Abstract: This paper presents an escaping route method in a trap situation (a case that the robot is trapped in a local minimum by the potential of obstacles). In this scheme, the APFs for path planning have a multiplicative and additive configuration between APFs for goal destination and APFs for obstacle avoidance unlike conventional configuration where APFs for obstacle avoidance is added to APFs for goal destination. The virtual escaping route method proposed to allow a robot to escape from a local minimum in trap situation where the total forces consist of repulsive forces by obstacles and attractive force by a goal are zero.

1. INTRODUCTION

Potential field methods may cause a robot to be trapped in local minima generated by the same potential functions. Compared with extensive studies focused on the derivation of optimal potential field functions and their applications, very few of attempts have been made at the proposition of analytical design guidelines to evade possible local minima.

To solve this problem, the authors in [1] have proposed a new potential function configuration and have presented analytical guidelines for designing potential functions to avoid local minima for a number of representative scenarios based on the proposed framework for path planning. Specifically the following cases are addressed: a narrow passage problem and a trap situation. The framework enables the robot to reach a goal while avoiding any obstacles under representative scenarios. Different from previous studies on local path planning based on APFs, the purpose of this study is to explore the analysis of path planning design based on artificial potential functions for a number of representative scenarios.

In this paper, as an extension of paper [1], we present a set of analytical design guidelines of its nature for a trap situation (a case that the robot is trapped in a local minimum by the potential of obstacles). The goal is not to tackle all possible local minima and collision problems in APF configurations, instead we focus our attention on a set of analytical guidelines for designing APFs for a number of representative scenarios in path planning problem.

2. PROBLEM STATEMENT

One of the most popular path planning methods is based on potential function utilization where a robot is modeled as a moving particle (named a point robot) inside APFs that reflects free collision space structure into the robot workspace. Each APF is generated by superposing an attractive potential that attracts the robot to the goal configuration and a repulsive potential, which repels robot far away from existing obstacles. The negative gradient of the generated global potential field is interpreted as an artificial force acting on the robot and causing variations on its movement. Here the task of the robot is to move toward a goal (e.g., a light source) while avoiding obstacles encountered.

3. PROPOSED ESCAPING METHOD

3.1 Proposed APFs

Before we describe artificial potential fields, a relative position vector between the robot and the goal is defined as

$$\psi^o = P - P_{\text{goal}} \tag{1}$$

where $P_{\text{goal}}$ is the target position.

Attraction towards the goal is modeled by an attractive field, which in the vicinity of obstacles, draws the charged robot towards the goal. The simple APFs for goal destination are modeled as follows:

$$U^c = c_\psi (1 - e^{-\psi^o}) \tag{2}$$

where $c_\psi$ and $I_\psi$ are the strength and correlation distance for goal destination. The first term $c_\psi$ in the right side of (2) acts to make $U^c$ zero when $\psi^o = 0$.

Its corresponding force is then given by the negative gradient of (2).

$$F^c = -\nabla U^c = -\frac{2 c_\psi \psi^o}{I_\psi} e^{-\psi^o/I_\psi} \tag{3}$$

Relative position vectors between the robots and the obstacles are defined as

$$\psi^o_j = P - O_j \tag{4}$$

where $O_j$ is the position of obstacle $j$ which is a neighbor of the robot. Collisions between the obstacles and the robot are avoided by the repulsive force between them, which is simply the negative gradient of the potential field. We employ the algorithm that prevents collisions with obstacles by calculating the repulsive potential, based on the shortest to an object. The simple APFs for obstacle avoidance are modeled as following.

$$U^o_j = \sum_{i \in N_j} \left( c_o e^{-\frac{\psi^o_j}{I_o}} \right) \tag{5}$$

where $c_o$ and $I_o$ are the strength and correlation distance for obstacle avoidance. $N_j$ denotes the set of indexes of those obstacles which are neighbors of the robot. Its corresponding force is then given by the negative gradient of (5).

$$F^o = -\nabla U^o = \sum_{i \in N_j} \left( \frac{2 c_o \psi^o_j}{I_o} e^{-\psi^o_j/I_o} \right). \tag{6}$$

