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EXISTENCE OF SUBPOLYNOMIAL ALGEBRAS IN $H^*(BG, Z/p)$

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1. Introduction

Let G be a finite group. We denote BG a classifying space of G, which has a contractible universal principal G bundle EG. The stable type of BG does not determine G up to isomorphism. A simple example [due to N. Minami] is given by $Q_{4p} \times Z/2$ and $D_{2p} \times Z/4$ where p is an odd prime, Q_{4p} is the generalized quarternion group of order $4p$ and D_{2p} is the dihedral group of order $2p$. However the paper [6] gives us a necessary and sufficient condition for BG_1 and BG_2 to be stably equivalent localized at p. The local stable type of BG depends on the conjugacy classes of homomorphisms from the p-groups Q into G. This classification theorem simplifies if G has a normal sylow p-subgroup. Then the stable homotopy type depends on the Weyl group of the sylow p-subgroup.

DEFINITION 1.1. Two subgroups $H, K < G$ are called pointwise conjugate in G if there is a bijection of sets $H \xrightarrow{\alpha} K$ such that $\alpha(h) = g_h^{-1} h g_h$ for $g_h \in G$ depending on $h \in H$.

If we assume G has a normal Sylow p-subgroup P, we set $G = P \rtimes H$ for p'-group H by Zassenhouse theorem, and $G = P \cdot H, H \cap P = \{1\}$. Let $W_G(P)$ denote the Weyl group of $P < G$, i.e. $W_G(P) = N_G(P)/P \cdot C_G(P)$ where $N_G(P)$ is the normalizer and $C_G(P)$ is the centralizer. Then $W_G(P) \leq Out(P)$.

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THEOREM 1.2. [6] *Let G_1 and G_2 be finite groups with normal Sylow p-subgroups P_1 and P_2. Then BG_1 and BG_2 have the same stable homotopy type, localized at p, if and only if $P_1 \cong P_2$ (say P) and $W_{G_1}(P)$ is pointwise conjugate to $W_{G_2}(P)$ in $Out(P)$.*

By using this theorem, the first author found two groups G_1, G_2 such that $H^*(BG_1, Z/p)$ is isomorphic to $H^*(BG_2, Z/p)$ in $\mathcal{U}$, the category of unstable modules over the Steenrod algebras $\mathcal{A}$, but not isomorphic as graded algebras over Z/p.

EXAMPLE 1.3. [5] Let p, l be different odd primes such that $p \equiv 1 \pmod{l}$. We set P be an elementary abelian p-group of rank l^2, i.e. $P = (Z/p)^{l^2}$. Then $Out(P) = GL_{l^2}(F_p)$. We take $H_1 \cong (Z/l)^3$ and $H_2 \cong U_3(F_l)$ (3×3 upper triangular matrices over F_l) so that H_1 is not isomorphic to H_2 in $GL_{l^2}(F_p)$. The groups H_1 and H_2 act on P by matrics multiplication. Now we set $G_i = P \rtimes H_i (i = 1, 2)$. Then $W_{G_i}(P) = P \cdot H_i / P \cdot C_{G_i}(P) \cong H_i / H_i \cap C_{G_i}(P) = H_i$. Pointwise conjugacy is an immediate consequence from the double coset formula for induced representations. Thus by the theorem 1.2, BG_1 is stably equivalent to BG_2 localized at $p > 2$.

However if we take $p = 7$ and $l = 3$, we can show $H^*(BG_1, Z/7)$ is not isomorphic to $H^*(BG_2, Z/7)$ as graded algebras over $Z/7$. From now on, we consider $G_1 = P \rtimes H_1$ and $G_2 = P \rtimes H_2$ where $P = (Z/7)^9, H_1 \cong (Z/3)^3$ and $H_2 = U_3(F_3)$ and $H_1, H_2 < GL_9(F_7)$. From the cohomology of groups, we have $H^*(BG_i, Z/7) = H^*(BP, Z/7)^{H_i} = (Z/7[y_1, \cdots, y_9] \otimes E[x_1, \cdots, y_9])^{H_i}$ where $|x_j| = 1, |y_j| = 2$ and $y_j = \beta x_j, \beta$ is the Bockstein homomorphism ($i = 1, 2, j = 1, \cdots, 9$).

