The Design and Simulation of a Fuzzy Logic Sliding Mode Controller (FLSMC) and Application to an Uninterruptible Power System Control

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Abstract: A Fuzzy Logic Sliding Mode Control or FLSMC for the uninterruptible power system (UPS) is presented, which is tracking a sinusoidal ac voltage with specified frequency and amplitude. The FLSMC algorithm combines feedforward strategy with the Variable Structure Control (VSC) or Sliding Mode Control (SMC) and fuzzy logic control. The control function is derived to guarantee the existence of a sliding mode. FLSMC has an advantage that the stability of FLSMC can be proved easily in terms of VSC. Furthermore, the rules of the proposed FLSMC are independent of the number of system state variables because the input of the suggested controller is fuzzy quantity sliding surface value. Hence the rules of the proposed FLSMC can be reduced. The simulation results illustrate that the purposed approach gives a significant improvement on the tracking performances. It has the small overshoot in the transient and the smaller chattering in the steady state than the conventional VSC. Moreover, its can achieve the requirements of robustness and can supply a high-quality voltage power source in the presence of plant parameter variations, external load disturbances and nonlinear dynamic interactions.

Keywords: Sliding Mode Control; Fuzzy Logic; Uninterruptible Power System.

1. INTRODUCTION

An uninterruptible power system (UPS) have been used widely of computer network, instrumentation equipment and factory automation systems. The proposed scheme of the UPS, as shown in Fig. 1, consists of a rectifier, battery charger, booster, pulse width modulation (PWM) inverter, LC filter, current controlled loop and voltage controlled loop. The current controlled loop is designed to control the capacitor charge current quickly and effectively, especially when a sudden load occurs. The voltage controlled loop is designed to achieve accurate voltage tracking and minimize the total harmonic distortion in the presence of load disturbance and plant parameter variation. Such performances are usually difficult to achieve by using a linear controller.

In the recent years, the cross combination of intelligent control methods has been a hot spot of the study. Researcher attach more and more attention to it as variable structure control (VSC) or sliding mode control (SMC) has the characteristics of deflection, decoupling and rapid response and in particular, the advantages of a variable structure control is invariant to system parameter variations and disturbances when the sliding mode occurs [1-2]. In the sliding mode the dynamic behavior of the system becomes equivalent to that of an unforced system of lower order and the closed-loop response becomes insensitive to those changes in plant parameters which act within channels implicit in the nominal control input. The idea of VSC is to force the system to slide along a predetermined switching plane and is a variable high speed switching feedback control. This variable structure control law provides an effective and robust means of controlling nonlinear plants. The sliding mode operation results in a control system that is robust to model uncertainties, parameter variations and disturbances. The VSC attains the conventional goals of control such as stabilization, tracking and regulation. Although the conventional VSC approach has been applied successfully in many applications, but it may result in a steady state error when there is load disturbance in it. The Fuzzy Logic Sliding Mode Control or FLSMC approach is presented in [4-5], combines the feedforward path in the IVSC to solve this problem. Although, the FIVSC method can give a better tracking performance than IVSC method does at steady state, its performance during transient period needs to be improved.

In this paper, the Fuzzy Logic Sliding Mode Control or FLSMC approach is presented. This approach, which is the extension of FIVSC approach, incorporates a fuzzy logic to improve the dynamics response for command tracking. The proposed approach is more suitable for the UPS which is tracking a sinusoidal ac voltage with specified frequency and amplitude. The design and simulation of the UPS, which is tracking a sinusoidal command input using the FLSMC approach is described. Some simulation results are presented to demonstrate the superior tracking performance of our controller over the MIVSC and IVSC approaches. The proposed approach is fairly robust to plant parameter variations and load disturbances.

![Fig. 1. The functional configuration of the UPS.](Image)

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2. DYNAMIC MODELING OF THE PWM INVERTER WITH OUTPUT FILTER

The block diagram of the proposed L-C output filter is show in Fig. 2, the current loop for the UPS which consists of PWM inverter circuit and power MOS as the switching device. In the PWM circuit, let sinusoidal input be \( e_a = A_m \sin(\omega t) \), then its approximate output voltage is

\[
V_m = \frac{V_{DC} A_m}{2A_d} \sin(\omega t)
\]  

(1)

where \( V_{DC} \) is the dc supply voltage from booster and \( A_d \) is the triangular peak value. Thus, the mode of the PWM inverter can be synthesized to be an amplifier with constant gain

\[
K_d = \frac{V_m}{e_a} = \frac{V_{DC}}{2A_d}
\]  

(2)

The current-controlled loop is designed so that the current of the capacitor can respond quickly and effectively. The mathematical model of the current-controlled loop for the UPS is shown in Fig. 3, where \( K_p \) is the equivalent gain of the PWM inverter and \( K_{pc} \) is the compensated gain of the current loop.

