Wavelet Based Noise Variance Estimation and Its Application to Image Restoration

*Sinyoung Jun, Jinyoung Youn, Yoonjong Yoo, and Joonki Paik
*Graduate School of Advanced Image Science, Multimedia, and Film, Chung-Ang University

Abstract

The wavelet transform has an advantage over the Fourier transform for representing in homogeneous functions that have discontinuities, sharp peaks, like images. This paper proposes a noise variance estimation method by utilizing the wavelet transform.

I. Introduction

For estimation of noise variance many approaches have been proposed in the literature [1]. Major contribution of this paper is that it can selectively use the proper frequency components using wavelet transform.

II. Background Mathematical Theory

The discrete wavelet transform (DWT) of $f(x,y)$ of size $M \times N$ is defined as

$$W_{\phi}(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \phi_{j,m,n}(x,y), \quad (2)$$

and

$$W_{\psi}(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \psi_{j,m,n}(x,y),$$

for $i = H, V, D$, \quad (3)

where index $i$ identifies the direction of each wavelet, such as horizontal(H), vertical(V), and diagonal(D) [3].

III. Wavelet-Based Noise Variance Estimation

Fig. 1 shows the DWT for degraded image $g(m,n)$ [2, 3], where $d^h(m,n)$ is assumed to haul only noise and edge components because its low pass component is filtered twice by directional high pass filters [1, 3].

Fig. 1. Block diagram of the wavelet transform and our algorithm

For extracting the purely noise component, we need to remove edge components in $d^h(m,n)$. Since it is hard to completely separate noise and edge components, we use 1-level horizontal and vertical wavelets, that is $d^h(m,n)$ and $d^v(m,n)$. The proposal noise variance estimation algorithm based
edge separation is summarized as follows.

Step 1: Perform 1-level DWT of the degraded image. Step 2: Perform the DWTs of $d^H(m,n)$ and $d^V(m,n)$ as shown in Fig. 2. Step 3: Make the 2-level wavelets edge map using threshold and interpolation. Step 4: Make the 1-level diagonal wavelets $d^H(m,n)$'s edge map in Fig. 2. Step 5: Noise variance is estimated by computing the energy of the edge-removed diagonal wavelet.

![Diagram of wavelets](image)

**Fig. 2.** 2-level wavelets for horizontal component $d^H(m,n)$ and vertical component $d^V(m,n)$

**IV. Experimental Results**

Experiments are conducted on a gray scale test image Lena of size $256 \times 256$. The threshold value for the 2-level wavelet edge map is determined as

$$
5 \times \max[d^H(m,n),d^V(m,n)] + 4 \times \min[d^H(m,n),d^V(m,n)] / 9
$$

Table 1 shows the results obtained by using the proposed method, and Table 2 shows comparison with other method [4]. Our method is adjacent for real noise variance.

**Table 1. Estimation noise variance**

<table>
<thead>
<tr>
<th>SNR</th>
<th>Real noise variance</th>
<th>Estimation noise variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 dB</td>
<td>3.2198e+003</td>
<td>3.1714e+003</td>
</tr>
<tr>
<td>10 dB</td>
<td>321.9476</td>
<td>320.3596</td>
</tr>
<tr>
<td>20 dB</td>
<td>32.1947</td>
<td>32.0097</td>
</tr>
<tr>
<td>30 dB</td>
<td>3.2195</td>
<td>3.2856</td>
</tr>
<tr>
<td>40 dB</td>
<td>0.3219</td>
<td>0.3806</td>
</tr>
<tr>
<td>50 dB</td>
<td>0.0322</td>
<td>0.0848</td>
</tr>
</tbody>
</table>

**Table 2. Comparison by the other method**

<table>
<thead>
<tr>
<th>Noise variance</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other method</td>
<td>0.08</td>
<td>0.17</td>
<td>0.44</td>
<td>0.88</td>
<td>1.76</td>
</tr>
<tr>
<td>Proposed method</td>
<td>0.15</td>
<td>0.25</td>
<td>0.54</td>
<td>1.07</td>
<td>2.12</td>
</tr>
</tbody>
</table>

We use Wiener filter as an image restoration filter, where noise power is replaced by noise variance [3]. Signal power of the degraded image is used as the original power spectrum.

![Image of degraded and restored images](image)

**Fig3.** (a) Degraded image by $7 \times 7$ uniform blur and 50 dB additive noise and (b) the restored image using wiener filter.

**V. Conclusion**

In this paper we presented a new wavelet-based for noise variance estimation [2, 3]. The primary advantage of the method is acceptable accuracy and simplicity. The proposed algorithm can accurately estimate a wide range of noise variance as shown in Table 2. A comparison with several previously published estimation methods indicates the improved accuracy of the proposed algorithm.

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**Reference**


