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Conditions for Manipulation of Object with Multiple Contacts by Intelligent Jig System

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Abstracts A manipulation of a multiple contacted object by a Rotational Base and Single-jointed Finger mechanism(RBSF mechanism) is discussed. The manipulation is characterized by multiple contacts on an object and large motions of the object with sliding contacts. The kinematics and dynamics allowing sliding at multiple contacts are explored. The conditions for manipulation of an object at multiple contacts by the RBSF mechanism, which cannot exert arbitrary contact forces because it has a fewer number of joints than is required for active control, is presented.

Keywords Robotics, Intelligent Jig, dynamics, multiple contact, sliding contact

1. INTRODUCTION

An important issue in flexible intelligent systems of manufacturing and assembly is to automate their operations. Automated operations are currently limited to welding and painting. The task of positioning workparts has not been automated.

We have worked to develop an intelligent jig system which can automatically manipulate heavy workparts and make the relocation of the jig unnecessary, and have proposed a new kind of mechanism, a Rotational Base and Single-jointed Finger mechanism, or RBSF mechanism (Yashima,1993). The base rotates around a center axis, and each finger has a single prismatic joint with one degree of freedom. The object is manipulated by the cooperation of the fingers and base. The manipulation of the RBSF mechanism is characterized by multiple contacts between the object and the fingers, and large motions of the object using sliding at the contact points. This manipulation can increase the load carrying capability due to multiple contacts on the object.

There are some problems encountered in manipulating the object by the RBSF mechanism. The kinematic and dynamic analysis of the object with multiple contacts has not been discussed because of the analytical difficulties resulting from multiple contacts. The RBSF mechanism cannot exert arbitrary contact forces at the object because it has fewer joints than are required for active control of contact forces. In this paper, we show the RBSF mechanism system is determinant, using linear complementality problem. In addition, we show the conditions for generating the object acceleration and angular acceleration.

2. KINEMATICS AND DYNAMICS

2·1 *Kinematic Constraints*

Consider m-fingers in contact with a rigid object as shown in Fig.1. Let Σ_o and Σ_θ be the coordinate frames at the object center of mass and at the rotational axis of the base, respectively. Fix a coordinate frame on the object surface at the ith contact point, Σ_{ci}. Let the x axis of Σ_{ci} be defined as the tangent to the object surface, while the z axis is in the direction of inward normal, and ${}^{ci}\boldsymbol{t}_{ci}, {}^{ci}\boldsymbol{n}_{ci} \in \Re^2$ are unit vectors in the direction of x, z axis relative to Σ_{ci}, respectively. Define ${}^B\boldsymbol{x}_o$, ${}^B\boldsymbol{x}_{fi} \in \Re^2$ to be the positions of the origins of Σ_o and Σ_{ci} respectively relative to the inertial frame Σ_B, and φ, $\theta \in \Re$ to be the orientation of Σ_o and Σ_θ, respectively. The rotation matrices ${}^B\boldsymbol{R}_o$, ${}^B\boldsymbol{R}_{ci}$, ${}^B\boldsymbol{R}_\theta \in \Re^{2\times2}$ specify the orientations of Σ_o, Σ_{ci}, and Σ_θ relative to Σ_B, respectively. Let ${}^o\boldsymbol{c}_{oi}(\xi_{oi}) \in \Re^2$ be a position of ith contact point relative to Σ_o and $\xi_{oi} \in \Re$ be a relative position of ith contact point on the object surface. Unless otherwise specified, the notation is as follows: vectors are represented by the upper left index showing the frame.