3.2. Total APFs for path planning

Following configuration for total potential is proposed to overcome such local minimum problems. The total potential has a multiplicative and additive structure between the potential for goal destination and the potential for obstacle avoidance.

$$U^{\text{total}} = \frac{1}{c_\psi} U^c + U^o$$

where

$$U^o_j = \sum_{i \in N_j} \left( c_o e^{-\frac{\psi^o_j}{I_o}} \right) (1 - e^{-\frac{\psi^o_j}{I_o}}) - c_\psi e^{-\frac{\psi^o_j}{I_\psi}} \tag{7}$$

Its corresponding force is

$$F^{\text{total}} = -\nabla U^{\text{total}} = \sum_{i \in N_j} \left( \frac{2 c_o \psi^o_j}{I_o} e^{-\psi^o_j/I_o} \right) (1 - e^{-\psi^o_j/I_o})$$

$$+ \sum_{i \in N_j} \left( c_o e^{-\psi^o_j/I_o} \right) \left( -\frac{2 c_\psi \psi^o_j}{I_\psi} e^{-\psi^o_j/I_\psi} - \frac{2 c_o \psi^o_j}{I_o} e^{-\psi^o_j/I_o} \right). \tag{8}$$

Figure 1 shows that each robot starting from different initial points reaches the target near an obstacle while avoiding obstacles. As for the mathematical analysis of the above case, see paper [1] proposed by the authors.

![Fig.1. Robot trace to a goal near obstacles using (8)](image-url)
3.3. Trap situation: a case that the potential of the goal is overwhelmed by the potential of two obstacles.

We propose the virtual escaping route that allows a robot to escape from a local minimum in trap situation where the total forces composed of repulsive forces by obstacles and attractive force by a goal are zero. Figure 1 shows that a robot is trapped in a local minimum by repulsive force from three obstacles and attractive force from the goal. A local minimum is identified when the following four conditions are all satisfied.

\[
\begin{align*}
|F_{\text{rep}}| &< a_i, \\
|\theta - \sum \theta_i^r| &< a_i, \\
|\theta^r_i - \theta^g| &> a_i, \\
|P_i - P_i^r (k-1)| &< a_i
\end{align*}
\]

where \(a_i\), \(a_1\), and \(a_2\) are positive constants close to zero, and \(a_2\) is a positive degree close to zero degree. \(\theta^r_i\) is the angle between a robot and a goal and \(\theta^g\) is the angle between a robot and each obstacle. \(N^r\) denotes the set of indexes of those obstacles which trap the robot and can be identified as obstacles satisfying \(|\psi^r_i| < a_2\), where \(a_2\) is a positive constant.

In Fig. 2, \(\theta^1\) and \(\theta^2\) are angles between a robot and the nearest obstacle from the root that is located in the left and right sides from a view of the robot, respectively. \(\theta^r = \theta^1 - \theta^2\) and \(\theta^g = \theta^1 + \theta^2\).

If a robot satisfies four conditions in (9), the robot chooses a left or right route according to \(\min(\psi^{lir}, \psi^{rir})\) where \(\psi^{lir}\) and \(\psi^{rir}\) are the distance between the goal and the nearest obstacle in the left and right sides from a view of the robot, respectively.

\[
\psi^{lir} = \sqrt{(x_i - x_{goal})^2 + (y_i - y_{goal})^2}
\]

\[
\psi^{rir} = \sqrt{(x_i - x_{goal})^2 + (y_i - y_{goal})^2}
\]

where \(\psi^{lir}\) and \(\psi^{rir}\) are the distance between a robot and the nearest obstacle in the left and right sides from a view of the robot, respectively. The obstacle closest according to \(\min(\psi^{lir}, \psi^{rir})\) is denoted as \(O_i\). In Fig. 2, the robot chooses the right path due to \(\psi^{rir} < \psi^{lir}\).

![Fig. 2. virtual escaping route](image)

3.4. Analysis of the APFs for trap situation.

Before we describe the virtual escaping route, a relative position vector between a robot and the nearest obstacle that traps the robot is defined as

\[
\psi_i^r = P - O_i
\]

where \(O_i\) is a position of the obstacle that traps the robot and is the nearest to the goal.

