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ON THE *R*-SEMIDEVELOPABLE SPACES

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In this paper, a class of spaces, called r-semidevelopable space is introduced by a natural way. This class of spaces lies between the class of semidevelopable spaces and the class of cushioned pair-semidevelopable spaces. We show some properties of the r-semidevelopable spaces.

A topological space X is said to be semidevelopable [1] if there is a sequence of (not necessarily open) covers of X, $\gamma=\{\gamma_n\}_{n=1}^{\infty}$ such that for each $x\in X$, $\{St(x,\gamma_n)\}_{n=1}^{\infty}$ is a neighborhood base at x. In this case, γ is called a semidevelopment for X.

A semidevelopment $\gamma=\{\gamma_n\}_{n=1}^{\infty}$ of X is said to r-semidevelopment if each $x\in X$ and closed set F not containing x, there exists an integer m such that $Int(St(x,\gamma_m)\cap Int(St(F,\gamma_m)=\phi$. A topological space X is said to be r-semidevelopable if there exists a r-semidevelopment for X.

By a cushioned pair-semidevelopment [2] for X we shall mean a pair of semidevelopments (γ,δ) such that γ_n is cushioned in δ_n for each n. A topological space X is said to be cushioned pair-semidevelopable if and only if there exists a cushioned pair-semidevelopment of X. Unless otherwise stated no separation axioms are assumed.

It is trivial that r-semidevelopable spaces is semidevelopable. The following theorem shows the relation between the r-semidevelopable spaces and the cushioned pair-semidevelopable spaces.

THEOREM 1. *Every cushioned pair-semidevelopable space is r-semidevelopable.*

Proof. Let (γ,δ) be a cushioned pair-semidevelopment. We can assume that γ_{n+1} refines γ_n for each n [2]. Let $x\subset X$ and F be closed set not containing x. Since $\delta=\{\delta_n\}_{n=1}^{\infty}$ is a semidevelopment, there exists an integer m such that $St(x,\delta_m)\subset\complement F$. Thus we have $x\notin St(F,\delta_m)$. For such m, we have $C\dotplus(St(F,\gamma_m))\subset St(F,\delta_m)$. Therefore we obtain $x\in\complement Cl(St(F,\gamma_m))$.

Since $\gamma=\{\gamma_n\}_{n=1}^{\infty}$ is also a semidevelopment, there exists an integer m such that $x\in Int(St(x,\gamma_{m'}))\cap \mathscr{C}Cl(St(F,\gamma_m))$. If we take $k=\max\{m,m'\}$, then we have $Int(St(x,\gamma_k))\cap Int(St(F,\gamma_k))=\phi$. Hence $\gamma=\{\gamma_n\}_{n=1}^{\infty}$ is a r-semidevelopment.

COROLLARY 2. *Every cushioned pair-semidevelopable space is reguar.*

A space X is stratifiable [5] if and only if to each closed subset $F\subset X$ one can assign a sequence $\{U_n\}_{n=1}^{\infty}$ of open subsets of X such that

(a) $F\subset U_n$ for each n,

(b) $\bigcup_{n=1}^{\infty}(ClU_n)=F$,

(c) $U_n\subset V_n$ whenever $U\subset V$.

A correspondence $F\longrightarrow\{U_n\}_{n=1}^{\infty}$ is a dual stratification for the space X whenever it satisfies the three conditions.

In [4], Chu showed that every cushioned pair-semidevelopable space is stratifiable. We have the same result in r-semidevelopable spaces.

THEOREM 3. *Every r-semidevelopable space is stratifiable.*

Proof. Let X be a topological space with a refining r-semidevelopment $\gamma=\{\gamma_n\}_{n=1}^{\infty}$ for X. For any closed subset $F\subset X$, let $U_n=Int(St(F,\gamma_n))$. Then $F\longrightarrow\{U_n\}_{n=1}^{\infty}$ is a dual stratification for X. For each $x\in F$, we have $x\in Int(St(x,\gamma_n))\subset Int(St(F,\gamma_n))=U_n$. Therefore we have (a) $F\subset U_n$ for each n. For the condition (b), assume that $y\notin F$, there exists an integer m such that $Int(St(y,\gamma_m))\cap Int(St(F,\gamma_m))=\phi$. Therefore y does not belong to $Cl\ U_m$. Thus we have $\bigcap_{n=1}^{\infty}(Cl\ U_n)\subset F$. Since it is clear that $\bigcap_{n=1}^{\infty}(Cl\ U_n)\supset F$, we obtain (b) $\bigcap_{n=1}^{\infty}(Cl\ U_n)=F$. Since $\gamma=\{\gamma_n\}_{n=1}^{\infty}$ is a refining r-semidevelopment, it is easily shown that (c) $U_n\subset V_n$ if $U\subset V$.

COROLLARY 4. *Every cushioned pair-semidevelopable spaceis stratifiable.*

J.G. Ceder [3] showed that every stratifiable T_1-space is paracompact. Alexander [1] has shown that every semidevelopable T_0-space is T_1. Thus we have the following Corollary.

COROLLARY 5. *Every r-semidevelopable T_0-space is paracompact.*

REMARK. (1) If X be a semimetric space such that for each $x\in X$ and closed set F not containing x, there exists an integer m such that $Int\left(S\left(x, \frac{1}{m}\right)\right) \cap Int\left(S\left(F, \frac{1}{m}\right)\right)=\phi$, then X is a r-semidevelopable T_0-space. (The converse of (1) is also true.)

(2) If X is a metric space, γ_n is the collection of all spheres of radius less than $\frac{1}{n}$, then $\gamma=\{\gamma_n\}_{n=1}^{\infty}$ is a r-semidevelopment.

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