Stability Analysis of the Optimal Semi-Trailer Vehicles

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Abstract: Stability of truck and trailer are the most significance in Thai automotive industry. This paper presents the mathematical model of a six-degree-of-freedom semi-trailer vehicle. Search method was implemented to obtain the optimum design variables of the trailer which are the distance from the fifth wheel to the centroid of the trailer and the distance from the centroid of the trailer to the trailer axle. The objective function is to minimize the steady side slip velocity, steady-state yawing velocity and steady-state angle between the tractor and the trailer.

From the calculation, the optimum distance from the fifth wheel to the centroid of the trailer and the optimum distance from the centroid of the trailer to the trailer axle are 5.50 and 3.25 meters respectively. The stability of the optimal semi-trailer vehicle was also examined in steady state. The steady side slip velocity, yawing velocity and the angle between tractor and trailer are also obtained using linearization technique under unit step disturbance of the tractor front wheel steering angle.

Keywords: a six degree of freedom semi-trailer system, optimum design, side slip velocity, yawing velocity linearization technique

1. INTRODUCTION

Recent years, the need of motor vehicle safely has been increased in the modern automobile industries. The handling of semi-trailer of articulated vehicles presents the steady state characteristic of the vehicle as over steer and under steer terms. The active safety – oriented direct moment control technology to improve heavy duty truck stability [1]. The measurement method of the equivalent cornering stiffness which dominates the vehicle stability was presented by Kitahara and Sakai [2]. The breaking system for a non-rigid vehicle such as tractor and semi-trailer combination was analyzed by Shilton [3]. The factors effect the breaking system are steering inertia. And front tyre cornering stiffness. In this paper, the nonlinear programming optimization technique is presented to calculate the optimization design variables which are the distance from the fifth wheel to the centroid of the trailer and the distance from the centroid of the trailer to the trailer axle.

2. THEORETICAL ANALYSIS

The articulated semi-trailer vehicle is made up of two units. The two axle vehicle supports the front end of a trailer on fifth wheel. Three degree of freedom tractor and trailer are required to describe the system when traveling at a steady forward speed as shown in Figure 1.

![Diagram of tractor and trailer](image)

For small disturbances of the vehicle at a constant forward speed, the forward motion equation is ignored and the forward speed \( U \) is assumed constant. The simplified equation of motion of tractor and trailer can be written as:

\[
\begin{align*}
(m_1 + m_2) & \ddot{V} - m_2 \cos \psi \dot{\theta} = (m_1 + m_2)U \dot{\theta} \\
- m_2 d \ddot{\theta} - m_2 \sin \psi \dot{\theta}^2 - X_1 \cos \delta - X_2 - X_3 \cos \psi \\
& + f_3 (\alpha \sin \psi) \\
(m_1 + m_2) \ddot{V} - m_2 \cos \psi \dot{\theta} = -(m_1 + m_2)U \dot{\theta} \\
& + m_2 \sin \psi \dot{\theta}^2 + f_1 (\alpha_1) + f_2 (\alpha_2) \\
& + f_3 (\alpha_3) \cos \psi + X_1 \sin \delta + X_3 \sin \psi \\
- m_2 d \ddot{\theta} & - \dot{f}_1 (\alpha_1) \\
- \dot{f}_2 (\alpha_2) & - \dot{f}_3 (\alpha_3) \cos \psi + aX_1 \sin \delta - dX_3 \sin \psi \\
& = m_2 \cos \psi \ddot{U} + m_2 \cos \psi \dot{\theta} (l_1 + m_2 d^2) \dot{\theta} \\
& + m_2 \cos \psi \ddot{\theta} - (h + e) f_3 (\alpha_3) \\
\end{align*}
\]

For small disturbances of the vehicle at a constant forward speed, the forward motion equation is ignored and the forward speed \( U \) is assumed constant. The simplified equation of motion of tractor and trailer can be written as:

\[
\begin{align*}
(m_1 + m_2) & \ddot{V} - \left(\frac{C_1 + C_2 + C_3}{U}\right) \dot{V} - \\
(m_2 d + e) & \ddot{\theta} + \left(m_1 + m_2\right) U \frac{aC_1 - be_2 - (d + e + h)C_3}{U} \dot{\theta} \\
& + \left(m_2 e^2 - \frac{(h + e)}{U} C_3 D - C_3\right) \psi = -C_1 \delta
\end{align*}
\]
\[
\begin{align*}
&\left[-m_2d\frac{aC_1-bC_2-dC_3}{C_3}V
+\left[\left(m_1+m_2(d+e)\right)d_m-m_2dU\right]-\frac{a^2C_1+b^2C_2+d(d+e+g)C_3}{U}\right]V
+\left[-m_2dD^2+\frac{d(d+e)C_3}{U}\right]V=0 \quad (6)
\end{align*}
\]

\[
\begin{align*}
&\left[-m_2d\frac{h+eC_3}{C_3}V
+\left[\left(m_1+m_2(d+e)\right)d_m-m_2dU\right]-\frac{(h+e+g)C_3}{U}\right]V
+\left[-(l_2+m_2e)\right]^2+(h+e)\frac{C_3}{C_3D}V=0 \quad (7)
\end{align*}
\]

At steady state \(DV, D\theta\) and \(D\psi\) are equal to zero. The static characteristic of semi-trailer can be written as:

\[
\begin{align*}
\left[-C_1+C_2+C_3\right]V
+\left[\left(m_1+m_2\right)U-aC_1+bC_2-(d+e+g)C_3\right]V
-C_3\psi=-C_3\delta \quad (8)
\end{align*}
\]

\[
\begin{align*}
\left[aC_1-bC_2-dC_3\right]V
+\left[-m_2dU-\left(a^2C_1+b^2C_2+d(d+e+g)C_3\right)\right]V
+dC_3\psi=-aC_3\delta \quad (9)
\end{align*}
\]

\[
\begin{align*}
\left[\frac{h+e}{C_3}\right]V
+\left[-m_2dU-\frac{(h+e+g)C_3}{U}\right]V
 +(h+e)C_3\psi=0 \quad (10)
\end{align*}
\]

For a steady angle of steering where \(\delta\) is the angle of steer of the front wheels.

