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THE n-SECTIONING OF AN ANGLE

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The impossibility of trisecting an angle using ruler and compass alone is well-known. It is nevertheless possible to trisect an angle using methods other than ruler and compass ([1, 3]), but simple techniques are not widely known. Here we present a method which can be used not only to trisect but also in fact to n-sect any angle.

In the x-y plane, imagine a horizontal line H while it moves from the $y=0$ to $y=1$ position at a constant speed, simultaneously a ray R from the origin O rotates counter-clockwise around O from the x-axis to the y-axis position at a constant angular speed. Call the curve of the intersection of the line H and the ray R created by the above motions the angle section curve or just ASC.

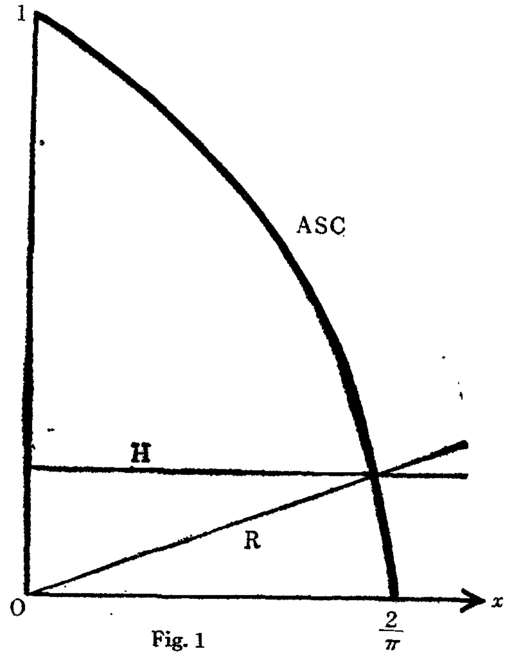

Fig. 1

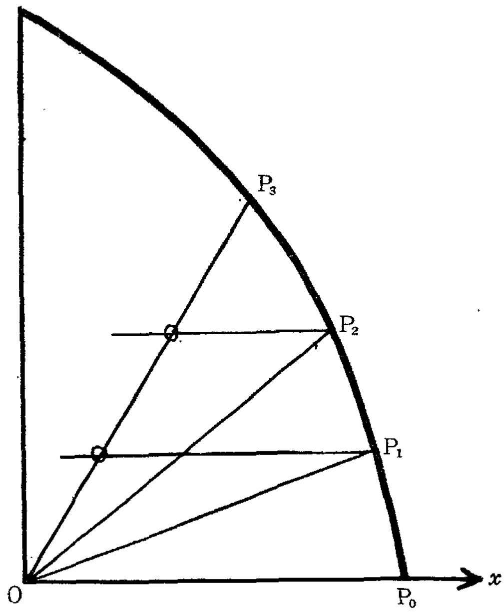

Fig. 2

Since the equations of H and R are $y=t$ and $y=x\tan(\pi t/2)$, respectively, in terms of a parameter t, varying from $t=0$ to $t=1$, the equation of the ASC is

$$(*) \qquad y=x\tan(\pi y/2),$$

and Fig. 1 shows this curve.

It is easy to see that if we can n-sect a right angle and any acute angle, then we can n-sect any angle greater than a right angle. Suppose θ is a given angle less than or equal to a right angle. Let P_0 denote the intersection of the ASC and the x-axis and let P_n be a point on the ASC such that $\angle P_nOP_0=\theta$. $\left(\text{Actually } P_0=(\frac{2}{\pi},\ 0)\right)$. To n-sect θ, we n-sect the segment OP_n and draw parallels, parallel to the x-axis, through these $n-1$ division points of OP_n, by routine methods using ruler and compass. If these parallels meet the ASC at $P_1, P_2, \cdots, P_{n-1}$, in order of increasing heights toward P_n, the rays OP_i, $i=1, 2, \cdots, n-1$, are the desired n-secting lines of $\angle P_nOP_0=\theta$, because as the moving line H at a constant speed passes through P_i and then P_{i+1}, the time elapsed will be $1/n$ th of the time required for H to pass from the x-axis to P_n, and during the same time interval the ray R will have swept through $1/n$ th of the angle θ. Fig. 2 shows the trisection of θ.

Our method of n-sectioning of an angle is based on the construction of the ASC, and we can construct this curve by a computer with plotter or even a desk calculator with the card reader and hand plotting. Program in such a way to make a computer plots 10^m points whose coordinates are

$$(y\cot(\pi y/2),\ y),\ \text{where } y=k/10^m,\ k=1, 2, \cdots, 10^m.$$

The exponent m depends on the degree of accuracy desired.

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