Path Tracking Control Using a Wavelet Neural Network for Mobile Robots

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Abstract - In this paper, we present a Wavelet Neural Network (WNN) approach to the solution of the tracking problem for mobile robots that possess complexity, nonlinearity and uncertainty. The neural network is constructed by the wavelet orthogonal decomposition to form a wavelet neural network that can overcome the problems caused by local minima of optimization and various uncertainties. This network structure is helpful to determine the number of the hidden nodes and the initial value of weights with compact structure. In our control method, the control signals are directly obtained by minimizing the difference between the reference track and the pose of a mobile robot that is controlled through a wavelet neural network. The control process is a dynamic on-line process that uses the wavelet neural network trained by the gradient-descent method. Through computer simulations, we demonstrate the effectiveness and feasibility of the proposed control method.

1. Introduction

The localization and path tracking problems for mobile robots have been given great attention by automatic control researchers in the recent literatures[1]-[5]. Motion control of mobile robots is a typical nonlinear tracking control issue and has been discussed with different control schemes such as PI, GPC based EKF etc. In order to provide the control schemes with some degree of robustness, Neural Networks (NN) based controllers have also been proposed in the past[4]-[6]. Neural networks have become an attractive tool to model the complex nonlinear systems due to its inherent ability to approximate arbitrary continuous functions. On the other hand, an amount of research has been done on applications of Wavelet Neural Networks (WNNs), which combine the capability of artificial neural networks in learning from processes and the capability of wavelet decomposition, for identification and control of dynamic systems[7]-[10]. The WNNs can further result in a convex cost index to which simple iterative solutions such as the gradient descent rules are justifiable and are not in danger of being trapped in local minima when choosing the orthogonal wavelets as the activation functions in the nodes. In this paper, we present a WNN approach to the solution of the tracking problem for mobile robots that possess complexity, nonlinearity and uncertainty. This network structure is helpful to determine the number of the hidden nodes and the initial value of weights with compact structure. In our control method, the control signals are directly obtained by minimizing the difference between the reference track and the pose of a mobile robot that is controlled through a WNN. The control process is a dynamic on-line process that uses the WNN trained by the gradient-descent(GD) method. Through computer simulations, we demonstrate the effectiveness and feasibility of the proposed control method. In Section II, a kinematic model of mobile robot is described. WNN control structure and design method are described in Section III, and computer simulations are given in Section IV. Finally Section V presents a brief conclusion.

II. Kinematic Model of Mobile Robot

The robot used in this paper is composed of two driving wheels and four casters and is fully described by a three dimensional vector of generalized coordinates $X$ constituted by the coordinates $(x, y)$ of the midpoint between the two driving wheels, and by the orientation angle $\theta$ with respect to a fixed frame as shown in Fig. 1. We have the equation for motion dynamics as the follows:

\[ \Delta \theta_t = \frac{R_{\text{pos}} - L_{\text{pos}}}{d} \]
\[ u_k = (R_{\text{pos}}, L_{\text{pos}}), \]  
\[ x_0 = (x, y, \theta) \]

where $u(k)$ is the control variable which is each displacement of right and left wheels.

\[ x_{k+1} = x_k + \left( \frac{1}{4d} \right) A + B \sin \theta_k + \left( \frac{1}{2} \right) A + B \cos \theta_k \]
\[ y_{k+1} = \left( \frac{1}{4d} \right) A + B \cos \theta_k + \left( \frac{1}{2} \right) A + B \sin \theta_k \]
\[ \theta_{k+1} = \theta_k + \left( \frac{1}{d} \right) B \]

where, $A = R_{\text{pos}} + L_{\text{pos}}, B = R_{\text{pos}} - L_{\text{pos}}$

III. Wavelet Neural Network Controller

3.1 WNN structure

In the our WNN structure, $N_i$ input, multidimensional wavelets, and two output structure are considered as shown in Fig 2. In Fig 2., $N_i$ input is composed of errors and past errors between reference trajectory and controlled trajectory, and output $R_{\text{pos}}$ and $L_{\text{pos}}$ are control variables. Each control variable is as the follows.

