Abstracts A new target adaptive guidance (TAG) algorithm is proposed to engage the aim point formed by adding a bias to the information from an infrared (IR) seeker for improving passive homing guidance effectiveness. The TAG algorithm utilizes an observability enhancing mid-course guidance algorithm to obtain convergent estimates of state variables involved particularly in range channel otherwise unavailable from passive sensors. Simulation results indicate that the TAG algorithm provides improved terminal effectiveness without computational complexities.

Keywords TAG, Passive homing, Observability enhancing guidance

I. INTRODUCTION

When an infrared (IR) seeker with a gyro-stabilized detector [1] is used to track an aerial target, the seeker tends to track the hottest spot of the target such as a point on engine exhaust plume rather than fuselage so that the homing guidance algorithm based on the measurements from the seeker could provide low guidance effectiveness to the missile system. This effect becomes significant for small missiles with short fusing distance. In order to improve the effectiveness, it is needed to modify the homing guidance command by adding a bias-type command adapted to geometric relations of the missile - target homing engagement scenarios. The adaptation includes change of the guidance aim point from the point tracked by the seeker to a vulnerable point of the target located on the fuselage, and identification of the target flight direction. The resulting guidance is known as target adaptive guidance (TAG).

This paper proposes a new and accurate TAG algorithm for passive homing missile applications. The proposed algorithm utilizes the guidance law of [3] as a mid-course guidance scheme to generate an observability enhancing trajectory and to obtain accurate guidance parameters involved in the TAG algorithm as well as the mid-course guidance algorithm. The planar guidance law established in this paper can be easily extended to three-dimensional problems. This paper is organized as follows. In Section II, presented are system descriptions and a new TAG algorithm followed by brief review of the observability enhancing guidance law and a nonlinear state estimator called the modified gain extended Kalman filter (MGEKF)[4]. Closed-loop guidance is formulated by placing the proposed guidance law and the estimator in cascade and the guidance performance is tested by a series of Monte Carlo simulation runs in Section III. Finally, conclusions are presented in Section IV.

II. TAG WITH BEARINGS-ONLY MEASUREMENTS

A. System Descriptions

Passive homing guidance using an IR seeker can be formulated as target tracking with bearings-only measurements [3, 5]. In this section, dynamics of the planar bearings-only target tracking are established in Cartesian coordinates. The six-element state vector \( \mathbf{x} \) under the assumption of a zero-lag missile autopilot is composed of missile to target relative position, relative velocity, and target acceleration vectors. The continuous system dynamics are represented as follows.

\[
\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} + D\mathbf{u}_T
\]

where \( \mathbf{x} = (X, Y, \dot{X}, \dot{Y}, A_{r_t}, A_{R_t})^T \),

\[
A = \begin{bmatrix} 0 & I_2 & 0 \\ 0 & 0 & I_2 \\ 0 & 0 & -2I_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -I_2 \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ I_2 \end{bmatrix}
\]

\( A_m = (A_{m_t}, A_{m_R})^T \) denotes the missile acceleration vector, and \( U_T = (U_{r_t}, U_{R_t})^T \) denotes the target acceleration command vector. Target accelerations \( A_{r_t} \) and \( A_{R_t} \) are developed through a first-order lag
with a time constant of \( \frac{1}{\lambda} \). The engagement geometry is depicted in Fig. 1 with seeker line-of-sight (LOS) angle for the hottest spot of the target and the aim point of TAG located \( \ell \) meters ahead of the spot. The aim point is assumed to be along the target velocity vector neglecting the target angle of attack.

![Fig. 1. Missile - target engagement geometry](image)

The LOS angle measurement at time \( t = t_i \) can be written as

\[
z(t_i) = \beta(t_i) + \nu(t_i)
= \tan^{-1}\left(\frac{X(t_i)}{Y(t_i)}\right) + \nu(t_i) \tag{3}
\]

where \( \nu(t_i) \) is a white Gaussian noise sequence with zero-mean and variance of \( \sigma^2 \). The measurement noise includes range-independent and scintillation noises. Due to detection of temperature difference between the target and environment, the effect of IR noises on target tracking is known to be insignificant compared to that of radar seeker noises which are sensitive to changes in radar cross section induced by engagement geometry changes.

The intensity of IR measurement is inversely proportional to the square of range such that the signal to noise ratio (SNR) is in general low at target lock-on, and it increases as range decreases. For lock-on-before-launch missiles, target lock-on is assisted by human operators and the low SNR may be compensated by human intelligence, however this becomes a problem for lock-on-after-launch missiles. Threshold for target lock-on should be carefully determined from the relation between SNR and false alarm rate for such missiles. [9] describes characteristics of IR radiation through the earth’s atmosphere and their modeling methods.

