

## UNIFORM CONSISTENCY OF A HISTOGRAM DENSITY ESTIMATOR AND MODAL ESTIMATION

BY BOCK KI KIM

Let  $-\infty = Y_0 < Y_1 < Y_2 < \cdots < Y_n < Y_{n+1} = +\infty$  be an ordered sample of  $n$  independent observations,  $X_1, X_2, \dots, X_n$  of a random variable  $X$  with distribution function  $F(x)$  and density  $f(x)$  continuous on its support set  $S(f) = \{x : f(x) > 0\} = (a, b)$ . As a nonparametric histogram estimator of the density function  $f(x)$ , consider an estimator  $f_n(x)$  of the form:

$$f_n(x) = k_n/n(Y_{A_n(x)+k_n} - Y_{A_n(x)})$$

where  $\{A_n(x)\}$  is a suitably chosen sequence of non-negative integer-valued indexing random variables; and  $\{k_n\}$  is also an appropriately defined sequence of positive integers which depends only on the sample size  $n$ . J. Van Ryzin (1973) has given conditions under which the above estimators are pointwise consistent. In this paper we establish conditions under which such a histogram density estimator is uniformly consistent almost surely. When the density has a unique mode, the results are used to obtain a strongly consistent estimator of the mode similar to that of Venter (1967).

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## PRO-AFFINE ALGEBRAIC GROUPS, AN APPROACH FROM HOPF ALGEBRAS

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Theory of pro-affine algebraic groups is examined in terms of the Hopf algebras of polynomial functions on these groups.

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