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\[ R_{\text{peak}} = \Phi (E, \gamma) = \sum_{j=1}^{N} \phi_{1j}(z_{1j}) + a_{10} + \sum_{k=1}^{N} a_{1k} e_{k} \]
\[ L_{\text{peak}} = \Phi (z, \gamma) = \sum_{j=1}^{N} c_{2j} \phi_{2j}(E) + a_{20} + \sum_{k=1}^{N} a_{2k} e_{k} \]
where,
\[ \phi_{1j}(z_{1j}) = \sum_{k=1}^{N} \phi (z_{1j}) ; \text{ with } z_{1j} = \frac{e_{j} - m_{1j}}{d_{1j}} \]
\[ \phi (z) = -z \exp \left( -\frac{1}{2} z^{2} \right) \]: motor wavelet
\[ \gamma = \{ a_{10}, a_{1j}, c_{1j}, m_{1j}, d_{1j} \} : \text{ WNN parameters} \]

![Fig 2 Wavelet neural network structure](image)

### 3.2 WNN controller

Usually, a WNN structure is used for the modeling of the dynamic systems. In our control system, we design the direct adaptive control system using WNN structure. The purpose of our control system is to minimize the state errors \( E(e_{x}, e_{y}, e_{\theta}) \) between reference trajectory and controlled trajectory of a mobile robot. For this purpose, we train the WNN parameters using the gradient-descent method. The overall control system is shown in Fig. 3. A WNN controller calculates the control input \( u(k) = [R_{\text{pass}}(k) L_{\text{pass}}(k)]' \) by training the inverse dynamics of plant iteratively. But, the updating of WNN parameters through the variation rate \( J(\gamma, y) \) in the gradient-descent method is not able to be calculated directly we train the parameters of WNN through the transformation of the output error \( e_{y} \) of plant.

![Fig 3 Direct adaptive control system](image)

**Training Procedure:**
- Definition of the following cost function so as to train the WNN controller based on direct adaptive control technique.

\[ C = \frac{1}{2} ((z_{r} - x)^{2} + (y_{r} - y)^{2} + (\theta_{r} - \theta)^{2}) \]

where, \( e_{x} = x_{r} - x \)
\( e_{y} = y_{r} - y \)
\( e_{\theta} = \theta_{r} - \theta \).

- Calculation of the partial derivative of the cost function with respect to the parameter set of a WNN controller, \( \gamma \).

\[ \frac{\partial C}{\partial \gamma} = -e_{x}\frac{\partial x}{\partial \gamma} - e_{y}\frac{\partial y}{\partial \gamma} - e_{\theta}\frac{\partial \theta}{\partial \gamma} = -e_{x}\frac{\partial x}{\partial u}\frac{\partial u}{\partial \gamma} \]

\[ = -J(u) \frac{\partial u}{\partial \gamma} \]

where, \( J(u) = \frac{\partial y}{\partial u} \) is the feedforward Jacobian of plant and as following.

\[ J(u) = \begin{bmatrix} \begin{pmatrix} -\frac{1}{d} & R_{\theta} \cos \theta \end{pmatrix} & \begin{pmatrix} \frac{1}{d} \end{pmatrix} & \begin{pmatrix} L_{\theta} \sin \theta \end{pmatrix} & \begin{pmatrix} \frac{1}{d} \end{pmatrix} & \begin{pmatrix} -L_{\theta} \sin \theta \end{pmatrix} & \begin{pmatrix} \frac{1}{d} \end{pmatrix} & \begin{pmatrix} -L_{\theta} \sin \theta \end{pmatrix} \end{bmatrix} \]

The partial derivative \( \frac{\partial u}{\partial \gamma} \) of the control input \( u \) with respect to the parameters of a WNN controller \( \gamma \) can be calculated by using the equations from Eqn. (9) to Eqn. (13).

- Updating of WNN parameters. The minimization is performed by the following iterative GD method:

\[ \gamma(n+1) = \gamma(n) - \eta \frac{\partial C(n)}{\partial \gamma(n)} \]

where, \( \eta \) is the learning rate of a WNN.

