Robust Kalman filtering for the TS Fuzzy State Estimation

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Abstract - In this paper, the Takagi-Sugeno (TS) fuzzy state estimation scheme, which is suggested for a steady state estimator using standard Kalman filter theory with uncertainties. In that case, the steady state with uncertainty can be represented by the TS fuzzy model structure, which is further rearranged to give a set of uncertain linear model using standard Kalman filter theory. And then the unknown uncertainty is regarded as an additive process noise. To optimize fuzzy system, we utilize the genetic algorithm. The steady state solutions can be found for proposed linear model then the linear combination is used to derive a global model. The proposed state estimator is demonstrated on a truck-trailer.

1. Introduction

The design of a Kalman filter relies on having an exact dynamic model of the system under consideration in order to provide optimal performance when the design contains relatively small modeling errors [1]. However, most dynamical systems in the world have severe nonlinearity. In order to design of optimal filters, many recent works have dealt with the problem of robust filter design for all admissible uncertainties by minimizing the upper bound on the variance of estimation error. The uncertainty is parameterized in terms of a norm-bounded parameter matrix [2, 3]. Conventionally, the extended Kalman filter has been proposed for state estimation by using a linearization of the nonlinear systems around the present estimate through an application of linear filter theory [4]. However, few works have studied the estimation problem for nonlinear uncertain systems. In this paper, the uncertain nonlinear system is represented by T-S fuzzy model structure, which is further rearranged to give a set of uncertain linear systems. The unknown uncertainty is regarded as an additive process noise and then the time-varying variance of the process noise is calculated. This model is designed for a local linear state space model using standard Kalman filter theory. Finally, the proposed state estimator is demonstrated on a truck-trailer.

2. Fuzzy State Estimation

2.1 Preliminaries and problem formulation

Nonlinear systems can be approximated as locally linear systems in much the same way that nonlinear functions can be approximated as piecewise linear functions. Consider a nonlinear dynamical system of the form:

\[ x(k+1) = f(x(k), u(k)) \]

This nonlinear system can be approximated by a T-S fuzzy model, which is composed of a set of fuzzy inference rules. Nonlinear dynamical system with uncertainties can be represented by fuzzy linear models of the form:

\[ IF \quad z(i)(k) \in \tilde{P}_i \quad \text{THEN} \quad z(i+1)(k) = A_i z(i)(k) + B_i u(k) + C_i(w(k)) \]

where \( \tilde{P}_i \) is the fuzzy set, \( L \) is the number of fuzzy rules, \( z(i)(k) \) is measured output, \( A_i \) and \( B_i \) are known constant matrices, the process noise \( w(k) \) is white with PSD \( S_w \), the measurement noise \( v(k) \) is white with PSD \( S_v \), and the process noise and measurement noise are uncorrelated, \( \Delta A_i(k) \Delta A_i^T(k) \leq 1 \) is uncertain time-varying matrix, and \( x(k), z(i), w(k) \) are premise variables.

Now we define \( L \) discrete time signals \( x(k) \) and \( y(k) \). The final output of the fuzzy system is inferred as follows:

\[
\begin{align*}
  x(k+1) &= \sum_{i=1}^{L} \mu_i(x(k))[A_i + \Delta A_i x(k)] + B_i u(k) + G_i w(k) \\
  y(k) &= \sum_{i=1}^{L} \mu_i(x(k))[G_i x(k) + v(k)]
\end{align*}
\]

where

\[ A_i(k) = \sum_{i=1}^{L} \mu_i(x(k))A_i \]

\[ \Delta A_i(k) = \sum_{i=1}^{L} \mu_i(x(k))\Delta A_i \]

\[ w_i(x(k)) = \sum_{i=1}^{L} \mu_i(x(k))w_i(x(k)) \]

\[ F_i(z_i(k)) \]

\[ \mu_i(x(k)) = \sum_{i=1}^{L} \mu_i(x(k)) \]

with \( w_i(x(k)) \geq 0 \) for all \( i \), we get the following form:

\[ \sum_{i=1}^{L} \mu_i(x(k)) = 1 \]

