Structure of CA which the complement vector is acyclic


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요. 약

본 논문에서는 2개의 직전자를 가지는 선형 MACA부터 유도된 여원 CA의 행동에 대한 분석을 한다.

1. Introduction

An analysis of the state-transition behavior of group cellular automata (briefly CA) was studied by many researchers ([1], [5], [7], [9], [11]). The characteristic matrix of group CA is nonsingular. But the characteristic matrix of nongroup CA is singular. Although the study of the class of machines with singular characteristic matrix has not received due attention. However some properties of nonsingular CA have been employed in several applications ([6], [8], [10], [11]).

In this paper, we present a detailed analysis of the behavior of complemented CA derived from a linear multiple-attractor CA with two predecessor (briefly TPMACA) by replacing the XORs at some (or all) of the cells. Also, we give the specific features displayed in the state-transition behavior of the complemented CA C resulting from inversion of next-state logic of some (or all) of the cells of a TPMACA C. We call C' the CA corresponding to C. Especially we investigate the behavior of the complemented CA which the complement vector is an acyclic state lying on some nonzero-tree as the complement vector of a linear TPMACA.
2. Background

A CA consists of a number of interconnected cells arranged spatially in a regular manner [2], where the state-transitions of each cell depend on the states of its neighbors. The CA structure investigated by Wolfram can be viewed as a discrete lattice of sites(cells), where each cell can assume either the value 0 or 1. The next state of a cell is assumed to depend on itself and on its two neighbors (3-neighbourhood dependency). The cells evolve in discrete time steps according to some deterministic rule that depends only on logical neighbourhood. In effect, each cell consists of a storage element (D flip-flop) and a combinatorial logic implementing the next state function.

If the next-state function of a cell is expressed in the form of a truth table, then the decimal equivalent of the output is conventionally called the rule number for the cell [2].

Neighbourhood state: 111 110 101 100 011 010 001 000
Next state: 0 1 0 1 1 0 1 0 (rule 90)
Next state: 1 0 0 1 0 1 1 0 (rule 150)

The top row gives all eight possible states of the three neighboring cells (the left neighbor of the \(i\)th cell, the \(i\)th cell itself, and its right neighbor) at the time instant \(t\). The second and third rows give the corresponding states of the \(i\)th cell at time instant \(t+1\) for two illustrative CA rules. On minimization, the truth tables for the rules 60, 90, 102, 10, 204 and 240 result in the following logic functions, where \(\oplus\) denotes XOR logic and \(q_i(t)\) denotes the state of the \(i\)th CA cell at the \(i\)th time instant, \(q_{i-1}(t)\) and \(q_{i+1}(t)\) refer to the state of its left and right neighbors.

- rule 60: \(q_{i+1}(t) = q_{i-1}(t) \oplus q_i(t)\)
- rule 90: \(q_{i+1}(t) = q_{i-1}(t) \oplus q_{i-r}(t)\)
- rule 102: \(q_{i+1}(t) = q_i(t) \oplus q_{i+r}(t)\)
- rule 150: \(q_{i+1}(t) = q_{i-1}(t) \oplus q_{i-2} \oplus q_{i+1}(t)\)
- rule 204: \(q_{i+1}(t) = q_i(t)\)
- rule 240: \(q_{i+1}(t) = q_{i-1}(t)\)

Definition 2.1. [17] i ) Linear CA: if the next-state generating logic employs only XOR logic, then the CA is called a linear CA; otherwise it is called a non-linear CA.

ii ) Complemented CA: Complemented CA employs XNOR logic for one or more CA cells.

iii ) Group CA: A CA is called a group CA if all the states in its state-transition diagram lie on cycles, otherwise it is referred to as a non-group CA.

iv ) Reachable state: In the state-transition diagram of a non-group CA, a state having at least one in-degree is called a reachable state, while a state with no in-degree is called a non-reachable state.

v ) Cyclic state: Reachable states which lie on cycles are called cyclic states.

vi ) Attractor: A state having a self-loop is referred to as an attractor. An attractor can be viewed as a cyclic state with unit cycle length.

vii ) Depth: The maximum number of state transitions required to reach the nearest cyclic state from any non-reachable state in the CA state-transition diagram is defined as the depth of the non-group CA.

viii ) Level and Predecessor: Level of a state \(S_i\) is defined as the minimum number of time steps required to reach a cyclic state starting from \(S_i\). The state \(S_i\) can be viewed as \(i\)-level
3. Behavior of complemented CA derived from a linear TPMACA

In this section we present the behavior of complemented CA derived from a linear TPMACA. Especially we investigate the behavior of complemented CA derived from a linear TPMACA C which the complement vector is an acyclic state lying on some nonzero tree of C′.

