Simulation of a piezoelectric flextentional sonar transducer using a coupled FE-BEM

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Abstract

A piezoelectric flextentional sonar transducer has been simulated using a coupled FE-BEM. The dynamics of the sonar transducer is modelled in three dimensions and is analyzed with external electrical excitation conditions. The numerical results are available such as steady-state displacement modes, underwater directivity patterns. It is shown that the present barrel-stave sonar transducer of the piezoelectric material produces flextentional displacements which could be related with higher output power, lower quality factor and omnidirectional beam pattern than other types of sonar transducers.

1. Introduction

Ocean acoustic tomography requires wide bandwidth, compact, and effectively low frequency sources of sound [1]. This paper describes the modelling process for such a sonar transducer using a coupled finite element-boundary element method (FE-BEM). Flextentional sonar transducers are widely used as a high-power projector. It is a compact sound source efficiently over a broad frequency range [2]. The precise dimensions of a flextentional sonar transducer could be optimally predicted by the analysis of the flextentional transducer dynamics. The choice of the active element depends on structural types and electromechanical efficiency of the transducer. A flextentional transducer such as a barrel-stave type has been modelled in three dimensions. The main aim of this paper is to simulate the structural dynamics of the flooded piezoelectric flextentional sonar transducer using a coupled FE-BEM. Different results for analyses are produced; displacement modes and directivity patterns.

2. Numerical Method

The following equation (1) is the integral formulation of the piezoelectric equations modelling of a sonar transmitter submerged into the water [3]:

\[
\begin{align*}
F & \quad \text{Applied Mechanical Force} \\
Q & \quad \text{External Electrical Charge} \\
\varphi & \quad \text{Elastic Displacement} \\
\Phi & \quad \text{Electric Potential}
\end{align*}
\]
\[(F) + [L](A^T)^{-1}p_{inc} = [K_{nn}](a) + [\rho_f \omega^2(L)(A^T)^{-1}B^T](a) - \omega^2[M](a) + j\omega[R](a) + [K_{ss}](\Phi) - (Q) = [K_{ss}](a) + [K_{ss}](\Phi)\]

$\hspace{0.2cm} p_{inc}$ Incident Pressure
$[K_{nn}]$ Elastic Stiffness Matrix
$[K_{ss}]$ Piezoelectric Stiffness Matrix
$[K_{ss}] = [K_{ss}]^T$
$[K_{ss}]$ Permittivity Matrix
$[M]$ Mass Matrix
$[K]$ Dissipation Matrix
$[L]$ Coupling Matrix at the Fluid-Structure Interface
$A^T$ Fluid BEM Matrix [A]
$B^T$ Fluid BEM Matrix [B]
$\omega$ Angular Frequency
$\rho_f$ Fluid Density
$\sqrt{-1}$

Incident pressure of the equation (1) is zero in case of the present sonar transmitter. The isoparametric formulation for 3 dimensional structural elements is well documented by Allik H. et al. [4]. Each 3 dimensional finite element is composed of 20 quadratic isoparametric nodes and each node has nodal displacement (u, v, w) and electric potential (\(\Phi\)) variables. Table 1 and Table 2 show property values of the materials used for the flextentional sonar transducer.

Table 1. Piezoelectric Material Properties of PZT4 (Axially Polarized Properties)

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Kg/m$^3$</td>
<td>7500</td>
</tr>
<tr>
<td>$C_e$</td>
<td>N/m$^4$</td>
<td>3.06E+10</td>
</tr>
<tr>
<td>$C_t$</td>
<td>N/m$^4$</td>
<td>-5.2</td>
</tr>
<tr>
<td>$\varepsilon_{33}$</td>
<td>N/Vm</td>
<td>-5.2</td>
</tr>
<tr>
<td>$\varepsilon_{33}$</td>
<td>N/Vm</td>
<td>15.1</td>
</tr>
<tr>
<td>$\varepsilon_{33}$</td>
<td>N/Vm</td>
<td>12.7</td>
</tr>
<tr>
<td>$\varepsilon_{33}$</td>
<td>N/Vm</td>
<td>12.7</td>
</tr>
<tr>
<td>$C_{11}$</td>
<td>N/m$^4$</td>
<td>6.46E-9</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>N/m$^4$</td>
<td>6.46E-9</td>
</tr>
<tr>
<td>$C_{13}$</td>
<td>N/m$^4$</td>
<td>5.62E-9</td>
</tr>
<tr>
<td>$C_{11}$</td>
<td>N/m$^4$</td>
<td>0.69</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>N/m$^4$</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Table 2 Properties of other materials used for the flextentional sonar transducer

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$ [Kg/m$^3$]</th>
<th>Young's Modulus $Y$ [N/m$^2$]</th>
<th>Poisson's Ratio $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.22</td>
<td>1.411E5</td>
<td>-</td>
</tr>
<tr>
<td>Aluminium</td>
<td>2750</td>
<td>70.0E9</td>
<td>0.34</td>
</tr>
<tr>
<td>Steel</td>
<td>7850</td>
<td>207.0E9</td>
<td>0.29</td>
</tr>
</tbody>
</table>

For very-low-frequency (below 2KHz) and higher-power applications flextentional transducers are generally used (Fig. 1). Here a stack of ceramic ring, as it expands and contracts as a result of the applied alternating voltage, exerts an oscillatory force on a pair of thick metal "barrel staves" [5]. A bolt holds the staves together and pre-stresses the ceramic stack so that even under high drive the ceramic and any bonds between components remain in compression. With this construction the relatively small linear motion of the ceramic stack is converted into a much larger change in the volume of the staves, so that moderate power levels are possible.