From these definitions, we define \( L \) discrete time signals and it can be defined as

\[ x_i(k) = \mu_i(x(k))x(k), \quad z_i(k) = \mu_i(z_i(k)) \]

2.2 Filtering using the fuzzy model

In this section we modify the Kalman filter for the system given by using (3) and (5). Suppose we are given the linear discrete time system of the form.

\[ x(k+1) = Ax(k) + p_z(k)\Delta z(k) + p_z(k)Bw(k) + p_z(k)\mu \]

\[ y(k) = C\Delta x(k) + p_z(k)v(k) \]

where the \( p_z(k) = (\Delta A + u(k)) \) is the overall process noise with uncertain uncertainty matrix; thus we can treat the uncertain linear discrete system by adjusting this process noise variance.

We combine the Kalman filter for the local systems given in (7) to obtain a state estimator for the TS fuzzy model given in (2). The state \( \hat{x} \) of the system can be derived by assuming a recursive estimator of the form. The predicted state is represented as

\[ \hat{x}^*(k) = (I - K(k)C(k))\hat{x}^*(k-1) + K(k)v(k) \]

\[ \hat{x}^*(k+1) = A\hat{x}^*(k) + Bw(k) \]

where \( ^* \) is superscript to indicate a quantity before the measurement is taken into account, and \( ^+ \) superscript to indicate a quantity after the measurement is taken into account.

Because of the modified state \( \hat{x}^*(k) \), the measurement residual is defined as

\[ v(k) = y(k) - \hat{x}^*(k) \]

where \( v(k) \) is the measurement residual, and \( K(k) \) is a Kalman gain whose matrices are to be determined.

The unknown process noise with uncertainty \( w(k) \) are inferred by a double-input single-output fuzzy system, for which the \( j \)th fuzzy IF-THEN rule is represented by

\[ IF x_1 = A_{j1} \quad \text{and} \quad x_2 = A_{j2}, \quad THEN \quad y_j = \bar{w}_j \]
where two premise variables \( z_1 \) and \( z_2 \) are the measurement residual \( v(k) \) and change rate \( \dot{v}(k) \), respectively, consequent variable \( y_3 \) is the overall process noise, and \( A_k \) is fuzzy set. It has the Gaussian membership function with center \( z_{ij} \) and standard deviation \( \sigma_{ij} \)

\[
f(z_i, \bar{z}_{ij}, z_{ij}) = \exp \left[ -\frac{1}{2} \frac{(z_i - \bar{z}_{ij})^2}{\sigma_{ij}^2} \right]
\]

(12)

In this paper, the GA methods will be applied to optimize the parameters and the structure of the system, using the product-sum inference method, singleton fuzzifier, center average defuzzifier, and Gaussian membership function. That is, the defuzzified output of the fuzzy model based on the overall process noise with unknown uncertainty is given by

\[
w_k = \frac{1}{\sum_i A(z_{ij})} \sum_i A(z_{ij}) \times A(z_{ij})
\]

(13)

According to the approximation theorem by the GA, the overall process noise is optimized.

We define the estimator error \( \hat{z} \) and its covariance with \( P \) as,

\[
\hat{z} = x - \hat{x}, \quad P = E(\hat{z}\hat{z}^T)
\]

(14)

The steady state Kalman filter presented can be used to estimate the states of each of the \( L \) dynamic systems given in (7). This will give us \( L \) local steady state estimates. Due to the estimated term \( \hat{v} \), the covariance matrix of \( P(k+1) \) becomes

\[
P(k+1) = A_k P(k) - K_k C_k P(k) A_k^T + Q_k \hat{v}_k \quad \hat{v}_k = C_k [w_k + \hat{v}_k]
\]

(15)

When the unknown uncertainty is employed, the conventional Kalman filter has to be modified.

