Dynamic Modelling and Control of Solid-Oxide Fuel Cell

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Abstract - In this paper, the dynamic models of SOFC are suggested. It consists of electrochemical model, thermal model, voltage equation and several loss equations. Control problem on tracking steady voltage by air flow is discussed and an adaptive controller is designed to withstand the variation of stack current. Simulation is done to prove the solution of control algorithms.

1. INTRODUCTION

Fuel cells are attractive as the electric power production of the future because they are modular, efficient, and environmentally friendly. However, fuel cells are dynamic devices which will affect the dynamic behavior of the power system to which they are connected and hence analysis of such a behavior requires an accurate dynamic model.

Two types of fuel cells are likely to be used as power plants namely solid-oxide fuel cells (SOFC) and molten carbonate fuel cells (MCFC). Each has a specific dynamic model. Most of the published models, however, concentrate on standalone fuel cells. The model proposed in this paper includes the electrochemical and thermal aspects of chemical reactions inside the stack of SOFC and voltage losses due to activation, concentration, ohmic losses are account for.

In this paper, the dynamic model of SOFC will be suggested and dealt on some control problem. Simulation will also be done to prove the applicability of the solution of control algorithms.

2. MAIN SUBJECT

2.1 Dynamic Modelling for SOFC

The proposed dynamic model of SOFC is based on the chemical, thermal and electrical principles and has multiple input and output as shown in Fig.1.

The two inputs are fuel \( H_2 \) and air \( O_2 \) and three outputs are DC voltage \( V_d \), water \( H_2O \) and heat \( T \).

The electrochemical model of fuel, water and air will be represented by the component material balance equations as follows.

\[
\begin{align*}
\dot{x}_1 &= -\frac{1}{r_1} x_1 + \frac{1}{r_1 K_1} N_1 - \frac{K_1}{r_2 K_1} I_s : \text{fuel flow} \\
\dot{x}_2 &= -\frac{1}{r_2} x_2 + \frac{K_2}{r_2 K_2} I_s : \text{water flow} \\
\dot{x}_3 &= -\frac{1}{r_3} x_3 + \frac{1}{r_3 K_3} N_3 - \frac{K_3}{r_3 K_3} I_s : \text{air flow} \\
\tau_1 &= \frac{W}{K_1 R T} , \quad \tau_2 = \frac{W}{K_2 R T} , \quad \tau_3 = \frac{W}{K_3 R T} \\
K_1 &= \frac{N_1}{x_1} , \quad K_2 = \frac{N_2}{x_2} , \quad K_3 = \frac{N_3}{x_3}
\end{align*}
\]

where, \( x_1, x_2, x_3 \) are mole fractions, \( r_1, r_2, r_3 \) are time constants and \( K_1, K_2, K_3 \) are molar constants with fuel, water and air, respectively. \( N_1, N_2, N_3 \) are flow rates at the input cell and \( N_1, N_2 \) are reaction rates of fuel and air, respectively. \( \hat{F} = 0.25NF \) is a constant dependent on Faraday’s constant \( \hat{F} \) and number of electrons \( N \) in the reaction, \( I_s \) is a stack current, \( W \) is a compartment volume, \( R \) is a gas constant and \( T \) is a cell temperature.

The thermal model will be represented by the energy balance equations as follows.

\[
M_c C_p \dot{T} = q_c W_c + \sum Q_i : \text{thermal dynamics}
\]

where, \( M_c \) is a mass, \( C_p \) is a heat capacity, \( W_c \) is a volume, \( q_c \) heat generation of the cell unit and \( Q_i \) is a total heat between cell and separators.

The stack output voltage and ohmic, concentration, activation losses will be represented by the Nernst equation as follows.

\[
V_{dc} = V_0 - \eta_{ohm} - \eta_{con} - \eta_{act}
\]

\[
V_0 = N_0 \left[ E_0 + \frac{RT}{2F} \ln \frac{x_1 \sqrt{x_3}}{x_2} \right] : \text{output voltage}
\]
\[ \eta_{ohm} = r \frac{J}{x} \text{ at } r = a \exp \left[ \frac{b}{T_0} \left( \frac{1}{T} - \frac{1}{T_0} \right) \right] \] : ohmic loss

(9)

\[ \eta_{con} = \frac{RT}{nF} \ln \left( \frac{1}{I_L} \right) \] : concentration loss

(10)

\[ \eta_{act} = \frac{RT}{nF} \ln \left( \frac{I_L}{I_0} \right) \approx a + b \ln(I_L) \] : activation loss

(11)

where,

- \( V_0 \) : open-circuit reversible potential
- \( N_0 \) : number of cells in stack
- \( E_0 \) : standard reversible cell potential
- \( T_0 \) : constant temperature
- \( r \) : ohmic resistance
- \( a, b \) : constant coefficients
- \( n \) : number of electrons in reaction
- \( I_L \) : limiting current
- \( a, b \) : Tafel constant and Tafel slope

The above all equations are based on the following assumptions.

1) Stack is fed with hydrogen and air.
2) A uniform gas distribution among cells.
3) There is no heat transfer among cells.
4) The ratio of pressures between the interior and exterior of the channel is large enough to consider orifice choked.

