Automatic Blood Pressure Control
Using PI Controller with $H_\infty$ Loop-Shaping

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Abstract: In this paper, we show a new form of blood pressure controller combined PI control with $H_\infty$ loop-shaping. Hypertensive patients or post-operative patients need to maintain normally blood pressure. Exact regulation of blood pressure is needed for maintaining variable blood pressure of preventing complications. The regulation of blood pressure is achieved by injecting drugs, and usually sodium nitroprusside is used as those kinds of drugs. It is necessary to control the infusion rate sodium-nitroprusside carefully to achieve the desired blood pressure. It has been known that regulation of blood pressure by automatic controller is more effective than regulation of blood pressure by human operators. The control of blood pressure has many constraints and uncertainties. Most of biological system has the time-varying variables and the side effects such as increased risk of sepsis and organ failure. To solve such a problem, we design a new robust PI controller using $H_\infty$ loop-shaping to decrease noise effects that come out from human body and errors for time delay. The system with designed controller shows more stable control of mean blood pressure and more robust performance for uncertainties. Validation methods for the control performance are confirmed to computer simulations.

Keywords: blood pressure control, PI controller with $H_\infty$ loop-shaping

1. INTRODUCTION

The need for automatic controllers arises in many clinical situations requiring more accurate, reliable, and autonomous regulation of physiological variables. Among other things, the control of mean arterial blood pressure (MAP) is very important in a clinical part. Hypertensive patients, operative patients, and post cardiac operation patients need the accurate regulation of MAP. It was proved that the automatic system of blood pressure regulation had better efficiency than clinical staffs. A popular means to reduce blood pressure is to reduce vascular resistance by infusing a vasodilating drug. Sodium nitroprusside (SNP) is a clinically proven vasodilating drug. When administered intravenously, SNP relaxes the muscle of the peripheral vasculature and reduces the blood pressure[1].

Since the late 1970’s, blood pressure control systems have been developed. Sheppard[2] used a modified PID controller, but this controller could not cope with individual differences of response to hypotensive drugs. Jackson et al.[3] developed a microprocessor-based control which used SNP to correct malignant hypertension. The PI controller was able to achieve MAP of 106mmHg. Ma et al.[4] developed an adaptive controller using SNP. They used Zigler-Nichols step response method for computing the PI controller parameters.

Recent researches are to try to keep blood pressure at an abnormal level with the intention of facilitating surgical operations or medical treatments. And it is required to suggest a controller that is strong to constraints and uncertainties.

In this paper, we show a new form of blood pressure controller combined PI control with $H_\infty$ loop-shaping. The idea of the proposed method is to design a controller that minimizes the signal transmission from load disturbances and measurement noise to process input and output. The designed controller shows more stable control of mean blood pressure and more robust performance for uncertainties.

2. CONTROLLER DESIGN

First of all, we introduce a characteristics of biological system,

- having time constant
- having different time delay to each objects
- being unstable in general feedback control system because body system can be a nonminimum phase system
- being constraints for the amplitude of inputs and the response rate of outputs
- being nonlinear system.

Therefore, when we regulate some characteristic components in biological system, it is required stabilities and reliabilities. The purpose of our study is to control the blood pressure by a hypotensive drug. PI controller is the most commonly used in automatic blood pressure control because it has a lot of versatility, high reliability, and ease of design. Also PI controllers have various forms, and Ma et al. [4] proposed a PI controller as
where $K_p$ is the gain of the controller, $T_i$ is the integrated time. These PI controller parameters are computed using Zigler-Nichols step response method.

A particular $H_\infty$ optimization problem guarantees closed –loop stability and a level of robust stability at all frequencies.

The proposed technique uses only the basic concept of loop-shaping methods, and then a robust stabilization controller for the normalized coprime factor perturbed system is used to construct the final controller.

The singular values of the nominal plant are shaped, using a precompensator $W_1$ and a post compensator $W_2$, to give a desired open-loop shape. The nominal plant $G$ and the shaping functions $W_1$, $W_2$ are combined to form the shaped plant, $G_s$, where $G_s = W_1GW_2$.

Suppose $G(s)$ has a stabilizable and detectable state-space given by

$$G(s) = C(sI - A)^{-1}B + D = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$  \hspace{1cm} (2)

Then a left coprime factorization of $G(s)$ is given by

$$[N \ M] = \begin{bmatrix} A + HC & B + HD & H \\ R^\frac{1}{2}C & R^\frac{1}{2}D & R^\frac{1}{2} \end{bmatrix}$$  \hspace{1cm} (3)

We have to choose weighting functions to improve the performance closed-loop system. The weighting functions are selected to look for the open-loop singular value. And they are also selected to present good closed-loop performance for open-loop singular value curve of plant that is extended by weight function.

