Robust Sinusoidal Tracking of High Performance Torsional Plants

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Abstract: In this paper, we study the tracking performance of a torsion disk system where the plant is required to track a triangular-type command signal with a small steady state error and delay. We investigate the tracking performance of the traditional inner/outer loop approach and underline its limitations in high performance applications. We then design a more advanced controller using the mixed sensitivity robust control approach and show that the tracking performance of the system can be improved substantially. The success of the design, even for the case of lightly damped plants such as the one considered in this paper, is largely the result of the proper weights selection used in the mixed sensitivity design. The main contribution of this paper is, therefore, the development of design guidelines for the weights selection when accurate tracking of periodic reference signals are desired.

Keywords: Linear systems, sinusoidal tracking, torsion plants, noncollocated control, robust control

1. INTRODUCTION

Servo motors in a typical industrial environment are linked to their end effectuators by transmission mechanisms having a finite stiffness. The elastically coupled two-mass motor/load system introduces finite zeros and a pair of complex conjugate poles in the transfer function of the system plant and thus brings up the problem of mechanical resonance. The problem is more noticeable in servo systems with position feedback sensors attached to the load side, e.g., to the end effectuators. In this case, the closed loop system will encompass unmodeled torsional resonance modes. The same resonance phenomenon appears in servo systems with a feedback sensor attached to the motor shaft. In such a case, the range of stable loop gains is severely limited along with the overall drive performance. Moreover, the load impact or the reference step input may provoke weakly damped oscillations of the link.

High servo loop gains are required to decrease the error in tracking of fast and steep trajectories. Nevertheless, high control loop gains cannot be practiced due to mechanical resonance modes frequently encountered as unmodeled dynamics within the servo system plant. This paper provides means of improving robustness of existing servo loop controllers with respect to torsional resonance, enabling significant increase of stable control loop gains.

Steady state tracking performance of servo systems is often specified in terms of standard command signals such as steps, ramps, etc. However, in certain applications it becomes necessary to specify a set of sinusoidal tracking performance. Design of control systems to track periodic signals falls in the area of repetitive control. This has been an active area of research in the last two decades where several successful applications have been reported in the literature. However, application of the repetitive control to certain high performance problems, such as those encountered in the turn-around sinusoidal tracking of the image mirror system of weather satellites or the ones appearing in connection with the high bandwidth nano-positioning systems, have been challenging and not quite satisfactory. In contrast, the mixed sensitivity robust design approach taken more recently by several researchers have resulted in systems with a much improved tracking performance. However, selection of the optimal weights needed to carry out this design has proved challenging in most cases. In this paper, we outline a guideline for selecting the parameters of the weighting functions associated with the mixed sensitivity design when small steady state sinusoidal tracking error and delay are the design goal. We then apply our results to a torsional plant and show that the steady state tracking performance is substantially improved over the conventional method.

2. SYSTEM DYNAMICS

Two possible configurations for the torsion disk system considered in this paper is shown in Figure 1. This is a two degree-of-freedom system consisting of two disks supported by a torsionally flexible shaft which is suspended vertically on anti-friction ball bearings. The shaft is driven by a brushless servo motor connected via a rigid belt, with a negligible tensile flexibility, and pulley system with a 3:1 speed reduction ratio. An encoder located on the base of the shaft is used to measure the angular displacement of the lower disk. The upper disk is connected to its encoder by a rigid belt/pulley with a 1:1 speed ratio. Both disks are allowed to rotate freely, i.e., they are not clamped. The system includes a disturbance drive system capable of outputting approximately 1.2[N-m/V] torque per applied input voltage to the upper disk. This drive may be used to take into account the effect of the neglected viscous friction associated with the system.