Lemma 3.1. Let F be a level i state in the α(≠0)-tree of a linear TPMACA C and β be an attractor of C. Then (β ⊕ \(T^{i-1}F\)) lies on a cycle of length 2 of the complemented CA C′ corresponding to C.

Theorem 3.2. Let F be a level i state in the α(≠0)-tree of a linear TPMACA C and β be an attractor of C. Then (β ⊕ \(α ⊕ T^{i-1}F\)) and β ⊕ \(T^{i-1}F\) lie on the same cycle of length 2 of C′, where C′ is in Lemma 3.1.

Corollary 3.3. Let F be a level i state in the α(≠0)-tree of a linear TPMACA C and β be an attractor of C. Then the sum of two different cyclic states which lie on the same cycle of length 2 of C′ is always α.

Lemma 3.4. Let C be a linear TPMACA with depth d and F be a state at the level \(i(0 < i \leq d)\) of a non-zero tree in C as a complemented vector. Then \(T^{i-1}F\) lies on a cycle of length 2 in the complemented CA C′ corresponding to C.

Lemma 3.5. Let C be a linear TPMACA with depth d and F be a state at the level \(i(0 < i \leq d)\) in the α(≠0)-tree
of \( C \) as a complemented vector. Then \( a \oplus \overline{T}^{i-1}F \) lies on a cycle of length 2 in the complemented CA \( C' \) corresponding to \( C \).

Theorem 3.6. Let \( F \) be a state at the level of \( i \), \( 0 < i < \text{depth} \), in the \( a( \neq 0) \)-tree of a linear TPMACA \( C \) as a complemented vector. Also let \( C' \) be the complemented CA corresponding to \( C \). Then the following hold:

(a) If \( x \) is a state at level \( (i + 1) \) in the \( a \)-tree of \( C \), then \( x \) is also a level \( (i + 1) \) state in the \( \overline{T}^{i-1}F \)-tree of \( C' \).

(b) If \( y \) is a state at level \( (i + 2) \) in the \( a \)-tree of \( C \), then \( y \) is also a level \( (i + 2) \) state in the \( (a \oplus \overline{T}^{i-1}F) \)-tree of \( C' \).

(c) The state \( 0 \) of \( C \) is rearranged at level \( i \) in the \( \overline{T}^{i-1}F \)-tree of \( C' \).

(d) The state \( F \) is a level \( (i - 1) \) state in the \( \overline{T}^{i-1}F \)-tree of \( C' \).

Theorem 3.7. Let \( F \) be a state at the level of \( i \), \( 0 < i < \text{depth} \), in the \( a( \neq 0) \)-tree of a linear TPMACA \( C \) as a complemented vector. Then the following hold:

(a) If \( x \) is a state at level \( (i + j) \) in the \( a \)-tree of \( C \), then \( x \) is also a level \( (i + j) \) state in the \( \overline{T}^{i-1}F \)-tree of \( C' \), where \( j \) is an odd number such that \( i + j \leq \text{depth} \).

(b) If \( y \) is a state at level \( (i + j) \) in the \( a \)-tree of \( C \), then \( y \) is also a level \( (i + j) \) state in the \( a \oplus \overline{T}^{i-1}F \)-tree of \( C' \), where \( j \) is an even number such that \( i + j \leq \text{depth} \).

Lemma 3.8. Let \( x \) and \( y \) be the level 1 states in the \( a \)-tree and \( \beta \)-tree of a linear TPMACA \( C \) respectively. Then \( x \oplus y = a \oplus \beta \).

Theorem 3.9. Let \( F \) be a state at the level \( i \) in the \( a( \neq 0) \)-tree of a linear TPMACA \( C \) as a complemented vector. Then the following hold:

(a) If \( x \) is a state at level \( (i + j) \) in the \( \beta( \neq a) \)-tree of \( C \), then \( x \) is also a level \( (i + j) \) state in the \( \beta \oplus a \oplus \overline{T}^{i-1}F \)-tree of \( C' \), where \( j \) is an odd number such that \( i + j \leq \text{depth} \).

(b) If \( y \) is a state at level \( (i + j) \) in the \( \beta( \neq a) \)-tree of \( C \), then \( y \) is also a level \( (i + j) \) state in the \( \beta \oplus \overline{T}^{i-1}F \)-tree of \( C' \), where \( j \) is an even number such that \( i + j \leq \text{depth} \).

(c) If \( w \) is a state in the \( \beta( \neq a) \)-tree of \( C \) such that the level of \( w \) is lower than \( i \), then \( w \) gets rearranged at level \( i \) in the \( \beta \oplus \overline{T}^{i-1}F \)-tree of \( C' \).

(d) The states at level \( i \) of \( C \) get rearranged at level up to \( (i - 1) \) of \( C' \).

References