Steady sideslip velocity.

\[
\frac{V}{\delta}=\left[U\left(m_1l_2+m_2h[a+d]\right)\right]l_2+L_2bC_2
+\left[U^2\left[m_1l_2(aC_1-bC_2)+m_2h[aC_1-bC_2]+d(C_1+C_2)\right]\right]
+\left[L_2^2C_1C_2\right] \quad (11)
\]

Steady state yawing velocity response.

\[
\frac{\frac{\delta}{\delta}}{\delta}=\left[UQC_2hL_2\right]
+\left[U^2\left[m_1l_2(aC_1-bC_2)+m_2h[aC_1-bC_2]+d(C_1+C_2)\right]\right]
+\left[L_2^2C_1C_2\right] \quad (12)
\]

3. OPTIMIZATION TECHNIQUE

The objective function is to minimize steady side slip velocity steady state yawing velocity and steady state angle between tractor and trailer

\[
Y = w_1 \frac{V}{\delta} + w_2 \frac{\theta}{\delta} + w_3 \frac{\psi}{\delta} \quad (14)
\]

The constraints equation

\[
l_{\min} \leq (e+h) \leq l_{\max} \quad (15)
\]

Exhaustive search technique was used to obtain the optimum semi-trailer which are distance between the center of the fifth wheel to the centroid of the trailer (\(e\)) and the distance between the trailer axle to the center of mass of the trailer (\(h\)).

4. SIMULATION RESULTS

The optimum design of a six-degree of freedom semi-trailer vehicle in this work is to minimize the steady side slip velocity steady state yawing velocity and the steady state angle between the tractor and the trailer using search method. The design variables are the distance from the fifth wheel to the centroid of the trailer ; \(e\) and the distance from the centroid to the trailer axle , \(h\). The optimum distance from the fifth wheel to the centroid of the trailer (\(e\)) \(= 5.5 \text{ m.}\) and the optimum distance from the centroid of the trailer to the trailer axle (\(h\)) \(= 3.25 \text{ m.}\).

In this study, the static characteristics of the optimal semi-trailer vehicle were compared with the static characteristics of the commercial semi-trailer vehicle for 25,000 kg and 32,000 kg semi-trailer as shown in Fig.2 to Fig.7 respectively. The steady side slip velocity , the steady state yawing velocity and the steady state angle between tractor and trailer of the optimal semi-trailer for 6,000 kg tractor and 19,000 kg trailer are shown in Fig.2 , Fig.3 and Fig.4 respectively, The side slip velocity and the yawing velocity are increase with the increase in semi-trailer forward velocity. The angle between the tractor and the trailer of the optimal trailer is independent on the forward velocity. The angle between the tractor and the trailer of the optimal trailer is independent on the forward velocity. Fig.5 , Fig.6, and Fig.7 show the steady state slip velocity , the steady state yawing velocity and the steady state angle between tractor and trailer of the optimal semi-trailer for 6,000 kg tractor and 26,000 kg trailer. The optimal semi-trailer has better stability then the commercial semi-trailer as shown in Fig.2 to Fig.6.
Fig 2. Side slip Velocity per steer angle for 25,000 kg semi-trailer

Fig 3. Yawing velocity per steer angle for 25,000 kg semi-trailer

Fig 4. Angle between tractor and trailer per steer angle for 25,000 kg semi-trailer

Fig 5. Side slip Velocity per steer angle for 32,000 kg semi-trailer

Fig 6. Yawing velocity per steer angle for 32,000 kg semi-trailer

Fig 7. Angle between tractor and trailer per steer angle for 32,000 kg semi-trailer
5. CONCLUSION

The stability characteristics semi-trailer vehicles are simulated and can be concluded as:

1) The implementation of the search method to obtain the optimum distance from the fifth where to the centroid of the trailer and the distance from the centroid of the trailer to the trailer axle
2) The stability of the optimal semi-trailer was examined in term of sideslip velocity yawing velocity and the angle between tractor and trailer.
3) The optimal trailer has better stability characteristics than the commercial trailer.

6. NOMENCLATURE

U = Steady forward velocity of tractor and trailer vehicle.
V = Steady side ship velocity.
r = Steady yawing velocity.
ψ = Angle between tractor and trailer at steady state.
e = Distance from fifth where to the centroid of trailer.
h = Distance from the centroid of the of trailer to the trailer axle.
a = Distance from tractor centroid to the front axle.
b = Distance from tractor centroid to the rear axle.
m₁ = Mass of the tractor.
m₂ = Mass of the trailer.
l₁ = Moment inertia of mass of tractor.
l₂ = Moment inertia of mass of trailer.
C₁ = Tyre stiffness coefficient of tractor at front axle.
C₂ = Tyre stiffness coefficient of tractor at rear axle.
C₃ = Tyre stiffness coefficient of trailer axle.
f₁ = Tyre laterals force of tractor at front axle.
f₂ = Tyre laterals force of tractor at rear axle.
f₃ = Tyre laterals force of trailer axle.
α₁ = Tyre lateral displacement of tractor at front axle.
α₂ = Tyre lateral displacement of tractor at rear axle.
α₃ = Tyre lateral displacement of trailer axle.
δ = Angle of steer at front wheel.
W₁ , W₂ , W₃ = weighting factor.

REFERENCES