In this paper we show that there exists a subpolynomial algebra with 9 generators each of dimension 6 in $H^*(BG_i, Z/7)$ through computing the Poincaré series of $Z/7[y_1, \cdots, y_9 : 2]^{H_i} (i = 1, 2)$.

In section 2, we describe the generators of the invariant elements under H_1, H_2 of $H^*(BG_i, Z/7)$ in dimension 6. In the third section, we calculate the Poincaré series of the invariant subpolynomial algebra in $H^*(BG_i, Z/7)$ by using Molien's theorem. Finally we show that there exists a subpolynomial algebra with 9 generators each of dimension 6 in $H^*(BG_i, Z/7)$.

2. Invariants in $H^*(BP, Z/7)^{H_i}$.

First we describe the invariant elements under H_1 and H_2 in dimension 6.

(1) generators of invariant in $H^6(BP, Z/7)^{H_1}$

$$d_1 = {y_1}^3 + {y_2}^3 + {y_3}^3 + {y_4}^3 + {y_5}^3 + {y_6}^3 + {y_7}^3 + {y_8}^3 + {y_9}^3$$
$$d_2 = y_1y_3y_2 + y_4y_6y_5 + y_7y_9y_8$$
$$d_3 = y_1y_7y_4 + y_2y_8y_5 + y_3y_9y_6$$
$$d_4 = y_1y_5y_9 + y_2y_6y_7 + y_3y_4y_8$$
$$d_5 = y_1y_8y_6 + y_2y_9y_4 + y_3y_7y_5$$
$$d_6 = y_1y_3y_5 + y_2y_1y_6 + y_3y_2y_4 + y_7y_9y_2 + y_8y_7y_3 + y_9y_8y_1 + y_4y_6y_8 + y_5y_4y_9 + y_6y_5y_7$$
$$d_7 = y_1y_3y_8 + y_2y_1y_9 + y_3y_2y_7 + y_4y_6y_2 + y_5y_4y_3 + y_6y_5y_1 + y_7y_9y_5 + y_8y_7y_6 + y_9y_8y_4$$
$$d_8 = y_1y_3y_4 + y_2y_1y_5 + y_3y_2y_6 + y_7y_9y_1 + y_8y_7y_2 + y_9y_8y_3 + y_4y_6y_7 + y_5y_4y_8 + y_6y_5y_9$$
$$d_9 = y_1y_3y_7 + y_2y_1y_8 + y_3y_2y_9 + y_7y_9y_4 + y_8y_7y_5 + y_9y_8y_6 + y_4y_6y_1 + y_5y_4y_2 + y_6y_5y_3$$
$$d_{10} = y_1y_3y_6 + y_2y_1y_4 + y_3y_2y_5 + y_7y_9y_3 + y_8y_7y_1 + y_9y_8y_2 + y_4y_6y_9 + y_5y_4y_7 + y_6y_5y_8$$
$$d_{11} = y_1y_3y_9 + y_2y_1y_7 + y_3y_2y_8 + y_7y_9y_6 + y_8y_7y_4 + y_9y_8y_5 + y_4y_6y_3 + y_5y_4y_1 + y_6y_5y_2$$
$$d_{12} = y_1y_4y_9 + y_2y_5y_7 + y_3y_6y_8 + y_7y_1y_6 + y_8y_2y_4 + y_9y_3y_5 + y_4y_7y_3 + y_5y_8y_1 + y_6y_9y_2$$
$$d_{13} = y_1y_4y_8 + y_2y_5y_9 + y_3y_6y_7 + y_7y_1y_5 + y_8y_2y_6 + y_9y_3y_4 + y_4y_7y_2 + y_5y_8y_3 + y_6y_9y_1$$
$$d_{14} = {y_1}^2y_2 + {y_2}^2y_3 + {y_3}^2y_1 + {y_4}^2y_5 + {y_5}^2y_6 + {y_6}^2y_4 + {y_7}^2y_8 + {y_8}^2y_9 + {y_9}^2y_7$$
$$d_{15} = {y_1}^2y_4 + {y_4}^2y_7 + {y_7}^2y_1 + {y_2}^2y_5 + {y_5}^2y_8 + {y_8}^2y_2 + {y_3}^2y_6 + {y_6}^2y_9 + {y_9}^2y_3$$
$$d_{16} = {y_1}^2y_3 + {y_3}^2y_2 + {y_2}^2y_1 + {y_4}^2y_6 + {y_6}^2y_5 + {y_5}^2y_4 + {y_7}^2y_9 + {y_9}^2y_8 + {y_8}^2y_7$$
$$d_{17} = {y_1}^2y_7 + {y_7}^2y_4 + {y_4}^2y_1 + {y_2}^2y_8 + {y_8}^2y_5 + {y_5}^2y_2 + {y_3}^2y_9 + {y_9}^2y_6 + {y_6}^2y_3$$
$$d_{18} = {y_1}^2y_5 + {y_5}^2y_9 + {y_9}^2y_1 + {y_2}^2y_6 + {y_6}^2y_7 + {y_7}^2y_2 + {y_4}^2y_8 +$$