Each contact point between a fingertip and an object must be maintained during sliding in order to realize the manipulation of the object. The position of the ith fingertip is identical to that of the ith contact point on the object surface, as follows:

$$\boldsymbol{x}_o + \boldsymbol{R}_o\boldsymbol{c}_{oi}(\xi_{oi}) = \boldsymbol{x}_{fi}. \qquad (1)$$

On the other hand, the position of the ith fingertip can be written as:

$$\boldsymbol{x}_{fi} = \boldsymbol{R}_\theta [r_i \ l_i]^T \qquad (2)$$

where r_i and l_i are x and z values of ith fingertip position relative to Σ_θ.
Differentiating Eqs.(1) and (2) gives

$$\dot{\boldsymbol{x}}_o + \omega_o \boldsymbol{D}\boldsymbol{R}_o\boldsymbol{c}_{oi} + \boldsymbol{R}_o\left(\frac{\partial \boldsymbol{c}_{oi}}{\partial \xi_{oi}}\right)\dot{\xi}_{oi} = \boldsymbol{R}_\theta \begin{bmatrix} -l_i & 0 \\ r_i & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{l}_i \end{bmatrix} \qquad (3)$$

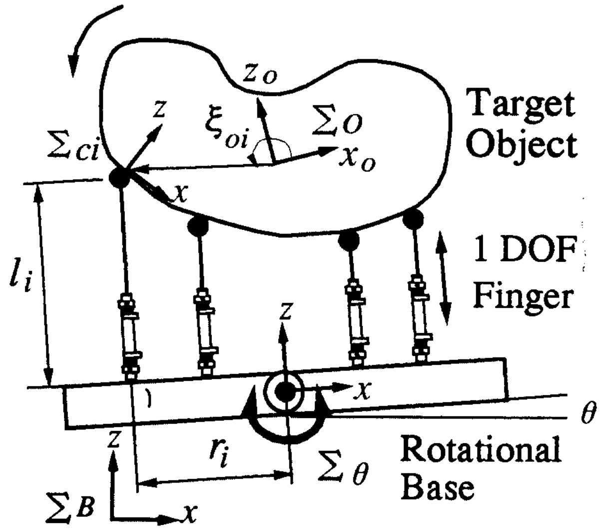

Fig. 1 Manipulating an object using RBSF mechanism.

where ω_o is the angular velocity of the object relative to Σ_B, and $\boldsymbol{D}$ is the orthogonal rotation matrix.

Aggregating Eq.(3) for $i = 1 \sim m$ using the Jacobian matrix $\boldsymbol{J} \in \Re^{2m \times (m+1)}$ for the RBSF mechanism gives kinematic constraint equation:

$$\boldsymbol{G}^T \dot{\boldsymbol{X}}_o + \boldsymbol{A}_o \dot{\boldsymbol{\xi}}_o = \boldsymbol{J}\dot{\boldsymbol{\Theta}} \tag{4}$$

where $\dot{\boldsymbol{X}}_o \triangleq \left[\dot{\boldsymbol{x}}_o^T,\ \omega_o\right]^T \in \Re^3$, $\dot{\boldsymbol{\xi}}_o \triangleq \left[\dot{\xi}_{o1} \cdots \dot{\xi}_{om}\right]^T \in \Re^m$, $\dot{\boldsymbol{\Theta}} \triangleq \left[\dot{\theta}\ \dot{l}_1 \cdots \dot{l}_m\right]^T \in \Re^{m+1}$, and $\boldsymbol{A}_{oi} \triangleq \boldsymbol{R}_o\left(\frac{\partial \boldsymbol{c}_{oi}}{\partial \xi_{oi}}\right) \in \Re^2$. The matrix $\boldsymbol{A}_o \in \Re^{2m \times m}$ is the block diagonal matrix of $\boldsymbol{A}_{oi}$, and we define

$$\boldsymbol{G}^T \triangleq \begin{bmatrix} \boldsymbol{E}_2 & \boldsymbol{D}\boldsymbol{R}_o\boldsymbol{c}_{o1} \\ \cdots & \cdots \\ \boldsymbol{E}_2 & \boldsymbol{D}\boldsymbol{R}_o\boldsymbol{c}_{om} \end{bmatrix} \in \Re^{2m \times 3}.$$

2·2 Kinematic Phase

Differentiating ξ_{oi} represents a relative velocity between the ith fingertip and the ith contact point on the object surface, *i.e.* the sliding velocity. $\dot{\xi}_{oi} = 0$ indicates that there is non-sliding at the ith contact point and $\dot{\xi}_{oi} \neq 0$ indicates that there is sliding at the ith contact point. The kinematic phase, therefore, can be divided into three phases in accordance with the number of non-sliding contact points, s.