![Figure 1 Sonar transducer prototypes for frequencies below 2KHz](image)

2.1 Modelling of a barrel-stave typed piezoelectric sonar transducer

Instead of using the piezoelectric ceramic itself in a flexural mode, it is possible to devise flextentional structures in which the high stress but low strain generation of the ceramic in the thickness mode is transformed into larger displacements by means of some type of level action (Fig. 1 ~ Fig. 3). A stack of piezoelectric ceramic operating in the
thickness mode is connected to a surrounding elliptical shell like a barrel-stave. When the stack extends, along the major axis of the ellipse, the shell moves inwards along the minor axis, thus producing a large volume displacement overall. In general terms, the resonance frequency of such a transducer depends principally on the major and minor axes, wall thickness, and material properties of the shell, with the stack itself having a lesser influence [6]. The bandwidth is also dependent primarily on the parameters of the shell. Maximum eccentricity leads to the maximum bandwidth, but has the lowest power output, whilst least bandwidth and highest power occurs for the least eccentric shape [6]. The maximum pressure which an elliptical shell can withstand is also dependent on its shape and thickness, and is therefore related to its resonance frequency. The size of the flexextensional transducer is generally much less than a wavelength in water at their resonance frequency. It therefore radiates approximately omni-directionally in the plane perpendicular to the major axis. A compressive pre-stress needs to be applied to the stack for higher power output from a compact size. This is usually done by applying pressure to the minor axis of the shell, thus extending the major axis, and inserting the stack into the extended shell. On release of the pressure, the relaxation of the shell applies the necessary force to the stack.

Figure 2 External view of the flexextensional sonar transducer (a) and their corresponding finite mesh elements in 3 dimensions (b) and (c).

Figure 3 Three dimensional view of the flexextensional sonar transducer within the fluid domain (a) and the internal materialistic composite of the modelled sonar transducer (b).

The piezoelectric flexextensional sonar transducer has been totally divided into 608 elements with 3280 nodes. The solid-fluid interfacing surface elements are 320 with 992 nodes. Only one fourth of the total elements are used for formulation of the global coefficient matrix because of the symmetry of the structure as shown in Fig. 3 (b). The resulted size of the global coefficient matrix is 3876 by 3876. One important point for loading of electric
3. Results and Discussions

The coupled FE-BE method has been programmed with Fortran language running at a supercomputer Cray C90. Calculation is done with double precision and the program is made for three dimensional structures. It is a common practice to have the size of the largest element to be less than \( \lambda/3 \). In this paper the interest frequency of the acoustic radiation is less than 2KHz, so that \( \lambda/3 \) is about 0.25m. Fig. 5 shows the displacement modes of the one fourth of the total structure at 900Hz. The figures are plotted with hidden lines in series for 1/20 intervals of one cycle, so that the change of the structural displacement can be viewed in different phases. From the series of figures in different phases, it is clear to notice that the force generated by the active element is transferred to the aluminium stave through the end caps in the similar mechanism like an arm lever. Therefore the relatively small linear motion of the ceramic stack is converted into a much larger change in the volume of the staves.

Fig. 6 shows the beam patterns of the transducer at 900Hz in polar form (a) and in rectangular form (b). And Fig. 7 shows the same beam pattern in three dimension.

![Normalized Directivity Pattern at 800 Hz](image)

**Figure 6** Beam patterns of the flexentional piezoelectric sonar transducer at 900 Hz in polar form (a) and in rectangular form (b).

![Beam patterns of the flexentional piezoelectric sonar transducer at 900 Hz in three dimension](image)

**Figure 7** Beam patterns of the flexentional piezoelectric sonar transducer at 900 Hz in three dimension.

4. Conclusion

The dynamics of the barrel-stave sonar transducer of the piezoelectric material had been simulated using a coupled FE-BEM. The flexentional displacement mode was temporarily figured to show the mode in different phases. This paper does not include the effect of hydrostatic pressure which is significantly important for deep water operation. More advanced structural design should be considered for deep-water application such as a free-flooded flexentional transducer [7]. In conclusion, this presented coupled FE-BEM code can be used for the design and the analysis of
Figure 5 Displacement modes of the one fourth piezoelectric flextentional transducer at 900Hz with different phases

Sonar transducers in many different aspects in material and in structure. Last 20 years have been spent for the development of other software design tools like ATILA [8,9], ANSYS [10] and PHOEBE [11] for sonar transducer design. ATILA and ANSYS use only infinite elements instead of boundary elements for radiation conditions in the fluid which often results in incorrectness of the results. PHOEBE uses boundary elements for the radiation condition but its calculation is done in...
single precision which also results in incorrectness of the results. The present coupled FE-BEM uses both boundary elements for radiation conditions in the fluid and double precisions for more correct computational results.

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References