From Eqns. (6) and (7), \( \frac{\partial u(n)}{\partial \gamma(n)} \) is the gradient of the controller output, \( u(n) \), with respect to parameters set, \( \gamma(n) \), and the components of this vector are as the follows:

- **parameter** \( a_{1,2n} \)

\[ \frac{\partial u(n)}{\partial a_{1,2n}} = 1 \]

- **direct connection parameters** \( a_{2k}, a_{2k} \)

\[ \frac{\partial u(n)}{\partial a_{2k}} = \frac{\partial u(n)}{\partial a_{2k}} = e_{k} \]

- **weights** \( c_{1}, c_{2} \)

\[ \frac{\partial u(n)}{\partial c_{1}} = \frac{\partial u(n)}{\partial c_{2}} = \Phi_{1}(E) \]

- **translations** \( m_{1,2k} \)

\[ \frac{\partial u(n)}{\partial m_{1,2k}} = -\frac{c_{1,2k}}{d_{1,2k}} \Phi_{1}(E) \]

where, \( \Phi_{1}(E) = \phi (x_{1,2k}) \phi (x_{2,2k}) \phi (x_{3,2k}) \cdots \phi (x_{3,2k}) \),

\[ \phi (x_{k}) = \frac{\phi (x_{k})}{\phi (x_{k})} = (x_{k} - 1)^{2} \exp \left( -\frac{1}{2} x_{k}^{2} \right) \]

- **dilations** \( d_{1,2k} \)

\[ \frac{\partial u(n)}{\partial d_{1,2k}} = -\frac{c_{3,2k}}{d_{1,2k}} \Phi_{1}(E) \]

\[ \frac{\partial u(n)}{\partial d_{1,2k}} = -\frac{c_{3,2k}}{d_{1,2k}} \Phi_{1}(E) \]
IV. Simulation Results

In this Section, we present simulation results to validate the control performance of proposed WNN controller for the tracking of mobile robots. The control objective for our tracking system is to minimizing the difference between the reference track and the pose of mobile robot that is controlled through a WNN controller. The parameters used in this simulation and simulation results are shown in Table 1. This simulation considers the tracking of a trajectory generated by the following displacements:

\[
R_{peak} = 1 \text{m/s}, \quad L_{peak} = 1 \text{m/s} (0 < t \leq 5) \\
R_{peak} = 8 \text{m/s}, \quad L_{peak} = 3 \text{m/s} (5 < t \leq 10) \\
R_{peak} = 3 \text{m/s}, \quad L_{peak} = 8 \text{m/s} (10 < t \leq 15) \\
R_{peak} = 1 \text{m/s}, \quad L_{peak} = 1 \text{m/s} (15 < t \leq 20)
\]

<table>
<thead>
<tr>
<th>Table 1 Parameters and simulation results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of wavelet function</td>
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<tr>
<td>Sampling time</td>
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<tr>
<td>Learning rate</td>
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<tr>
<td>Departure posture vector</td>
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<tr>
<td>Control result(MSE)</td>
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<tr>
<td>x</td>
</tr>
<tr>
<td>y</td>
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<tr>
<td>( \theta )</td>
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Fig. 4 shows the tracking control results of position \( x \), \( y \) and orientation angle \( \theta \) for a mobile robot, respectively. Also, Fig. 5 shows the control errors for tracking of a mobile robot, and Fig. 6 shows the feedforward Jacobian of a mobile robot system. From these results, we confirm that our WNN controller works well although the control error is increased in the four corners.

V. Conclusion

In this paper, a WNN controller based on direct adaptive control scheme was presented for the solution of the tracking problem for mobile robots. In our control method, the control signals are directly obtained by minimizing the difference between the reference track and the pose of mobile robot that is controlled through a wavelet neural network. The control process is a dynamic online process that uses the wavelet neural network trained by the gradient-descent method. In this work the WNN's parameters were randomly initialized. Through computer simulations, we verified the effectiveness and feasibility of our WNN control method although the control error is increased at corners.

[Reference]