The schematic diagram of the system can be drawn as shown in Figure 2. Using the free body diagram, the equations of motion can be written as

\[ J_1 \ddot{\theta}_1 + c_1 \dot{\theta}_1 + k_1 (\theta_1 - \theta_2) = T(t), \]
\[ J_2 \ddot{\theta}_2 + c_2 \dot{\theta}_2 + k_2 (\theta_2 - \theta_1) = 0. \]

Assuming zero initial conditions, these equations can be used.
to find the transfer functions
\[ G_1(s) = \frac{\theta_1(s)}{\theta_1(s)} = \frac{N(s)}{D(s)} \quad G_2(s) = \frac{\theta_2(s)}{\theta_1(s)} = \frac{k_1}{N(s)}, \]
where
\[ N(s) = J_2s^2 + c_2s + k_1, \]
\[ D(s) = s(J_1J_2s^3 + (c_1J_2 + c_2J_1)s^2 + (k_1J_1 + J_2) + c_1c_2s + k_1(c_1 + c_2)). \]

To determine the parameters, an identification experiment was conducted by displacing the upper disk by an angle of approximately 20 degrees and collecting data from the upper and lower encoders with the help of a data acquisition board and upon releasing the disk. The collected data were then used to estimate the natural frequency and damping of the poles. In turn, these values were used to calculate the inertia, the viscous damping coefficient, and the spring constant of the system. The values obtained are
\[ J_1 = 0.0108 \text{ [kg m}^2\text{]}, \quad J_2 = 0.0103 \text{ [kg m}^2\text{]}, \]
\[ c_1 = 0.007 \text{ [N rad/s]}, \quad c_2 = 0.001 \text{ [N rad/s]}, \]
\[ k_1 = 1.37 \text{ [N rad]} \]
resulting in the transfer functions
\[ G_1(s) = \frac{91.954[(s + 0.19)^2 + 11.15^2]}{s(s + 0.69)((s + 0.38)^2 + 15.71^2)}, \]
\[ G_1(s) = \frac{125.375}{(s + 0.19)^2 + 11.15^2}. \]

Finally, a test was conducted to calculate the hardware gain of the system. This value was found to be \( K_{hw} = 17.4 \).

3. TRADITIONAL CONTROL APPROACH

A traditional approach of designing a control system for the torsional plant has been to use a rate feedback as the inner loop, and a notch filter along with a PD controller as the outer loop. The measured outputs used for feedback are the angular displacements of the disks, and one of the controlled output chosen is the angular displacement of the lower disk. The latter variable is measured with a sensor whose output is rigidly coupled to the actuator input, thus making its control a colocated scheme. The inner loop is closed with a simple rate feedback in order to dampen the oscillatory mode. The outer loop, on the other hand, is designed using a notch filter to further attenuate the transmission of signals at the damped mode frequency (i.e., nearly canceling the poles with zeros). Finally, a PD control is used in the outer loop to achieve certain performance goals. The control system block diagram is shown in Figure 3.

Taking the conventional approach, an inner loop controller was first designed using a rate feedback with a gain of \( K_v = 0.01 \) to provide the greatest damping. Then the outer loop was designed using a fourth order notch filter
\[ G_n(s) = \frac{1333000[(s + 2.91)^2 + 13.3^2]}{[(s + 44.4)^2 + 44.4^2][(s + 177)^2 + 177^2]} \]
to provide a pair of complex conjugate zeros at the poles of the inner loop feedback. Finally, a PD controller was designed with the proportional constant of \( K_p = 0.06 \) and derivative constant of \( K_d = 0.001 \). The system was then tested with the sweep frequency of 0.1 to 10 Hertz and 400 counts amplitude and the sweep outputs data corresponding to the angular displacements of the upper and lower disks were collected. The noncollocated response obtained from the experiment shows that the system bandwidth is approximately 1.5 Hertz. It also shows that the notch antiresonance has somewhat been overcompensated for the damped response in the plant at about 2.1 Hertz. Finally, the system was tested by inputting a 10,000 count/second ramp trajectory of 2 seconds period and the steady state output responses data for both disks were collected as shown in Figure 4. The results show that the angular displacement for the lower disk has a steady state error of about 1000 counts and a delay of about 0.25 seconds. The angular displacement for the upper disk was found to have a steady state error of about 300 counts and a delay of about 0.2 seconds. It was also observed that adding an integral term to the controller worsens the tracking performance so that a three-term controller was not useful in this application.
The traditional approach to the design has several shortcomings. First, the sensitivity to changes in the upper disk inertia is reduced from the collocated control scheme but is still limited by the relatively low control gain. Secondly, the attainable control gain magnitude is limited by the modest gain margin of the open loop system. Thirdly, the static stiffness at both outputs is proportional to the proportional gain which is, in turn, limited by the gain margin of the system. Fourthly, although the noncollocated frequency response is better flattened through the system bandwidth than the collocated control (which translates to reduced oscillations in the transient response), the bandwidth is limited however by the relatively low control gain and hence the step rise time and ramp following lag are moderately large. Finally, the steady state error to a step input may be eliminated by addition of integral action but the attendant overshoot is often undesirable for high performance tracking applications.