Using the above all equations and Laplace transformation, a dynamic model of SOFC will be represented as fig.2.

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### 2.2 Adaptive Control for Tracking Problem

The simulation result for output voltage of the stack was presented in Fig.3 if the stack current should be maintained at constant value. The target system of SOFC has rated power of 100[kW], rated stack voltage of 286.3[V] and rated stack current of 300[A] and assume that the variation of temperature can be negligible. System parameters and several data need to simulation are arranged at Table.1.

![Fig.3 Simulation result for output voltage (V_d)](image)

#### Table.1 System parameters and data for simulation

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>VALUE/UNIT</th>
<th>SYMBOL</th>
<th>VALUE/UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 )</td>
<td>1.0(sec)</td>
<td>( T )</td>
<td>1000(°C)</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>2.0(sec)</td>
<td>( T_0 )</td>
<td>923(°C)</td>
</tr>
<tr>
<td>( \tau_3 )</td>
<td>1.5(sec)</td>
<td>( N_0 )</td>
<td>384</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>0.5</td>
<td>( E_0 )</td>
<td>0.8(V)</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>0.2</td>
<td>( a )</td>
<td>0.2</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>0.9</td>
<td>( \beta )</td>
<td>-2870</td>
</tr>
<tr>
<td>( \dot{N} )</td>
<td>12.0(mole/sec)</td>
<td>( J_L )</td>
<td>300[A]</td>
</tr>
<tr>
<td>( \dot{N} )</td>
<td>24.0(mole/sec)</td>
<td>( J_L )</td>
<td>500[A]</td>
</tr>
<tr>
<td>( K_p )</td>
<td>0.01</td>
<td>( a )</td>
<td>0.05</td>
</tr>
<tr>
<td>( R )</td>
<td>8.31[J/mole*K]</td>
<td>( b )</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Stack output voltage in Fig.3 can be converge to some steady value, however, a little error of output voltage will be exist by the variation of system parameters (especially, stack current). Hence, it is need to design adaptive controller to meet reference output voltage withstand to variation of parameters.

It is assume that the fuel flow \( \dot{N}_f \) is constant, stack current \( J_L \) is unknown parameter( \( \beta \) and air flow \( \dot{N}_f \) is control input \( \dot{J} \) to track output voltage \( V_d \) to reference voltage \( V_d \). For the control objective, define the tracking error as

\[ e = V_d - V_d \]

(12)

and estimation error as...
\( \bar{V} = \bar{v} - \bar{\theta} \)  

(13)

where, \( \bar{\theta} \) is a real constant value and \( \bar{\theta} \) is estimated value of unknown stack current. The Lyapunov function is chosen as

\[
U_L = \frac{1}{2} \varepsilon^2 + \frac{1}{2\gamma} \bar{\theta}^2
\]

(14)

where, \( \gamma \) is an adaptation gain. If the losses in eqn (9)-(11) are very little, then the derivative of \( U_L \) is

\[
U_L = e\dot{e} + \frac{1}{\gamma} \bar{\theta}\bar{\theta} = e V_d - \frac{1}{\gamma} \bar{\theta}\bar{\theta}
\]

(15)

\[
g_1 = \frac{1}{1 + \frac{1}{r_2} + \frac{N_1}{r_1K_{v_1}}}
\]

\[
g_2 = \frac{K_r}{r_1K_{v_1}} + \frac{K_r}{r_2K_{v_2}} - \frac{K_r}{2r_3K_{v_3}}
\]

\[
g_3 = \frac{1}{2r_3K_{v_3}}
\]

and \( c = \frac{N_{KT}}{2F} \). Now, the adaptive and control law are chosen as follows

\[
\bar{\theta} = \gamma g_3 : \text{adaptive law}
\]

(18)

\[
u = -\frac{1}{g_3} \left( k e + g_1 + g_2 \bar{\theta} \right) : \text{control law}
\]

(19)

where, \( k \) is a control gain. The derivative of \( U_L \) at eqn(2.10) will be as

\[
U_L = -ke^2 \leq 0
\]

(20)

Using Barbalat’s lemma [6], it can be shown that \( U_L(\bar{\theta}) \) tends to zero as \( t \rightarrow \infty \). Therefore, tracking error \( e \) will also converge to zero as \( t \rightarrow \infty \). As a result, the stability of the proposed adaptive control system can be guaranteed.

Simulation was done with \( k = 1, \gamma = 0.1, V_d = 28V \) and the results are depicted in Fig.4. It shows that stack output voltage is converge to reference voltage and parameter estimation value is also converge to constant value.

3. CONCLUSION

In this paper, the dynamic models of SOFC are presented and it consists of electrochemical model, thermal model, voltage equation and several loss equations. Control problem on tracking steady voltage by air flow is solved by proposed adaptive controller which designed to withstand to the variation of stack current. The appropriate adaptive and control law are designed and the effectiveness of the proposed controller is proved by simulation results. Experiments will be performed to confirm to this results in the near future.

![Fig.4 Simulation results for the proposed controller](image)

[REFERENCE]


- 2518 -