- When s$\geq$2, we can prove that there is non-sliding at all contact points, and the angular velocity of the object equals that of the base. Thus the object cannot rotate relative to the base. We call this phase ω_o *constraint phase*.
- When s=1, only one of the contact points is not in sliding contact, and the object rotates around that contact point. We call this phase *point constraint phase*. Though this phase can be used to cause the object to rotate through gravity, we do not consider the manipulation using this phase in this paper.
- When s=0, there is sliding at all contact points, and the angular velocity of the object does not equal that of the base. We call this phase ω_o *unconstraint phase*. Because the object can rotate relative to the base, this phase can be used to cause large rotational motions of the object utilizing sliding.

2·3 Dynamics and Frictional Constraints

The dynamics of the object at which the contact forces, $\boldsymbol{F}$, are applied by m-fingers can be written as:

$$\boldsymbol{M}_o \ddot{\boldsymbol{X}}_o + \boldsymbol{Q}_o = \boldsymbol{G}\boldsymbol{F} \tag{5}$$

where

$$\boldsymbol{M}_o \triangleq \begin{bmatrix} m_o\boldsymbol{E}_2 & \\ & I_o \end{bmatrix} \in \Re^{3\times3},\quad \boldsymbol{Q}_o \triangleq \begin{bmatrix} -m_o\boldsymbol{g} \\ 0 \end{bmatrix} \in \Re^3,$$

$m_o\boldsymbol{E}_2$ is the diagonal object mass matrix, I_o is the inertia of the object, $\boldsymbol{g}$ is the gravity vector, and $\boldsymbol{F} \triangleq \left[\boldsymbol{f}_1^T \cdots \boldsymbol{f}_m^T\right]^T \in \Re^{2m}$.

On the other hand, the dynamics of the RBSF mechanism can be written as:

$$\boldsymbol{M}_R(\boldsymbol{\Theta})\ddot{\boldsymbol{\Theta}} + \boldsymbol{Q}_R(\boldsymbol{\Theta}, \dot{\boldsymbol{\Theta}}) = \boldsymbol{\tau} - \boldsymbol{J}^T\boldsymbol{F} \tag{6}$$

where $\boldsymbol{M}_R \in \Re^{(m+1)\times(m+1)}$ is the inertia matrix of the RBSF mechanism, $\boldsymbol{Q}_R \in \Re^{m+1}$ is a vector of the gravity, Coriolis, and centrifugal forces, and $\boldsymbol{\tau} \in \Re^{m+1}$ is the vector of input base torques and input finger joint forces.

In accordance with the kinematic phase described above in section 2.2, the contact force, $\boldsymbol{F}$, can be expressed as follows:

- ω_o Constraint Phase

There is no sliding at any contact point, and the contact forces are constrained to lie within the friction cones on the assumption of Coulomb's Law. The contact forces can be written as:

$$\boldsymbol{F} = \boldsymbol{W}_n \boldsymbol{F}_n \tag{7}$$

where $\boldsymbol{W}_n \triangleq$ block diag $([\boldsymbol{n}_{11}\ \boldsymbol{n}_{12}] \ldots [\boldsymbol{n}_{m1}\ \boldsymbol{n}_{m2}]) \in \Re^{2m\times 2m}$,
$\boldsymbol{F}_n \triangleq [f_{n11}\ f_{n12} \cdots f_{nm1}\ f_{nm2}]^T \in \Re^{2m}$, $\boldsymbol{n}_{i1,2}$ is unit vector along the edge of the friction cone, and $f_{ni1,2}$ is the component of friction forces acting in the direction of the friction cone edge($f_{ni1,2} > 0$).

