.5(\alpha_k^i)^T(\Gamma_k^i)^{-1}(\alpha_k^i)] \quad (26.1)$$

$$\eta_k^i=\frac{1}{\rho}\lambda_k^i\bar{c}_i, \text{where } \rho=\sum_{j=1}^{2}\lambda_k^j\bar{c}_i \quad (26.2)$$

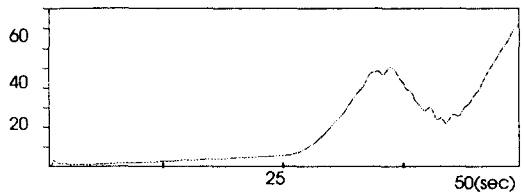
Step 4. ***Combination of Estimates***

$$X_{k|k}=\sum_{j=1}^{2}X_{k|k}^{j}\eta_k^j \quad (27.1)$$

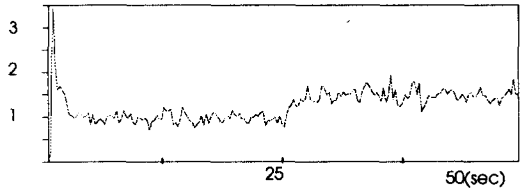
$$P_{k|k}=\sum_{j=1}^{2}\eta_k^j[P_{k|k}^j+(X_{k|k}^j-X_{k|k})(X_{k|k}^j-X_{k|k})^T] \quad 27.2)$$

4. SIMULATION RESULTS

The comparison between KCKF and KCKF-IMM is performed assuming single sensor in polar coordination. The target motion for this purpose is generated by mixed maneuvers with two different types of constant acceleration(CA) motions. For the first 100 sample periods, the CA maneuver satisfies the constraint of constant speed like $\vec{A}\cdot\vec{V}=0$. And for the last 100 samples, the CA maneuver does not satisfy the constraint like $\vec{A}\cdot\vec{V}\neq 0$. The initial position of target is assumed fixed altitude with range 2236 (m) and for the onset of outbreak from KC is initiated acceleration input 28.3 (m/sec^2). All filters in this simulation have same process and measurement noise covariances as $Q=1$ and $\Theta=1$, respectively. And the uncertainty of KC for KCKF is also selected as $\Omega=1$. The sampling interval is 0.25(sec.) Figure 1 shows the target position trajectory in Cartesian coordinate frame. As expected,

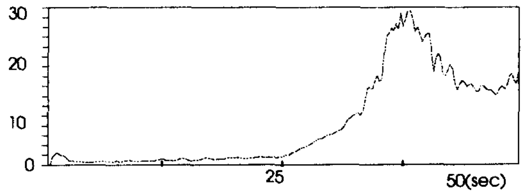


(a)

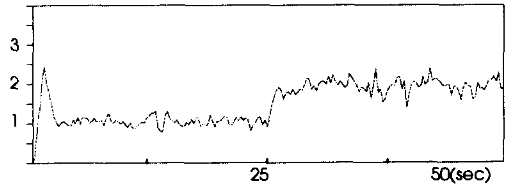


(b)

Fig. 3. Velocity RMSE of (a) KCKF and (b) KCKF-IMM



(a)



(b)

Fig 4. Acceleration RMSE of (a) KCKF and (b) KCKF-IMM

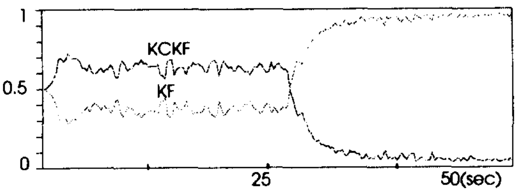


Fig. 5. Model probabilities in KCKF-IMM

the KCKF results in track loss for the last half filtering process according to the mismodeling. However, as we can see in Figure 2, Figure 3 and Figure 4, KCKF-IMM adapts the model matching by virtue of interacting model between the two model KCKF and KF. Figure 5 shows model transition standing for giving up about 90% of adherence of KC after last half periods.

5. CONCLUSIONS

An IMM approach consisting of a constant speed and constant acceleration model is proposed to track a maneuvering target. The additional KC which can be obtained from the nature of maneuver dynamics was employed as a pseudomeasurement with *Kalman* filter literature. The suitability of given KC was judged by the model *likelihoodness* which is updated by measurement error residual and its covariance.

IMM implementation for the single target with this approach promised the flexibility of additional constraint as previously mentioned. And this formulation can be applied for the multi-KC or multi-sensor target tracking by joining the data fusion or association which will be the next stage of this study.

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