An INS Filter Design Considering Mixed Random Errors of Gyroscopes

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Abstract: We propose a filter design method to suppress the effect of gyroscope mixed random errors at INS system level. It is based on the result that mixed random errors can be represented by a single equivalent ARMA model. At first step, the time difference of equivalent ARMA process is performed, which consider the characteristic of indirect feedback Kalman filter used in INS filter. Next, a state space conversion of time differenced ARMA model is achieved. If the order of AR is greater than that of MA, the controllable or observable canonical form is used. Otherwise, we introduce the state equation of which the state variable is composed of the ARMA model output and several step ahead predicts of that. At final step, a complete form state equation is presented. The simulation results shows that the proposed method gives less transient error and better convergence compared to the conventional filter which assume the mixed random errors as white noise.

Keywords: gyroscope, mixed random error, equivalent ARMA model, indirect feedback Kalman filter

1. INTRODUCTION

There is a kind of random errors in which a number of different forms of noises are mixed as commonly seen in gyroscopes. Various methods of modeling random errors have been studied in the past[1-4]. Recently, the author showed that a mixture of several noises such as white noise, random walk, quantization noise, Markov process, and other forms of general ARMA processes can be represented by a single equivalent ARMA model[5]. Previous results are focused on the modeling and suppressing the gyro random errors at sensor level and the filtering methods at INS system level are rarely studied.

In this paper, based on the result that mixed random errors can be represented by a single equivalent ARMA model, we propose a filter design method to suppress the effect of gyro random errors at INS system level. First, the time difference of equivalent ARMA process is performed which is included in the INS filters as state variable. The reason to perform the time difference is to consider the characteristic of indirect feedback Kalman filter used in the indirect feedback Kalman filter of which the linear state space model is presented. At this step, there are two directions which depend on the orders of AR and MA parts. If the order of AR is greater than that of MA, the ARMA model is converted to state equation easily using the controllable or observable canonical form. Otherwise, since the canonical form representation result in colored process noise, different type of model equation is needed. For this, we introduce the state equation of which the state variable is composed of the ARMA model output and several step ahead predicts of that. Third, a complete form state equation is presented. Via simulation results, we compare the proposed filter with the conventional filter which assumes the mixed random errors as white noise.

The paper is composed as follows. In section 2, we explain the conventional INS filter. In section 3, as main result, a new INS filter which considers mixed random errors is proposed. In section 4, some simulation results are presented. Finally we conclude in section 5.

2. PRELIMINAR

A conventional INS filter is explained. To compensate navigation error, indirect feedback Kalman filter is widely used. Via feedback of the state estimate to the linearized filter, the indirect feedback Kalman filter maintain the linearity of the error propagation equation which is acquired by perturbing the nonlinear INS equation[6]. If we consider only the random constant error among the gyroscope and accelerometer errors, the filter can be represented as follows and the order is 12.

\[
\begin{bmatrix}
\dot{f}_x \\
\dot{f}_y \\
\dot{f}_z \\
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & 2 \\
0 & 0 & 3 \\
\end{bmatrix}
\begin{bmatrix}
f_x \\
f_y \\
f_z \\
\end{bmatrix}
+ Gw_f
\]

where \( f_i \) is acceleration of vehicle, \( \Omega_i \) is earth rotation angular velocity and \( \rho_i \) is navigation frame rotation angular
velocity. To implement the filtering in the digital computer, the discrete version of Eq. (1) is needed.

\[
\begin{bmatrix}
    x_{f,t+1} \\
    x_{a,t+1}
\end{bmatrix} = \begin{bmatrix}
    I_{6d} & + \\
    0_{6d} & \Delta t
\end{bmatrix} \begin{bmatrix}
    x_{f,t} \\
    x_{a,t}
\end{bmatrix} + Gw_{f,t} + (2)
\]

\( \Delta t \) is time interval at which the update is executed. If we use velocity aided navigation, the measurement equation is as follows.

\[
h_t = \begin{bmatrix}
    1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
    x_{f,t} \\
    x_{a,t}
\end{bmatrix} + v_t
\]

\( v_t \) is velocity measurement noise and is assumed white noise.

3. INS FILTER CONSIDERING GYROSCOPE MIXED RANDOM ERRORS

We reconstruct the INS filter considering the gyroscope mixed random errors. The gyroscope noise in Eq. (2) is an extended vector of considering gyroscope 3 axes. Its coordinate transform matrix which transform gyroscope error into state space form using some due to the time difference compared to \( z_t \). If \( p > q + 1 \), \( \tilde{z}_t \) can be realized into state space form using observability or controllability canonical representation[6]. In the case \( p \leq q + 1 \), we introduce a difference realization method based on the fact that \( \tilde{z}_t \) can be represented as the sum of on step ahead predictor of \( \tilde{z}_t \) at \( t - 1 \) and white noise[7]. The resulting state space equation is as follows.

\[
\begin{bmatrix}
    x_{f,t+1} \\
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\end{bmatrix} + v_t
\]

\( v_t \) is velocity measurement noise and is assumed white noise.
4. SIMULATION

Through simulation, we compare the performance of the proposed filter compared to the conventional filter which assume the gyroscope mixed random errors simply as white noises. We use the order of ARMA(1,1) process as the equivalent ARMA model. The vehicle trajectory used is descending ‘S’ curve as shown in Fig. 1. The total flight time is 600 sec. As simulation conditions, we use 1 ft/sec initial velocity error, 0.1 deg initial attitude error, 1000 $\mu g$ accelerometer bias, 1 deg/h gyroscope and initial covariance corresponding initial errors. The white noise covariance of accelerometer is $(30 \, \mu g)^2$. The parameters of ARMA(1,1) are -0.98 for AR, 0.6 for MA and $(1 \, \text{deg}/h)^2$ for the white noise covariance. The white noise covariance of measurement is $(0.01 \, \text{ft/sec})^2$.

The simulation results show that the proposed filter gives less transient navigation error and faster convergence to zero compared to the conventional filter. In Fig. 2 and Fig. 3, the attitude error is presented. As seen in the figures, the proposed filter gives less transient yaw angle error and faster convergence to zero.

5. CONCLUSION

In this paper, based on the result that mixed random errors can be represented by a single equivalent ARMA model, we propose a filter design method to suppress the effect of gyro random errors at INS system level. At first step, considering the characteristic of the indirect feedback Kalman filter, the time differenced form of equivalent ARMA process is included in the INS filter. At next step, the time differenced ARMA process is represented as state space form. If the order of AR is greater than the one of MA, the ARMA model is converted to state equation easily using the controllable or observable canonical form. Otherwise, we introduce the state equation of which the state variable is composed of the ARMA model output and several step ahead predicts of that. Third, a complete form state equation is presented by including the system and measurement equations of the state space representation of time differenced ARMA process. Via simulation results, we showed that the proposed method gives less transient error and better convergence. From the results we can confirm that the proposed method is effective to reduce the effect of mixed random errors when they are not eliminated at sensor level.

REFERENCES