B. TAG and Mid-course Guidance

The objective of TAG is to generate a bias-type command based on \( \dot{\beta}_d \) of Fig. 1 besides conventional homing guidance commands. The bias-type command is required to guide the missile to the aim point located on the target fuselage. \( \beta_d \) and \( \dot{\beta}_d \) can be calculated from Fig. 1 as

\[
\begin{pmatrix}
\beta_d \\
\dot{\beta}_d \\
\end{pmatrix} = 
\begin{pmatrix}
tan^{-1}\left(\frac{\ell \sin \sigma_T}{r + \ell \cos \sigma_T}\right) \\
\frac{\ell (r \dot{\sigma}_T \cos \sigma_T - r \sin \sigma_T + \ell \dot{\sigma}_T)}{r^2 + 2 \ell \rho \cos \sigma_T + \ell^2} \\
\end{pmatrix} \tag{4}
\]

Note that \( \sigma_T \) is calculated from the target flight path angle and the seeker LOS angle. Since \( r, \dot{r}, \sigma_T, \) and \( \dot{\sigma}_T \) involved in \( \beta_d \) and \( \dot{\beta}_d \) are not available from the seeker, it is required to replace them with their estimates denoted as \( \hat{r}, \hat{\dot{r}}, \hat{\sigma}_T, \) and \( \hat{\dot{\sigma}}_T \) respectively. Passive homing guidance laws are commonly based on proportional navigation guidance (PNG) [2] and augmented proportional navigation guidance (APNG) [6] formulated to improve performance of PNG by including target acceleration perpendicular to the current LOS. However, it is proved in [3] that PNG and APNG do not provide observability to target tracking systems with bearings-only measurements including IR passive homing guidance systems. The lack of observability results from the absence of range measurements, and guidance principles which maintain small cross range deviations and thereby provide small LOS angular rates. Necessary and sufficient conditions for observability of the system described in Section II A with a constant acceleration command vector \( U_T \) are derived by a simple but effective approach in [8].

The results of [8] indicate that the system is unobservable for constant \( A_m \) or \( A_m = A_T \). Moreover, if the missile and target have the same time constants, the system is unobservable. [8] also treats a constant target acceleration case where constant \( \beta \) or constant \( A_m \) makes the system unobservable.

To enhance system observability, [3] suggests a closed-form guidance law which is tailored to provide initial heading error induced LOS angle oscillation without sacrificing guidance effectiveness. The guidance command \( A_m \) of [3] perpendicular to the current LOS is expressed as follows.

\[
A_m = NV_c \beta + \frac{N}{2} A_{\eta} + Fr\beta \tag{5}
\]

where \( N \) is the effective navigation ratio, \( V_c \) is the
closing velocity equivalent to $-\dot{r}$. $A_{r}$ is the target acceleration perpendicular to the current LOS, and $\mathbf{F}$ is a positive constant related to natural frequency of the LOS angle oscillation. As we notice from (5), the first two terms of the RHS of (5) compose APNG [6] while the last term provides the oscillation for large time-to-go ($t_{go}$), defined as $t_{go} = t_f - t$. As $t_{go}$ becomes smaller, (5) becomes closer to APNG. In practical passive homing applications, the variables involved in (5) should be replaced by their estimates obtained from a state estimator so that the guidance command becomes

$$A_{m} = N\hat{\beta} + \frac{N}{2} \dot{A}_{r} + \mathbf{F}\hat{\beta}$$  \hspace{1cm} (6)

where $\hat{\beta}$ implies the estimates of $\beta$ obtained from an estimator utilizing bearings-only measurements.

The following equation is proposed in this paper as a new TAG algorithm.

$$A_{m} = N\hat{\beta} + \frac{N}{2} \dot{A}_{r} + \mathbf{F}\hat{\beta}$$  \hspace{1cm} (7)

where $\hat{\beta}_d$ is obtained from (4) with the variables replaced by their estimates. As illustrated in Section III, $\hat{\beta}_d$ of (7) is insignificant for mid-course guidance phase where the $t_{go}$ is large however, $\hat{\beta}_d$ becomes significant at the terminal homing phase. This implies that the TAG algorithm of (7) employs (6) as a mid-course guidance law to enhance system observability for better state estimation while the performance of the guidance law in the terminal homing phase to engage the aim point becomes similar to that of APNG.

