

PROBABILISTIC LIMIT ABOUT PERMANENT OF (0, 1)-MATRICES

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Let $M(n, m, N)$ be the class of all $n \times m$ (0, 1)-matrices with a given number N of ones. Let P be a uniform distribution on $M(n, m, N)$. i.e. so that each of the $\binom{nm}{N}$ elements of $M(n, m, N)$ has the same probability $\binom{nm}{N}^{-1}$ to be chosen. Then permanent of matrix $\omega \in M(n, m, N)$, which is denoted by $\text{Per}(\omega)$, may be considered as a random variable on $M(n, m, N)$.

In this dissertation, the first and the second moment of $\text{Per}(\omega)$ is derived. When $m = \beta n$, $\beta \geq 1$ and $N(n)/(n^{3/2}) \rightarrow \infty$, asymptotic estimates of these quantities are found by the limiting distribution of generalized matching problem.

It turns out that the square of the first moment is asymptotic to the second moment, so we conclude that almost all matrices have asymptotically the same permanent which is the average of $\text{Per}(\omega)$.

The above theorem was proved by P.E. O'Neil in the case of $n=m$.

Next we show that, when $N = \frac{1}{2}nm$, the probability of the set of matrices whose permanent is equal to 0 is asymptotic to the probability of the set of matrices which have at least one zero row or at least $m-n+1$ zero columns.

The above statement means that almost all matrices whose permanent is equal to 0 have at least one zero row or at least $m-n+1$ zero columns.

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