$y_8{}^2y_3 + y_3{}^2y_4$
$d_{19} = y_1{}^2y_8 + y_8{}^2y_6 + y_6{}^2y_1 + y_2{}^2y_9 + y_9{}^2y_4 + y_4{}^2y_2 + y_3{}^2y_7 + y_7{}^2y_5 + y_5{}^2y_3$
$d_{20} = y_1{}^2y_6 + y_6{}^2y_8 + y_8{}^2y_1 + y_2{}^2y_4 + y_4{}^2y_9 + y_9{}^2y_2 + y_3{}^2y_5 + y_5{}^2y_7 + y_7{}^2y_3$
$d_{21} = y_1{}^2y_9 + y_9{}^2y_5 + y_5{}^2y_1 + y_2{}^2y_7 + y_7{}^2y_6 + y_6{}^2y_2 + y_4{}^2y_3 + y_3{}^2y_8 + y_8{}^2y_4$

(2) generators of invariant in $H^6(BP, Z/7)^{H_2}$

$\bar{d}_1 = y_1{}^3 + y_2{}^3 + y_3{}^3 + y_4{}^3 + y_5{}^3 + y_6{}^3 + y_7{}^3 + y_8{}^3 + y_9{}^3$
$\bar{d}_2 = y_1y_3y_2 + y_4y_6y_5 + y_7y_9y_8$
$\bar{d}_3 = y_1y_7y_4 + y_2y_8y_5 + y_3y_9y_6$
$\bar{d}_4 = y_1y_5y_9 + y_2y_6y_7 + y_3y_4y_8$
$\bar{d}_5 = y_1y_8y_6 + y_2y_9y_4 + y_3y_7y_5$
$\bar{d}_6 = y_1y_3y_5 + 2y_2y_1y_6 + 4y_3y_2y_4 + y_7y_9y_2 + 2y_8y_7y_3 + 4y_9y_8y_1 + y_4y_6y_8 + 2y_5y_4y_9 + 4y_6y_5y_7$
$\bar{d}_7 = y_1y_3y_8 + 4y_2y_1y_9 + 2y_3y_2y_7 + y_7y_9y_5 + 4y_8y_7y_6 + 2y_9y_8y_4 + y_4y_6y_2 + 4y_5y_4y_3 + 2y_6y_5y_1$
$\bar{d}_8 = y_1y_3y_4 + 2y_2y_1y_5 + 4y_3y_2y_6 + y_7y_9y_1 + 2y_8y_7y_2 + 4y_9y_8y_3 + y_4y_6y_7 + 2y_5y_4y_8 + 4y_6y_5y_9$
$\bar{d}_9 = y_1y_3y_7 + 4y_2y_1y_8 + 2y_3y_2y_9 + y_7y_9y_4 + 4y_8y_7y_5 + 2y_9y_8y_6 + y_4y_6y_1 + 4y_5y_4y_2 + 2y_6y_5y_3$
$\bar{d}_{10} = y_1y_3y_6 + 2y_2y_1y_4 + 4y_3y_2y_5 + y_7y_9y_3 + 2y_8y_7y_1 + 4y_9y_8y_2 + y_4y_6y_9 + 2y_5y_4y_7 + 4y_6y_5y_8$
$\bar{d}_{11} = y_1y_3y_9 + 4y_2y_1y_7 + 2y_3y_2y_8 + y_7y_9y_6 + 4y_8y_7y_4 + 2y_9y_8y_5 + y_4y_6y_3 + 4y_5y_4y_1 + 2y_6y_5y_2$
$\bar{d}_{12} = y_1y_4y_9 + y_2y_5y_7 + y_3y_6y_8 + y_7y_1y_6 + y_8y_2y_4 + y_9y_3y_5 + y_4y_7y_3 + y_5y_8y_1 + y_6y_9y_2$
$\bar{d}_{13} = y_1y_4y_8 + y_2y_5y_9 + y_3y_6y_7 + y_7y_1y_5 + y_8y_2y_6 + y_9y_3y_4 + y_4y_7y_2 + y_5y_8y_3 + y_6y_9y_1$
$\bar{d}_{14} = y_1{}^2y_2 + y_2{}^2y_3 + y_3{}^2y_1 + y_7{}^2y_8 + y_8{}^2y_9 + y_9{}^2y_7 + y_4{}^2y_5 + y_5{}^2y_6 + y_6{}^2y_4$
$\bar{d}_{15} = y_1{}^2y_4 + 4y_3{}^2y_6 + 2y_2{}^2y_5 + y_7{}^2y_1 + 4y_9{}^2y_3 + 2y_8{}^2y_2 + y_4{}^2y_7 + 4y_6{}^2y_9 + 2y_5{}^2y_8$
$\bar{d}_{16} = y_1{}^2y_3 + y_2{}^2y_1 + y_3{}^2y_2 + y_7{}^2y_9 + y_8{}^2y_7 + y_9{}^2y_8 + y_4{}^2y_6 + y_5{}^2y_4 + y_6{}^2y_5$