![Fig. 2. The block diagram of the L-C output filter.](image)

![Fig. 3. The block diagram of the current loop for the UPS.](image)

Then, the state dynamics can be expressed as

\[
i_e = -\frac{V_e}{L} + \frac{V_p}{L}
\]  

(3a)

\[
V_p = \frac{I_e}{C}
\]  

(3b)

where \( L \) is the inductance, \( C \) is the capacitance, \( i_e \) is the current through inductor, \( V_p \) is the voltage across capacitor, \( I_e \) is the load current and \( V_e \) is the equivalent output voltage of PWM.

The dynamic model of the UPS with a FLSMC as shown in Fig. 4, is derived as

\[
\dot{z} = -e_1 - \dot{x}_0
\]  

(4a)

\[
\dot{e}_1 = \dot{r} - \dot{V}_e = e_2
\]  

(4b)

\[
\dot{e}_2 = \dot{r} - \dot{V}_e = -\frac{1}{LC}K_pK_{pc}U + \frac{1}{L}K_pK_d(r - e_2) + \frac{1}{LC}V_e + \frac{1}{C} \int_0^t \dot{e}_1 dt
\]  

(4c)

where \( r \) is the input command, \( V_e \) represents output voltage, \( z = -\int c dt \) and \( e_1 = r - V_e \).

The simplified dynamic model of the UPS control system with the FLSMC approach can be described as

\[
z = z - e_1 = \dot{x}_0
\]  

(5a)

\[
\dot{e}_1 = e_1 + e_2
\]  

(5b)

\[
\dot{e}_2 = e_2 + \dot{r} + bU(k) + a_1(r - e_2) + a_2(r - e_1) + f.
\]  

(5c)

where \( a_1 = \frac{K_pK_{pc}}{L} \), \( a_2 = \frac{1}{LC} \), \( b = \frac{K_pK_d}{LC} \) and \( f \) are disturbances.

Note that the input command \( r \) in the above equation, as previously stated, is a sinusoidal ac voltage with specified frequency and amplitude, that is,

\[
r = R_m \sin(\omega t) \cdot \cos(\phi)
\]

The first and second time derivative of this input command are, respectively,

\[
\dot{r} = R_m \omega \cos(\phi) \quad \text{and} \quad \ddot{r} = -R_m \omega^2 \sin(\phi).
\]

3. DESIGN OF FLSMC SYSTEM

The structure of FLSMC is shown in Fig. 5. It combines the conventional VSC with an integral compensator and a feedforward path from the input command. The FLSMC system can be described by the following equation of state

\[
\dot{x}_i = x_{i+1}, \quad i = 1, \ldots, n-1
\]  

(6a)

\[
\dot{x}_n = -\sum_{i=1}^{n-1} a_i x_i + bU - f(t)
\]  

(6b)

\[
\dot{x}_0 = (r - x_i)
\]  

(6c)

where \( x_i \) is the output signal, \( r \) is the input command, \( a_i \) and \( b \) are the plant parameters and \( f(t) \) are disturbances.
The switching function, \( \sigma \) is given by

\[
\sigma = c_i (x_i - T x_i - r K_x) + \sum_{j=1}^{n} \alpha_j x_j
\]

(7)

where \( c_i > 0 \) constant, \( c_i = 1 \).

The design of such system involves

- the choice of the control function, \( U \) so that it gives rise to the existence of a sliding mode control
- the determination of the switching function, \( \sigma \) and the integral control gain using fuzzy rules
- the elimination of chattering phenomena of the control signal.

The control signal, \( U \) can be determined as follows, from Eq. (6) and Eq. (7), we have

\[
\dot{\sigma} = -c_i T (r - x_i) + \sum_{j=0}^{n} \alpha_j x_j + b U - f(t).
\]

(8)

Let \( \alpha_i = \alpha_i^0 + \Delta \alpha_i \); \( i = 1, \ldots, n \) and \( b = b^0 + \Delta b ; b^0 > 0, \Delta b > b^0 \)

where \( \alpha_i^0 \) and \( b^0 \) are nominal values of \( \alpha_i \) and \( b \) and \( \Delta \alpha \) and \( \Delta b \)

are the variations of \( \alpha_i \) and \( b \), respectively.