- ω_o Unconstraint Phase

There is sliding at all contact points. Generally the coefficient of kinetic friction is smaller than the coefficient of static friction. Therefore, to simplify the problem, we assume that kinetic frictional forces are negligible. The contact forces can be written as:

$$\boldsymbol{F} = \boldsymbol{W}_n \boldsymbol{F}_n \tag{8}$$

where $\boldsymbol{F}_n \triangleq [f_{n1} \ \cdots \ f_{nm}]^T \in \Re^m$ is the vector of the normal contact force, and we define

$$\boldsymbol{W}_n \triangleq \boldsymbol{R}_c \boldsymbol{n}_c \in \Re^{2m \times m} \tag{9}$$

where $\boldsymbol{R}_c$ and $\boldsymbol{n}_c$ are the block diagonal matrices of $\boldsymbol{R}_{ci}$ and ${}^{ci}\boldsymbol{n}_{ci}$.

Substituting Eq.(7) or Eq.(8) into Eqs.(5) and (6) yields the motion equations of the object and the RBSF mechanism in accordance with kinematic phase.

$$\boldsymbol{M}_o \ddot{\boldsymbol{X}}_o + \boldsymbol{Q}_o = \boldsymbol{G}\boldsymbol{W}_n \boldsymbol{F}_n \tag{10}$$

$$\boldsymbol{M}_R \ddot{\boldsymbol{\Theta}} + \boldsymbol{Q}_R + \boldsymbol{J}^T \boldsymbol{W}_n \boldsymbol{F}_n = \boldsymbol{\tau} \tag{11}$$

$$\boldsymbol{F}_n \geq 0. \tag{12}$$

2.4 Model Determinancy

Multifingered hands that have a sufficient number of joints are able to generate the desired object acceleration and angular acceleration by giving the joint torques obtained from the motion planning. However, this is not true in the case of the RBSF mechanism because of the specialization of this mechanism.

The forward problem of finding unique $\boldsymbol{F}_n$ and $\ddot{\boldsymbol{X}}_o$ in terms of $\boldsymbol{\tau}$ (*i.e.* model determinancy), in ω_o unconstraint phase, is discussed. First, consider the velocity and acceleration constraints for surface normal at the contact point. Eliminating $\dot{\boldsymbol{\xi}}_o$ from the velocity constraint Eq.(4) gives the following equation :

$$\boldsymbol{G}_1^T \begin{bmatrix} \dot{\boldsymbol{X}}_o \\ \dot{\boldsymbol{\Theta}} \end{bmatrix} = 0 \tag{13}$$

where we define

$$\boldsymbol{G}_1 \triangleq \begin{bmatrix} \boldsymbol{G}\boldsymbol{R}_c \boldsymbol{n}_c \\ -\boldsymbol{J}^T \boldsymbol{R}_c \boldsymbol{n}_c \end{bmatrix} \in \Re^{(m+4)\times m}. \tag{14}$$

Equation (13) shows the constraint conditions for surface normals at the contact points on the object and fingertips. Differentiating Eq.(13) yields the acceleration constraints as:

$$\boldsymbol{G}_1^T \begin{bmatrix} \ddot{\boldsymbol{X}}_o \\ \ddot{\boldsymbol{\Theta}} \end{bmatrix} + \dot{\boldsymbol{G}}_1^T \begin{bmatrix} \dot{\boldsymbol{X}}_o \\ \dot{\boldsymbol{\Theta}} \end{bmatrix} = 0. \tag{15}$$

Let $\boldsymbol{\gamma}_n \in \Re^m$ and $\boldsymbol{\delta}_n \in \Re^m$ be the relative velocities and the relative accelerations between the object and the fingertips at the contact points in the normal direction, respectively. Equations(13) and (15) can be rewritten as:

$$\boldsymbol{\gamma}_n \triangleq \boldsymbol{G}_1^T \dot{\boldsymbol{X}} \tag{16}$$

$$\boldsymbol{\delta}_n \triangleq \boldsymbol{G}_1^T \ddot{\boldsymbol{X}} + \dot{\boldsymbol{G}}_1^T \dot{\boldsymbol{X}} \tag{17}$$

where $\dot{\boldsymbol{X}} \triangleq \begin{bmatrix} \dot{\boldsymbol{X}}_o^T & \dot{\boldsymbol{\Theta}}^T \end{bmatrix}^T \in \Re^{m+4}$.

If the contacts between the object and the fingertips are maintained, $\boldsymbol{\gamma}_n = 0$ and $\boldsymbol{\delta}_n = 0$ are satisfied.