$\bar{d}_{17} = {y_1}^2 y_7 + 2{y_3}^2 y_9 + 4{y_2}^2 y_8 + {y_7}^2 y_4 + 2{y_9}^2 y_6 + 4{y_8}^2 y_5 + {y_4}^2 y_1$
$+2{y_6}^2 y_3 + 4{y_5}^2 y_2$
$\bar{d}_{18} = {y_1}^2 y_5 + 4{y_3}^2 y_4 + 2{y_2}^2 y_6 + {y_7}^2 y_2 + 4{y_9}^2 y_1 + 2{y_8}^2 y_3 + {y_4}^2 y_8$
$+4{y_6}^2 y_7 + 2{y_5}^2 y_9$
$\bar{d}_{19} = {y_1}^2 y_8 + 2{y_3}^2 y_7 + 4{y_2}^2 y_9 + {y_7}^2 y_5 + 2{y_9}^2 y_4 + 4{y_8}^2 y_6 + {y_4}^2 y_2$
$+2{y_6}^2 y_1 + 4{y_5}^2 y_3$
$\bar{d}_{20} = {y_1}^2 y_6 + 4{y_3}^2 y_5 + 2{y_2}^2 y_4 + {y_7}^2 y_3 + 4{y_9}^2 y_2 + 2{y_8}^2 y_1 + {y_4}^2 y_9$
$+4{y_6}^2 y_8 + 2{y_5}^2 y_7$
$\bar{d}_{21} = {y_1}^2 y_9 + 2{y_3}^2 y_8 + 4{y_2}^2 y_7 + {y_7}^2 y_6 + 2{y_9}^2 y_5 + 4{y_8}^2 y_4 + {y_4}^2 y_3$
$+2{y_6}^2 y_2 + 4{y_5}^2 y_1$

Now we compute the Poincaré series of $Z/7[y_1, \cdots, y_9 : 2]^{H_i}$, a subpolynomial algebra in $H^*(BG_i, Z/7)$ $(i = 1, 2)$.

3. Subpolynomial algebras in $H^*(BG_i, Z/p)$

We define Poincaré series of a graded module $\{M^k\}_{k \geq 0}$ by

$$P.S(M^k) = \sum_{k=0}^{\infty} (\mathrm{dim} M^k) t^k$$

the formal power series of t. Let V be a vector space with basis $\{x_1, \cdots, x_n\}$ over $Z/7$ and $G \subset GL(V)$. We identify $S(V^*)$ with the algebra $Z/7[x_1, \cdots, x_n]$, algebra of polynomial functions on V. $S(V^*)^G = \{f \in S(V^*) \mid gf = f$ for all $g \in G\}$ under the action of G on $S(V^*)$ given by $(gf)(v) = f(g^{-1}v)$, for $g \in G$, $v \in V$, $f \in S(V^*)$.