The control signal can be separated into

\[
U = U_{eq} + U_{fu}
\]

(9)

where the so called equivalent control \( U_{eq} \) is defined as the solution of Eq. (8) under the condition where there is no disturbances and no parameter variations,

that is \( \dot{\sigma} = 0, f(t) = 0, \alpha_i = \alpha_i^0, b = b^0 \) and \( U = U_{eq} \).

This condition results in

\[
U_{eq} = -c_i T (r - x_i) - \sum_{j=0}^{n} \alpha_j x_j / b^0.
\]

(10)

The transfer function when the system is on the sliding surface can be shown as

\[
H(s) = \frac{X_1(s)}{R(s)} = \frac{\alpha_n s^n + \alpha_{n-1} s^{n-1} + \ldots + \alpha_1 s + \alpha_0}{s^n + \alpha_{n-1} s^{n-1} + \ldots + \alpha_1 s + \alpha_0}
\]

(11)

Using the final value theorem, it can be shown from Eq. (12), that the steady-state tracking error due to a ramp command input is zero. This is the result of the integral action. Furthermore, the zeros of the transfer function Eq. (12), which is the result of the feedforward path will give rise to the improvement on tracking performance during the transient period. The transient response of the system can be determined by suitably selecting the poles of the transfer function.

Let \( S^0 + \alpha_0 S^{-1} + \ldots + \alpha_{n-1} S + \alpha_n = 0 \)

be the desired characteristic equation(closed-loop poles), the coefficient \( C_1 \) and \( T \) can be obtained by

\[
C_{n+1} = \alpha_1,
\]

\[
C_i = \alpha_{n-i},
\]

\[
T = \frac{\alpha_n}{\alpha_{n-1}}.
\]

4. DESIGN OF FUZZY CONTROLLER

By the definition

\[
U_{fu} = k_i (x_i - T x_i - r K_x) + k_m x_i + k_r (x_i - T x_i - r K_x) + \sum_{i=0}^{n} k_i x_i
\]

(13)

\( U_{fu} \) is required to guarantee the existence of the sliding mode under the plant parameter variations in \( \Delta \alpha \) and \( \Delta b \) and the disturbances \( f(t) \). Among them,

\[
k_i = \begin{cases} \alpha_i & \text{if } (x_i - T x_i - r K_x) \sigma > 0 \\ \beta_i & \text{if } (x_i - T x_i - r K_x) \sigma < 0 \end{cases}
\]

and

\[
k_{m,i} = \begin{cases} \alpha_{m,i} & \text{if } \sigma > 0 \\ \beta_{m,i} & \text{if } \sigma < 0 \end{cases}
\]

According to Eq. (8), we know

\[
\sigma = -c_i T (r - x_i) + \sum_{j=0}^{n} \alpha_j x_j + b U - f(t)
\]

and

\[
U = U_{eq} + k_i T (r - x_i) - \sum_{i=0}^{n} k_i x_i.
\]

(14)

Thus,

\[
\dot{\sigma} = -c_i T (r - x_i) + \Delta b U_{eq} + k_i T (r - x_i) + \sum_{i=0}^{n} k_i x_i
\]

\[
= -c_i T (r - x_i) + \frac{\Delta b}{b^0} c_i T (r - x_i) + \sum_{i=0}^{n} k_i x_i + \sum_{i=0}^{n} \beta_i x_i + \sum_{i=0}^{n} \alpha_i x_i
\]

\[
+ \left( k_i (x_i - T x_i - r K_x) + \sum_{i=0}^{n} k_i x_i \right)
\]
The condition for the existence of a sliding mode is known to be

$$\dot{\sigma}\dot{\sigma} < 0. \quad (15)$$

and then

$$\dot{\sigma}=\left[-\Delta \dot{\alpha} + a_0^N b^8 + c_i\alpha_0(l + \Delta b/b^8)^i + b_k\dot{x}_i - \dot{r}_k\right] \dot{\sigma}$$

$$+ \sum_{i=1}^{n} \left[-\Delta \alpha + a_0^N b^8 - c_i\alpha(l + \Delta b/b^8)^i + b_k \dot{x}_i \sigma \right]$$

$$+ [-\Delta \alpha + a_0^N b^8 + b_k \dot{x}_i \sigma]$$

$$+ [N + b_k \dot{x}_i] \dot{\sigma}$$

where

$$N = -(T_{x_0})(\Delta \dot{\alpha} + a_0^N b^8) + \Delta b/b^8[c_i(T-r-x_i)] - f(t)$$