4. WEIGHTS SELECTION FOR ROBUST SINUSOIDAL TRACKING

Keeping the aforementioned challenges in mind, we offer a new approach to the design with the hope of achieving a better tracking performance for the system. The approach taken in this paper is based on the more recent results available in the area of robust control theory. In particular, by formulating the problem as a mixed sensitivity robust control problem, we will show that the tracking performance of the system to a sinusoidal command signal can significantly be improved over the conventional approach. The mixed sensitivity robust control design considered in the next section requires the determination of certain frequency weights for the sensitivity function, complementary sensitivity function, and control effort. While the proper selection of weights requires a moderate effort in a typical tracking application, the optimal choice of the weights for high-performance applications, which are subject to stringent tracking performance requirements, is much more challenging. In this section, we outline a procedure for the proper selection of the weights in a given high performance sinusoidal tracking application. This procedure takes into account the usual tracking performance parameters including the bandwidth and sensitivity peak, as well as other parameters pertinent to tracking of sinusoidal reference signals such as the steady state error and delay. We then apply the results to the torsional plant discussed above in the next section and show that a superior tracking performance is achievable with the proposed approach.

Consider the feedback system shown in Figure 5 and let $S(s) = 1 + G(s)K(s)$ and $T(s) = 1 - S(s)$ be the sensitivity and complementary sensitivity transfer functions, respectively. In the $S/T$ mixed sensitivity design, the objective is to minimize the infinity norm

$$\alpha = \inf_{\omega} \left\| \frac{W_P S}{W_T T} \right\|_{\infty}$$

where $W_P(s)$ and $W_T(s)$ are the performance and the stability weights, respectively. These weights are often taken to be

$$W_P(s) = \left( \frac{s/\sqrt{M_S + \omega_H^2}}{s + \omega_H^2 \sqrt{A_S}} \right)^m,$$

$$W_T(s) = \left( \frac{s/\omega_B + 1/\sqrt{M_T}}{\sqrt{A_T s/\omega_B^2} + 1} \right)^n.$$

The amplitude responses of these weights and their inverses are shown in Figure 6.

To study the tracking performance of the feedback system in Figure 5, let $d = n = 0$ and consider the sinusoidal reference command

$$r(t) = A_r \cos \omega_r t, \quad \omega_r \ll \omega_B.$$

It then follows that the steady state error signal $e_{ss}(t) = r(t) - y_{ss}(t)$ can be written in the compact form

$$e_{ss}(t) = R_e \cos(\omega_r t + \phi_e)$$

where

$$R_e = A_r \sqrt{1 + \left| T(j\omega_r) \right|^2 - 2|T(j\omega_r)| \cos \omega_r \tau_e},$$

$$\phi_e = \arctan \left( \frac{|T(j\omega_r)| \sin \omega_r \tau_e}{1 - |T(j\omega_r)| \cos \omega_r \tau_e} \right),$$

and

$$\tau_e = \frac{\frac{2}{\omega_r} T(j\omega_r)}{\omega_r}.$$
represents the tracking delay. The objective is to select the parameters of the weighting functions \( W_P(s) \) and \( W_T(s) \) so that \( R_e \) and \( \tau_e \) are as small as possible over the tracking bandwidth. Guidelines for achieving this objective are outlined below.