Combining Eqs.(10) and (11) yields

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{Q} = \boldsymbol{u} + \begin{bmatrix} \boldsymbol{G}\boldsymbol{W}_n \\ -\boldsymbol{J}^T \boldsymbol{W}_n \end{bmatrix} \boldsymbol{F}_n \tag{18}$$

where

$$\boldsymbol{M} \triangleq \begin{bmatrix} \boldsymbol{M}_o & \\ & \boldsymbol{M}_R \end{bmatrix} \in \Re^{(m+4)\times(m+4)},$$

$$\boldsymbol{Q} \triangleq \begin{bmatrix} \boldsymbol{Q}_o \\ \boldsymbol{Q}_R \end{bmatrix} \in \Re^{m+4}, \text{ and } \boldsymbol{u} \triangleq \begin{bmatrix} 0 \\ \boldsymbol{\tau} \end{bmatrix} \in \Re^{m+4}.$$

Substituting Eqs.(9) and (15) into Eq.(18) gives

$$\boldsymbol{M}\ddot{\boldsymbol{X}} = \boldsymbol{Q}_u + \boldsymbol{G}_1 \boldsymbol{F}_n \tag{19}$$

where $\boldsymbol{Q}_u \triangleq \boldsymbol{u} - \boldsymbol{Q} \in \Re^{m+4}$.

Since $\boldsymbol{M}$ is the positive definite, Eq.(19) can be rearranged as:

$$\ddot{\boldsymbol{X}} = \boldsymbol{M}^{-1}\boldsymbol{Q}_u + \boldsymbol{M}^{-1}\boldsymbol{G}_1 \boldsymbol{F}_n. \tag{20}$$

Substituting Eq.(20) into Eq.(17) gives

$$\boldsymbol{G}_1^T \boldsymbol{M}^{-1} \boldsymbol{G}_1 \boldsymbol{F}_n + \boldsymbol{p} = \boldsymbol{\delta}_n \tag{21}$$

where $\boldsymbol{p} \triangleq \dot{\boldsymbol{G}}_1^T \dot{\boldsymbol{X}} + \boldsymbol{G}_1^T \boldsymbol{M}^{-1} \boldsymbol{Q}_u$.

The normal contact force, f_{ni}, and the relative acceleration, δ_{ni}, at ith contact point satisfy the following conditions: (a) If the contact is maintained, then $f_{ni} > 0$, $\delta_{ni} = 0$ are satisfied, (b) If the contact is violated, then $f_{ni} = 0$, $\delta_{ni} > 0$ are satisfied. Then $\boldsymbol{F}_n = [f_{n1} \ \cdots \ f_{nm}]^T$ and $\boldsymbol{\delta}_n = [\delta_{n1} \ \cdots \ \delta_{nm}]^T$ satisfy the complementarity conditions as follows:

$$\boldsymbol{F}_n \geq 0, \ \boldsymbol{\delta}_n \geq 0, \ \boldsymbol{F}_n^T \boldsymbol{\delta}_n = 0. \tag{22}$$

The problem of determining $\boldsymbol{F}_n$ satisfying the constraint conditions of Eqs.(21) and (22) can be transformed into the quadratic programming problem(Lötstedt,1981):

$$\min_{\boldsymbol{F}_n} \quad \frac{1}{2}\boldsymbol{F}_n^T \boldsymbol{G}_1^T \boldsymbol{M}^{-1} \boldsymbol{G}_1 \boldsymbol{F}_n + \boldsymbol{F}_n^T \boldsymbol{p} \tag{23}$$

$$\text{subj. to} \quad \boldsymbol{F}_n \geq 0. \tag{24}$$

Therefore, we can determine $\boldsymbol{F}_n$ and $\ddot{\boldsymbol{X}}_o$ uniquely for a given $\boldsymbol{\tau}$, when $\boldsymbol{G}_1 \in \Re^{(m+4)\times m}$ has full column rank.

3. DYNAMIC ANALYSIS OF CONTACT PROBLEMS

In this section, the dynamic analysis of contact problems that both of positive and negative object angular acceleration can be generated in ω_o unconstraint phase, is discussed.