By Molien's theorem

$$P.S(S(V^*)^G) = \frac{1}{|G|} \sum_{g \in G} \frac{1}{det(I - tg)}$$

a power series with positive integer coefficients. We apply Molien's theorem to compute the Poincaré series of the invariant subpolynomial algebra in $H^*(BP, Z/7)^{H_i}$. Since Molien's theorem is in characteristic 0, we extend the group H_i to the group $\widetilde{H}_i$ in $GL_9(\mathbf{F}_{\hat{7}})$ by Hensel's theorem.

THEOREM 3.1. (Hensel) [4] *Let $F(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n$ be a polynomial whose coefficients are p-adic integers. Let $F'(x) = c_1 + 2c_2 x + \cdots + nc_n x^{n-1}$ be the derivative of $F(x)$. Let a_0 be a p-adic integer such that $F(a_0) \equiv 0 \pmod{p}$ and $F'(a_0) \not\equiv 0 \pmod{p}$. Then there exists a unique p-adic integer a such that $F(a) = 0$ and $a \equiv a_0 \pmod{p}$.* □

Let $F(x) = 1 - x^3$ (mod $p = 7$). Then $F'(x) = -3x^2$. We take $a_0 = 2$ so that $F(2) = 1 - 2^3 \equiv 0 \pmod{7}$ and $F'(2) = -3 \cdot 2^2 \not\equiv 0 \pmod{7}$. Therefore there exists $w \in \mathbf{F}_{\hat{7}}$ such that $w^3 \equiv 1$, $w \equiv 2 \pmod{7}$ by Hensel's theorem. So we can lift the group H_i to $\widetilde{H}_i$ in $GL_9(\mathbf{F}_{\hat{7}})$ by replacing 2 by w. By abuse of notation we still use H_i instead of $\widetilde{H}_i$ in $GL_9(\mathbf{F}_{\hat{7}})$.

Now we have $H^*(BP, Z/7) = Z/7[y_1, \cdots, y_9] \otimes E[x_1, \cdots, x_9]$ where $|x_i| = 1$, $|y_i| = 2$ and $y_i = \beta x_i$, β is the Bockstein homomorphism. Then we calculate the Poincaré series of $Z/7[y_1, \cdots, y_9 : 2]^{H_i}$ in $H^*(BG_i, Z/7)$ $(i = 1, 2)$.

$$\begin{aligned} &P.S(Z/7[y_1, \cdots, y_9 : 2]^{H_1}) \\ &= \frac{1}{|H_1|} \sum_{\alpha_i \in H_1} \frac{1}{det(I - \alpha_i t^2)} \\ &= \frac{1 + 12t^6 + 186t^{12} + 331t^{18} + 186t^{24} + 12t^{30} + t^{36}}{(1 - t^6)^9} \\ &= 1 + 21t^6 + 339t^{12} + 2710t^{18} + 14010t^{24} + \cdots\cdots \end{aligned}$$

Similarly

$$\begin{aligned} &P.S(Z/7[y_1, \cdots, y_9 : 2]^{H_2}) \\ &= \frac{1}{|H_2|} \sum_{\beta_i \in H_2} \frac{1}{det(I - \beta_i t^2)} \\ &= \frac{1 + 12t^6 + 186t^{12} + 331t^{18} + 186t^{24} + 12t^{30} + t^{36}}{(1 - t^6)^9} \\ &= 1 + 21t^6 + 339t^{12} + 2710t^{18} + 14010t^{24} + \cdots\cdots \end{aligned}$$

In this Poincaré series, each coefficient corresponds to the number of invariant elements generated by y_i in each dimension $(i = 1, \cdots, 9)$.

From the process of the computations of Poincaré series, we show that there exists a subpolynomial algebra with 9 generators each of dimension 6 in $H^*(BG_i, Z/7)$, where $i = 1, 2$.