We can find the optimal Kalman gain by using (15),

\[
K_k = P(k) C_k^T (C_k P(k) C_k^T + 2 \hat{v}_k)^{-1}
\]

(16)

The predicted state is replaced by (16)

\[
\hat{x}_k = (I - K_k C_k) \hat{x}_k + K_k \hat{v}_k
\]

(17)

From (5), we can combine the local state estimates as

\[
\hat{x}_k = \sum_i \hat{x}_k
\]

(18)

2.3 Simulation results

In this section we consider state estimation for a discrete time model of a truck-trailer system. A noise-free representation of a truck-trailer system can be described as (6)

\[
\alpha(k+1) = \alpha(k) + \frac{VT^2}{L} \sin(\alpha(k)), \quad \beta(k+1) = \beta(k) + \frac{VT}{L} \sin(\alpha(k)), \quad \gamma(k+1) = \gamma(k) + \frac{VT^2}{L}
\]

(19)

where \( \alpha \) is the angle of the truck, \( \beta \) is the angle of the trailer, \( \gamma \) is the initial position of the rear of the trailer, and \( \delta \) is the position of the rear of the trailer, \( L \) is the length of the truck, \( T \) is the sampling time, \( V \) is the constant speed of backward movement of the truck, and \( \omega \) is the controlled steering angle (measured counter clockwise with respect to the truck orientation).

The following noisy fuzzy model, adapted from [7], can be used to represent the above system:

\[
\begin{align*}
IF \, x(k) \, IS \, F_1 & \quad THEN \quad \dot{x}(k) = A_1 \dot{x}(k) + B_1 u(k) + C_1 w(k) \\
IF \, x(k) \, IS \, F_2 & \quad THEN \quad \dot{x}(k) = A_2 \dot{x}(k) + B_2 u(k) + C_2 w(k)
\end{align*}
\]

(20)

The premise variable \( z \) is given as

\[
z(k) = \beta(k) + \frac{V(k) T}{2 \pi}
\]

(21)

The membership function in (20) are defined as \( F_1 = (0, 1) \) and \( F_2 = (x_T, 1) \) and we use following system parameters:

\[
A_1 = \begin{bmatrix} 1 & -VT & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 - VT & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}
\]

\[
\Delta A = \pm 0.3, \quad I = 2.8 \text{m}, \quad L = 5.5 \text{m}, \quad V = -1 \text{m/s}, \quad T = 0.5 \text{s}
\]

We will use the following matrices for the measurement noise covariance \( S_k = 0.2 \) and the overall process noise with uncertainty is following by (11)

\[
\begin{array}{c|c|c|c}
\text{Parameters} & \text{Values} \\
\hline
\text{Maximum Generation} & 200 \\
\text{Maximum Rule Number} & 50 \\
\text{Population Size} & 500 \\
\text{Crossover Rate} & 0.9 \\
\text{Mutation Rate} & 0.01 \\
\hline
\lambda & 0.75
\end{array}
\]

\[\text{(Fig. 1) Truck angle error (degrees)}\]

The two local state vectors of (7) are estimated according to (17), and are then combined according to (18) to obtain the global state estimate. Figure 1 shows that the simulation results of the proposed method. The dotted lines are measurement errors and the solid lines are estimation errors. And trailer position for a typical simulation with the initial conditions \( \alpha(0) = -45^\circ, \beta(0) = -45^\circ, \) and \( M(0) = -5\text{m} \).

3. Conclusion

The uncertain nonlinear systems via the TSK fuzzy system has been presented. The steady state with uncertain is represented by the TS fuzzy model structure, which is further rearranged to give a set of nonlinear model using standard Kalman filter theory. Then the unknown uncertainty is regarded as an additive process noise, it represented as a fuzzy system to compute the time-varying variance of the overall process noise. To optimize the employed fuzzy system, the genetic algorithm is utilized. Finally, the proposed state estimator is demonstrate on a truck-trailer.

[References]