1. Initially, let \( m = 1 \) and calculate \( \omega_B^* \) from the expression

\[
\omega_B^* \approx \omega_r \left( \frac{A_r}{R_e} \right)^{1/m}.
\]

(1)

If this value is too large, increase \( m \) and re-calculate \( \omega_B^* \).

2. Calculate \( M_T \) from

\[
M_T \approx \cos \omega_r \tau_e + \sqrt{\left( \frac{\omega_B^*}{\omega_r} \right)^{2m} - \sin^2 \omega_r \tau_e}
\]

(2)

using the values of \( m \) and \( \omega_B^* \) obtained in Step 1.

3. Let \( n = 1 \) and calculate \( \omega_{BT}^* \) from

\[
\omega_{BT}^* \approx \frac{\omega_r \sqrt{M_T}}{\tan \left( \frac{\omega_r \tau_e}{n} \right)}
\]

(3)

with the values of \( m \), \( \omega_B^* \), and \( M_T \) calculated in Steps 1 and 2. If \( \omega_{BT}^* \) is not large enough, increase \( n \) and recalculate \( \omega_{BT}^* \) from this expression till a satisfactory result is obtained.

4. Finally, calculate \( M_S \) from

\[
M_S \approx \left[ \frac{\omega_r}{\omega_B^* \tan \left( \frac{\omega_r \tau_e}{m} \right)} \right]^m \quad (\gamma \neq m\pi/2)
\]

(4)

where

\[
\gamma = \arctan \left( \frac{M_T \sin \omega_r \tau_e}{1 - M_T \cos \omega_r \tau_e} \right),
\]

(5)

or from

\[
M_S \approx \frac{\omega_r M_T \sin \omega_r \tau_e}{\omega_B^* [1 - M_T \cos \omega_r \tau_e]} \quad (M_T \cos \omega_r \tau_e \neq 1)
\]

(6)

if \( m = 1 \), using the values of \( \omega_B^* \), \( m \), and \( M_T \) calculated in Steps 1 and 2.

We now use the above procedure for finding the weights for the torsional plant. Since we expect our design to be at least as good as the traditional design, we take the traditional design performance as our reference. In the traditional design, the angular displacement of the upper disk was found to have a steady state error of about 300 counts and a delay of about 0.2 seconds. Therefore, we take \( R_e = 300 \) and \( \tau_e = 0.2 \). Also, we note that the period of the triangular command is 2 seconds which corresponds to the fundamental frequency of 0.5 Hertz. For the triangular wave, tracking of the first three harmonics is often sufficient. This consideration gives the minimum tracking bandwidth of about 1.5 Hertz. Therefore, we take \( \omega_r \approx 9.4 \) [rad/sec]. With \( A_r = 10000 \) and \( m = 1 \), from (1) we obtain \( \omega_B^* \approx 331 \) [rad/sec] which is too large. So, we increase \( m \) to a value of 2 and recalculate \( \omega_B^* \) from (1). We now obtain \( \omega_B^* \approx 52 \) [rad/sec] which is about 8.6 Hertz. This seems to be reasonable, so we proceed with the selection of the remaining parameters.

With the above values, from (2) we obtain \( M_T \approx 5.3 \) which seems to be slightly high but may be unavoidable due to the presence of the lightly damped modes associated with the plant. Next, we let \( n = 1 \) and from (3) we obtain \( \omega_{BT}^* \approx 67 \) [rad/sec]. Finally, from (4) we get \( M_S \approx 0.5 \) which is slightly lower than we might have expected.