The simple APF for \(O_i\) is modeled as following.

\[
U_i^r = c_i e^{-\frac{\psi_i^r}{\sigma_i}}
\]

(10)

Its corresponding force is then given by the negative gradient of (11).

\[
F_i^r = -\nabla U_i^r = \frac{2c_i \psi_i^r}{\sigma_i^2} e^{-\frac{\psi_i^r}{\sigma_i}}
\]

A relative position vector between a robot and a virtual point is defined as

\[
\psi_i^v = P - P_i
\]

(13)

where \(P_i\) is a position of the virtual point that is at the distance of \(d^r\) from the robot with angle \(\theta^r\) between the robot and the obstacle \(O_i\) as shown in Fig. 2. \(d^r\) is the distance of \(\psi_i^r\) when the robot is trapped by the obstacle and \(\theta^r\) is an angle for the escaping route.

If the robot goes toward the virtual point, the position of the virtual point is updated on the basis so that the virtual point maintains the distance \(d^r\) from the robot and the angle \(\theta^r\) between the robot and the obstacle \(O_i\) that makes the virtual escaping route.

The virtual point for the escaping route has attractive fields as follows.

\[
U_i^v = c_i (1 - e^{-\frac{\psi_i^v}{\sigma_i}})
\]

(14)

where \(c_i\) and \(l_i\) are the strength and correlation distance for an escaping route. The first term \(c_i\) is in the right side of (14) acts to make \(U_i^v\) zero when \(\psi_i^v = 0\).

Its corresponding force is then given by the negative gradient of (14).

\[
F_i^v = -\nabla U_i^v = \frac{2c_i \psi_i^v}{\sigma_i^2} e^{-\frac{\psi_i^v}{\sigma_i}}
\]

(15)

The total potential that the potential for the escaping route and the potential for obstacle avoidance are combined together is modeled as following.

\[
U = \frac{1}{c_i} \cdot U_i^v + U_i^r = (c_i e^{-\frac{\psi_i^v}{\sigma_i}}) (1 - e^{-\frac{\psi_i^r}{\sigma_i}}) + c_i
\]

(16)

Its corresponding force is

\[
F = -\nabla U = \frac{2c_i \psi_i^v}{\sigma_i^2} e^{-\frac{\psi_i^v}{\sigma_i}} (1 - e^{-\frac{\psi_i^r}{\sigma_i}}) + \frac{2c_i \psi_i^r}{\sigma_i^2} e^{-\frac{\psi_i^r}{\sigma_i}}
\]

(17)

4. SIMULATION

Figure 3 (a) shows an example of trap situation with no passage using the proposed configuration where a robot starts from \((0, 0)\). \(a_i = 0.001\), \(a_1 = 10^r\), \(a_2 = 0.02\), \(\theta = 70^r\), and \(\theta = 10^r\) are used. Small blank dots around the right obstacle make the virtual escaping route. If a robot meets another obstacle before the robot reaches the release point while the robot escapes a local minimum, a new escaping route is created on the basis of a new neighboring obstacle. Then, the robot is affected by the potential fields from the new neighboring obstacle and the virtual escaping route. If the robot goes the release point while following the virtual escaping route, it goes toward the goal by existing potential fields.

Figure 3(b) shows such an example where a robot starts from \((1, 0)\). Since the robot follows around the obstacles such as the characteristic of well-following approach, the proposed approach can be considered as an obstacle-following strategy on the basis of the virtual escaping route.

![Fig. 3. Virtual escaping route (a) x (0,0), (b) X (1,0)](image)

5. CONCLUSIONS

This paper presents a virtual escaping method based on APFs for local path planning. The paper proposes a framework for APFs to avoid local minima on a possible representative scenario. One of the main objectives is that the proposed multiplicative and additive configuration for total potential has reduced the non-feasible area of trapped region by local minima. Moreover, the proposed virtual escaping route enables a robot to escape from possible local minima where the total forces composed of repulsive forces by obstacles and attractive force by a goal are zero.

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REFERENCES