We mean an algebra A^* as a graded algebra over Z/p which satisfies
i) A^* is zero in odd degrees
ii) A^* is an integral domain
iii) A^* is commutative
Let $A^* \subseteq B^*$ be a pair of algebras. Then a derivation is a function $\partial : A^* \to B^*$ such that $\partial(x+y) = (\partial x) + (\partial y)$, $\partial(xy) = (\partial x)y + x(\partial y)$. The derivations are automatically linear since the ground field is Z/p.

LEMMA 3.2. [1] *Suppose that $\partial_1, \partial_2, \cdots, \partial_n : A^* \to B^*$ are derivations. Let $x_1, x_2, \cdots, x_n \in A^*$ and $\det(\partial_i x_j) \neq 0$; in other words $\partial_1, \partial_2, \cdots, \partial_n$ take linearly independent values on $x_1, x_2, \cdots, x_n$. Then $x_1, x_2, \cdots, x_n$ are algebraically independent over Z/p.* □

Now we apply this lemma to prove the following proposition.

PROPOSITION 3.3. *There is a subpolynomial algebra with 9 generators each of dimension 6 in $H^*(BG_i, Z/7)$, $(i = 1, 2)$.*

Proof. We set $A^* = B^* = Z/7[y_1, \cdots, y_9]$. Consider a derivation $\partial_i : A^* \to B^*$ where $\partial_i = \frac{\partial}{\partial y_i}$, $i = 1, 2, \cdots, 9$.

(i) case of $H^*(BG_1, Z/7)$

We choose 9 elements $z_1 = d_1$, $z_2 = d_{14}$, $z_3 = d_{15}$, $z_4 = d_{16}$, $z_5 = d_{17}$, $z_6 = d_{18}$, $z_7 = d_{19}$, $z_8 = d_{20}$ and $z_9 = d_{21}$ in $A^* \subset H^*(BG_1, Z/7)$. Then

$$\det \begin{pmatrix} \frac{\partial z_1}{\partial y_1} & \cdots & \frac{\partial z_9}{\partial y_1} \\ \cdots & \cdots & \cdots \\ \frac{\partial z_1}{\partial y_9} & \cdots & \frac{\partial z_9}{\partial y_9} \end{pmatrix} \neq 0 .$$

Thus $\partial_1, \partial_2, \cdots, \partial_9$ take linearly independent values on $z_1, z_2, \cdots, z_9$. By above Lemma 3.2, $z_1, z_2, \cdots, z_9$ are algebraically independent over $Z/7$. Therefore there exists a subpolynomial algebra $Z/7[z_1, \cdots, z_9]$ in $H^*(BG_1, Z/7)$ where $|z_i| = 6$.

(ii) case of $H^*(BG_2, Z/7)$

Similarly we choose 9 elements $\bar{z}_1 = \bar{d}_1, \bar{z}_2 = \bar{d}_{14}, \bar{z}_3 = \bar{d}_{15}, \bar{z}_4 = \bar{d}_{16}, \bar{z}_5 = \bar{d}_{17}, \bar{z}_6 = \bar{d}_{18}, \bar{z}_7 = \bar{d}_{19}, \bar{z}_8 = \bar{d}_{20}$ and $\bar{z}_9 = \bar{d}_{21}$ in $A^* \subset$

$H^*(BG_2, Z/7)$. Then

$$\det \begin{pmatrix} \frac{\partial \bar{z}_1}{\partial y_1} & \cdots & \frac{\partial \bar{z}_9}{\partial y_1} \\ \cdots & \cdots & \cdots \\ \frac{\partial \bar{z}_1}{\partial y_9} & \cdots & \frac{\partial \bar{z}_9}{\partial y_9} \end{pmatrix} \neq 0 ,$$

Thus $\partial_1, \partial_2, \cdots, \partial_9$ take linearly independent values on $\bar{z}_1, \bar{z}_2, \cdots, \bar{z}_9$. By Lemma 3.2, $\bar{z}_1, \bar{z}_2, \cdots, \bar{z}_9$ are algebraically independent over $Z/7$. Therefore there exists a subpolynomial algebra $Z/7[\bar{z}_1, \bar{z}_2, \cdots, \bar{z}_9]$ in $H^*(BG_2, Z/7)$ where $|\bar{z}_i| = 6$. □

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