The position of the contact point relative to the frame Σ_o is expressed as

$$\boldsymbol{c}_{oi} = -{}^o\boldsymbol{R}_{ci}(\alpha_{ni}\boldsymbol{n}_{ci} + \alpha_{ti}\boldsymbol{t}_{ci}) \tag{25}$$

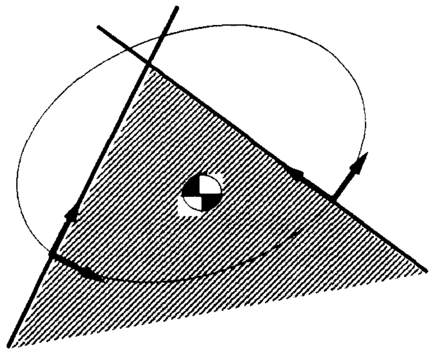

Fig. 2 The convex polygon

where α_{ni}, α_{ti} are scalar variables.

The motion equation for the rotation is written from Eqs.(5), (8), (9) as follows:

$$I_o\dot{\omega}_o = \sum(\boldsymbol{D}\boldsymbol{R}_o\boldsymbol{c}_{oi})^T\boldsymbol{R}_{ci}\boldsymbol{n}_{ci}f_{ni} \qquad (26)$$

Substituting Eq.(25) into Eq.(26) yields

$$I_o\dot{\omega}_o = -\sum\alpha_{ti}f_{ni} \qquad (27)$$

The conditions for causing the arbitrary convex shaped object to generate the positive and negative angular acceleration are shown as follows:

(1) The conditions for keeping contact : $f_{ni} \geq 0$

(2) The object center of mass is located within the convex polygon which is surrounded by the normals at the two contact points, as shown in Fig.2, to satisfy the following equation:

$$sgn(\dot{\omega}_o) = -sgn(\alpha_{ti}f_{ni}) \qquad (28)$$

Fig.3(a) shows the case that the object center of mass is within the convex polygon which is surrounded by the normals at the both ends of the fingers.

Fig.3(b) shows the case that the object center of mass is within the area which is surrounded by the normals at the contact point 2 and 3. Even if the the object center of mass is not within the convex polygon which is surrounded by the normals at the both ends of the fingers, the angular acceleration of the desired rotational direction can be derived.

4. CONCLUSIONS

We have analyzed the kinematics and dynamics of the object with multiple contacts by RBSF mechanism. We have confirmed that the RBSF mechanism model is determinant. We have obtained the conditions for causing the object to generate the angular acceleration.

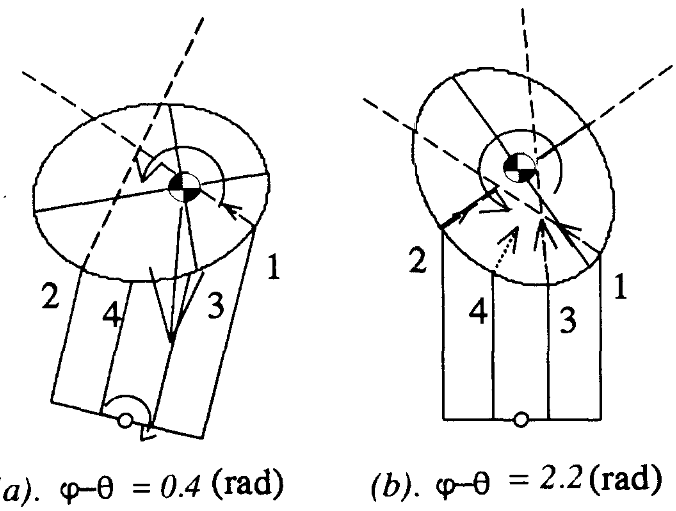

Fig. 3 The object is the ellipse of which the center of mass is largely shifted. The solid arrows indicate the acceleration ($\ddot{x}_o$) or the angular acceleration ($\dot{\omega}_o$ and $\ddot{\theta}$). The dotted arrows indicate the contacting forces. The broken arrows indicate the normal.

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