The $\gamma_{\infty}$ represented how robust stability is, and how good performance loop-shaping by weighting functions is. Then $\gamma_{\infty}$ is given by

$$\gamma_{\infty} = \inf_{K\text{ stabilizing}} \left\| \begin{bmatrix} W_1^{-1}K & (I - GK)^{-1}W_2^{-1} \end{bmatrix} \right\|$$ \hspace{1cm} (6)

In the procedure of loop-shaping, weighting functions $W_1$, $W_2$, that are design variable, are selected to satisfy open-loop shaping of the extended plant and to be not large the $\gamma_{\infty}$.

3. SIMULATION RESULTS

A mathematical model of the dynamic response of MAP to SNP was developed by Slate[5]. The developed model has a transfer function that represents the relationship between the change in the blood pressure and SNP infusion rate. The transfer function is

$$\frac{\Delta P(s)}{I(s)} = \frac{Se^{\alpha T_r} (1 + e^{-\frac{\tau}{T_r}})}{1 + \tau s}$$ \hspace{1cm} (7)

where $\Delta P(s)$ is change in blood pressure(mmHg), $I(s)$ is SNP infusion rate(ml/h), $S$ is drug sensitivity to SNP, $\alpha$ is recirculation fraction, $T_r$ is initial transport lag from injection site, $T_c$ is recirculation time delay, and $\tau$ is lag time constant resulting from the uptake, distribution, and biotransformation of the drug.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$S$</td>
<td>0.72 mmHg/ml/h</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
</tr>
<tr>
<td>$T_r$</td>
<td>30 s</td>
</tr>
<tr>
<td>$T_c$</td>
<td>45 s</td>
</tr>
<tr>
<td>$\tau$</td>
<td>40 s</td>
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Excessive injecting SNP caused toxic side effects. So operators set limits amount of injecting SNP. The acceptable range of the SNP infusion rate is
Using recursive least square algorithm, we can estimate the parameters of PI controller. It is estimated by [4]

\[ K_r = \frac{0.5 \left( \frac{T_e}{T_i} \right)}{K} \quad T_i = 3 T_d \]  \hspace{1cm} (9)

\[ K = \frac{b_3 + b_4}{1 - a} \quad T_d = \frac{T_n}{\ln a} \]

\[ T_e = (1 + n) T_s \quad n = \frac{b_1}{b_3 + b_4} \]  \hspace{1cm} (10)

where \( T_s \) is the sampling time (15s), parameters \( a, b_3, b_4 \) are given by Slate, \( a = 0.606, b_3 = 0.187, b_4 = 0.075 \) [9].

When we use \( 20 \text{mmHg} \) noise, Fig. 3 shows infusion rate with PI controller, and Fig. 4 shows with PI controller using \( H_o \) loop-shaping where \( \gamma_{\text{min}} = 2.79 \) (drug sensitivity is normal).

After the controller is operated, there is a delay of about 5 min before the blood pressure began to drop. This delay is the time while administrated SNP operates in human body.

![Fig. 3. Curve of infusion rate with PI controller.](image)

![Fig. 4. Curve of infusion rate with PI controller using \( H_o \) loop-shaping.](image)

![Fig. 5. Curve of infusion rate with PI controller for various sensitivities.](image)

![Fig. 6. Curve of infusion rate with PI controller using \( H_o \) loop-shaping for various sensitivities.](image)

When the patient’s drug sensitivity changes from low to high, the differences of the infusion rate with PI controller and PI controller using \( H_o \) loop-shaping are shown in Fig. 5 and Fig. 6, respectively.

And we use the infusion rates to regulate MAP. A dynamic model of the MAP of patient under influence of SNP can be represented as follows by Slate [5]

\[ \text{MAP} = P_i - P_a + r_n \]  \hspace{1cm} (11)

where \( P_i \) is the initial blood pressure, \( P_a \) is the change in blood pressure due to infusion of SNP (as shown (7)), \( r_n \) is a random noise. In here, it is assumed that \( P_a \) is a constant.

When MAP is changed rapidly, there is bad effect to patients. Therefore operators set limits the maximum acceptable rate of change of MAP same as the SNP infusion rate. The maximum change rate MAP is [4]
where the low-level MAP is limited at 80mmHg.

We assume that the initial blood pressure is 150mmHg and the objective was to reduce it to 110mmHg.

In Fig. 7 and Fig. 8, the MAP with PI controller and PI controller with $H_\infty$ loop-shaping is shown, respectively. We can see that the MAP with PI controller using $H_\infty$ loop-shaping approaches fast to the desired blood pressure level than PI controller case.

4. CONCLUSIONS

In this paper, we propose a new robust PI controller using $H_\infty$ loop-shaping to regulate the MAP that decreases noise effects that come out from human body. The system with designed controller shows more stable control of mean blood pressure using sodium nitroprusside and more robust performance for uncertainties. The MAP with proposed controller approaches fast to the desired blood pressure level. Validation methods for the control performance are confirmed to computer simulations.

REFERENCES