In order for Eq. (15) to be satisfied, the following conditions must be met,

$$\alpha \left\{ \begin{align*}
&= \inf \left[ \Delta \dot{\alpha} + a_0^N b^8 + c_i\dot{x}_i \right]/b \\
&= \sup \left[ \Delta \dot{\alpha} + a_0^N b^8 - c_i\dot{x}_i \right]/b \\
\end{align*} \right. \quad (17a)$$

$$\beta \left\{ \begin{align*}
&= \inf \left[ \Delta \dot{\alpha} + a_0^N b^8 + c_i\dot{x}_i \right]/b \\
&= \sup \left[ \Delta \dot{\alpha} + a_0^N b^8 - c_i\dot{x}_i \right]/b \\
\end{align*} \right. \quad (17b)$$

and where

$$k_{ni} = \left\{ \begin{align*}
&= \left[ \begin{align*}
\alpha_{ni} \left\{ \begin{align*}
&= \inf \left[ -N \right]/b \\
&= \sup \left[ -N \right]/b \\
\end{align*} \right. \\
&= \beta_{ni} \left\{ \begin{align*}
&= \inf \left[ \Delta \dot{\alpha} + a_0^N b^8 + c_i\dot{x}_i \right]/b \\
&= \sup \left[ \Delta \dot{\alpha} + a_0^N b^8 - c_i\dot{x}_i \right]/b \\
\end{align*} \right. \\
\end{align*} \right. \quad (17c)$$

Now we consider the effect of $\Delta k_i (i = \ldots, n)$, $\Delta k_i$ is the function is to eliminate the chattering phenomenon of the function theory. The subordinate function of $\sigma_n$ is shown in [7].

Firstly take positive constants $\alpha$ and $\beta$ normalize switching function $\sigma$ and its rate of change against time.

Suppose

$$\sigma_n = \alpha \sigma \quad (18)$$

$$\dot{\sigma}_n = \beta \dot{\sigma} \quad (19)$$

The input variable of the fuzzy controller is $\sigma_i \text{sign}(x_i - T_{x_0} - r_k) \cdot \sigma \text{sign}(x_i - T_{x_0} - r_k) \cdot \sigma \text{sign}(x_i)$ and $\dot{\sigma}_n \text{sign}(x_i)$ (i = \ldots, n), the output of the controller is $\Delta k_i$.

Secondly, define the language value of $\sigma_n$ and $\dot{\sigma}_n$ as P, Z, N; $\Delta k_i$ is language value as PB, PM, PS, ZE, NS, NM, NB; as well as their subordinate functions as in Figs. 6-8:

![Fig. 6. The subordinate function of $\sigma_n$.](image)

![Fig. 7. The subordinate function of $\dot{\sigma}_n$.](image)

![Fig. 8. The subordinate function of $\Delta k_i$.](image)

Define the following fuzzy control regularity Table 1:

<table>
<thead>
<tr>
<th>N</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>PB</td>
<td>PM</td>
</tr>
<tr>
<td>Z</td>
<td>PS</td>
<td>ZE</td>
</tr>
<tr>
<td>P</td>
<td>NS</td>
<td>NM</td>
</tr>
</tbody>
</table>

According to the above form, use the fuzzy calculation method introduced in [6] and gravity method to turn fuzzy output into precise control quantity

$$\Delta k_i = \left[ \Delta k_i \mu_{k_i} d\Delta k_i \right] / \left[ \mu_{k_i} d\Delta k_i \right] \quad (20)$$

where

$$\sigma_n \leq \frac{1}{3} \dot{\sigma}_n \leq \frac{1}{3} \quad \text{it is easy to get } \Delta k_i \text{; and when}$$

$$\sigma_n \leq \frac{1}{3} \dot{\sigma}_n \leq 0; \quad \sigma_n (N), \quad \dot{\sigma}_n (N,Z).$$

The subordinate function of $\Delta k_i (PB, PM)$ corresponding to is shown in [7].