The values we have chosen for the design are

\[
\begin{align*}
n &= 2, & M_S &= 1, & \omega_B^* &= 50, & A_S &= 10^{-6}, \\
m &= 1, & M_T &= 5, & \omega_{BT}^* &= 70, & A_T &= 10^{-3}.
\end{align*}
\]

(7)

These values yield the following weighting functions for the sensitivity and complementary functions

\[
W_P(s) = \left( \frac{s + 50}{s + 0.05} \right)^2, \quad W_T(s) = \frac{s/70 + 0.2}{10^{-3}s/70 + 1}.
\]

(8)

Inverse of these weights are shown in Figure 7.

### 5. ROBUST CONTROL APPROACH

The block diagram of the system for the robust control design is shown in Figure 8.

The first step in the design is to identify all the appropriate inputs and outputs that are needed for the computation of the generalized plant. The control input is \( u \), the measured outputs are \(-\theta_1 + \theta_2\), the exogenous inputs are \( r \) and \( d \),
and the exogenous outputs are \( z_1 = W_P (r - \theta_2) \) and \( z_2 = W_T \theta_2 \). Thus, the generalized plant is given by

\[
\begin{bmatrix}
  z_1 \\
  z_2 \\
  v_1 \\
  v_2
\end{bmatrix} =
\begin{bmatrix}
  -G_2 W_P & W_P & -K_{h_w} G_1 G_2 W_P \\
  G_2 W_T & 0 & K_{h_w} G_1 G_2 W_T \\
  -G_2 & I & -K_{h_w} G_1 G_2 \\
  0 & 0 & -K_{h_w} G_1
\end{bmatrix}
\begin{bmatrix}
  d \\
  r \\
  u
\end{bmatrix}.
\]

The controller, acting on the measured outputs, is given by

\[ u = Kv = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} r - \theta_2 \\
 -\theta_1 \end{bmatrix}. \]

Finally, the standard mixed sensitivity design was carried on the generalized plant and the resulting controller was simplified through a model reduction scheme. The controller obtained directly from the optimization was a ninth order controller, but the reduced order controller obtained subsequently through the optimal Hankel norm approximation was a seventh order and compatible to the one obtained through the conventional approach. However, the controller obtained through the robust control approach had a far superior performance compared to the conventional approach as can be clearly seen from Figure 9. This figure shows the closed loop response of the system to the same ramp trajectory used in testing the conventional design and the steady state output responses for the upper disk was obtained under the proposed controller. The results obtained show that the angular displacement of the upper disk gives a steady state error of about 100 counts and a delay of about 0.1 seconds. These results are encouraging and clearly show that the proposed method provides a superior performance, by a factor of at least 2, over the conventional method.

6. CONCLUSION

In this paper, we have developed a set of guidelines for selection of the weighting functions in the robust mixed sensitivity control design when the objective is to track a set of sinusoidal reference signals with high precision. We have applied our design methodology to a torsional plant and have shown that a superior tracking performance can be expected with the mixed sensitivity robust controller over the conventional approach of utilizing inner and outer loops with a rate feedback, notch filter, and PD controller. The results reported in this paper have a wide range of applications. It can compete with the traditional approach in applications such as machine tools, automobiles (cruise control), and spacecraft (attitude and gimbal control) where the plant is modeled as a rigid body and a reasonable tracking performance is required of the system. It provides a superior performance over the traditional approach in high performance applications where the rigid body model can no longer adequately represent the plant. The results obtained in this paper should prove valuable, particularly for the latter class of applications mentioned above.

Several questions remain unanswered and will be the subject of future work. For example, it is well known that the presence of plant’s non-minimum zeros has an adverse effect on the peak sensitivity gain and puts a restriction on the achievable tracking bandwidth. Therefore, selection of appropriate weights in the mixed sensitivity design problem, when tracking a sinusoidal reference signal, for the plants possessing non-minimum phase zeros would be an interesting area of future research.

References