C. State Estimation

As a state estimator used to generate the variables involve in the TAG algorithm described in (7), the modified gain extended Kalman filter (MEKF) is applied. The MEKF is chosen because of its bias-free characteristics and stochastic stability [4]. Advantages of the MEKF over the other filters such as pseudomeasurement filter and extended Kalman filter are also analyzed in [4]. The MEKF is formulated in the Cartesian coordinates where the filter model satisfies the system dynamics of (1) except that the target acceleration vector $\mathbf{A}_T$ used in the filter is governed by the Singer model [7] such as

$$\dot{\mathbf{A}}_T = -\mathbf{\lambda} \mathbf{A}_T + \mathbf{D} \mathbf{W}$$  \hspace{1cm} (8)

where $\mathbf{\lambda}$ is the assumed time constant of the target acceleration model, and $\mathbf{W} = (\mathbf{W}_x, \mathbf{W}_y)^T$ is a white Gaussian noise vector with zero-mean and power spectral density of $2\lambda \sigma_{a_{d}}^2 I_2$ with the assumed variance of target acceleration $\sigma_{a_{d}}^2$. The detailed algorithm of the MEKF is referred to Appendix B of [3]. With the observability results stated in Section IIIB and the MEKF structure, the TAG algorithm is tested through a series of Monte Carlo simulation runs in the next section.

III. SIMULATION RESULTS

The TAG law developed in the previous section and the MEKF are placed in cascade to form a closed-loop with the system dynamics described in (1) and the LOS angle measurement $z(t)$ of (3). The state estimates of the MEKF are used to realize the TAG law of (7) for practical applications.

The missile used in this study has a velocity of 680 m/s and the target has a velocity of 340 m/s and begins a maneuver at $t = 4$ s activated by a lateral acceleration command -30 m/s$^2$ perpendicular to the target velocity vector. $\lambda = 0.3$ (1/s) is used in (2) while $\lambda = 0.1$ (1/s) is used for the filter model expressed in (8). Initial heading angle of the target, $\sigma_{T}(0)$, is 60° while the missile has an initial heading error of $-5°$. The effective navigation ratio, $\mathcal{N} = 4$ is used and the aim point of the TAG algorithm is located 6 m ahead of the hottest spot of the target so that $\ell = 6$. The initial state vector for this scenario is given by $\mathbf{x}(0) = [0, 7000, 55, -466, 0, 0]$, and the initial state estimate $\hat{\mathbf{x}}(0)$ and covariance $\mathbf{P}(0)$ for the MEKF are assumed to be $\hat{\mathbf{x}}(0) = [0, 5000, -240, -636, 0, 0]$ and $\mathbf{P}(0) = \text{diag}[10^2, 3000^2, 300^2, 300^2, 288, 288]$. The LOS angle measurement is updated for every 20 ms and is corrupted by a white Gaussian noise sequence with zero-mean and variance of 1 (mrad)$^2$, and $\sigma_{a_{d}}^2 = 36$ (m/s)$^2$ is used for the assumed variance of target acceleration.

Fig. 2 shows the required missile maneuver to realize the TAG algorithm. $F = 3$ is selected to provide oscillatory motions of $\beta$ to enhance system observability via missile acceleration well within current missile maneuver capabilities. The true and estimated $t_{go}$ are plotted in Fig. 3, which indicate a good convergence property even for large initial errors.

The true and estimated $\hat{\beta}_d$ required to realize the bias-type guidance command for the TAG algorithm are plotted in Fig. 4. The proposed TAG algorithm
shows miss distance of 0.05 m to the aim point.

Fig. 2. Required missile acceleration

Fig. 3. True and estimated $f_{go}$

Fig. 4. True and estimated $\beta_d$

IV. CONCLUSIONS

In order to improve guidance effectiveness for IR passive homing missile systems, a new TAG algorithm is proposed. The TAG algorithm is composed of an observability enhancing mid-course guidance law to improve estimation performance for state variables involved in the algorithm, and a bias-type guidance command to cope with relocation of the aim point. The proposed TAG algorithm is developed by using modern control and estimation theory in an effort to avoid drawbacks of existing TAG algorithms related to accuracy and complexity of signal processing. Performance of the TAG algorithm is tested in a series of computer simulation runs, and the results indicate that the TAG algorithm in conjunction with the MGKF as a nonlinear state estimator provides accurate enough state estimates and improved terminal effectiveness without computational complexities.

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