Thus, points P1 and P2’s absissa are $\dot{\sigma}_n + \frac{2}{3} \dot{\sigma}_n + \frac{1}{3}$; P3 and P4’s absissas are $-\dot{\sigma}_n + \frac{2}{3} \dot{\sigma}_n + \frac{4}{3}$; then

$$\Delta k_i = -\frac{5}{2} \dot{\sigma}_n^2 - \frac{2}{3} \sigma_n^2 + \frac{2}{3}$$

$$-3 \dot{\sigma}_n^2 - \sigma_n^2 + \frac{1}{3}$$

Using the same method we get the precise output $\Delta k_i$ under other circumstances to be

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for \( \sigma \), \( \sigma_n \) is \( \sigma, \text{sign}(x_i-T_{X_i}-rK_{P}) \) and \( \bar{\sigma} \) is \( \sigma, \text{sign}(x_i-T_{X_i}-rK_{P}) \); for \( \dot{\sigma} \), \( \dot{\sigma}_n \) is \( \dot{\sigma}, \text{sign}(x_i) \) and \( \bar{\dot{\sigma}} \) is \( \dot{\sigma}, \text{sign}(x_i) \).

Finally, the control function of FLSMC approach for simulate is obtained as

\[
U=U_{eq}+k_{i}(x_{i}-T_{X_{i}}-rK_{P})+\sum_{i=1}^{n}k_{i}x_{i}+k_{1}k_{2}(x_{i}-T_{X_{i}}-rK_{P})+\sum_{i=1}^{n}k_{i}x_{i}
\]

Among them, \( U_{eq} \) is given by Eq. (10), \( k_{i} \) is given by inequality Eq. (17), \( k_{i} \) is given by Eq. (20), therefore \( U \) is a continuous function.

5. THE FLSMC SYSTEM FOR THE UPS

The block diagram of FLSMC system is shown in Fig. 9. The nominal values of the UPS parameters are listed in Table 2. The robustness of the proposed FLSMC approach against large variations of plant parameters and external load disturbances has been simulated for demonstration.

Take \( a=40 \), \( \beta=25 \), \( K_{S}, K_{p}, \sigma \), \( r \) as the ramp function and sampling time as 0.02 second. By considering operating points, one assumes the range of the plant parameter variations to be \( \Delta K_{P}[80 \% \ a_{1}, \Delta K_{S}[80 \% \ a_{2}], \Delta a_{1}[80 \% \ b_{1}], \Delta a_{2}[80 \% \ b_{2}], \Delta N[80 \% \ N] \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance</td>
<td>( C )</td>
<td>20</td>
<td>( \mu F )</td>
</tr>
<tr>
<td>Inductance</td>
<td>( L )</td>
<td>4.5</td>
<td>( mH )</td>
</tr>
<tr>
<td>Gain of Current loop</td>
<td>( K_{P} )</td>
<td>2.7</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>Gain of PWM</td>
<td>( K_{A} )</td>
<td>12.5</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>DC-bus</td>
<td>( V_{dc} )</td>
<td>300</td>
<td>( Volt )</td>
</tr>
</tbody>
</table>

Fig. 9. The block diagram of FLSMC system for the UPS.

6. SIMULATION RESULTS AND DISCUSSIONS

The simulation results of the dynamic response are shown in Fig. 10 and Fig. 11. Fig. 10, shows the trajectory of the output voltage. Where a sinusoidal command \( \{150\sin(120t)\} \) is introduced and the UPS is applied with a random variation of filter parameters \( L \) and \( C \), respectively, from \( 50\% \) to \( 500\% \) of the nominal value under resistive load of \( 50 \Omega \). These curves illustrate the robustness of the FLSMC for the UPS under various loads and abrupt disturbance. Fig. 11, shows the comparison of tracking errors under the same testing conditions. It is clear from the figures that FLSMC can track the sinusoidal command input very fast and extremely. Among others, the FLSMC approach gives the minimum tracking error.
7. CONCLUSIONS

A FLSMC design methodology for the UPS is presented in this paper. The FLSMC configuration is combines the nonlinear integral variable structure control with additional feedforward controller and fuzzy logic. Procedures are developed for choosing the control function for determining the coefficients of the switching plane and the integral control gain such that the system has desired properties. The application of FLSMC to the UPS at sinusoidal frequency has show that the proposed approach can improved the tracking performance and the output waveform of the controlled PWM inverter is much more smooth that that of the previous study methods, like the MIVSC and IVSC strategies. Furthermore, the simulation results demonstrate that the proposed approach can achieve the requirements of robustness and high-quality power supply. It is a practical law for the UPS control systems.

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