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Analytical Formulation for the Everett Function

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The Preisach model needs a density function or Everett function for the hysteresis operator to simulate the hysteresis phenomena. To obtain the function, many experimental data for the first order transition curves are required. However, it needs so much efforts to measure the curves, especially for the hard magnetic materials. By the way, it is well known that the density function has the Gaussian distribution for the interaction axis on the Preisach plane. In this paper, we propose a simple technique to determine the distribution function or Everett function analytically. The initial magnetization curve is used for the distribution of the Everett function for the coercivity axis. A major, minor loop and the initial curve are used to get the Everett function for the interaction axis using the Gaussian distribution function and acceptable results were obtained.

I. Introduction

The Preisach model is known as a most appropriate method to represent the magnetic hysteresis phenomena [1]. It is necessary, however, to know the distribution function to describe any hysteresis loop. This function stands for the density of the hysteresis operators in the Preisach plane. To obtain the function, many first order transition curves are required [2] and these can be found only experimentally. However, it is difficult to measure the curves, especially for the hard magnetic materials. Thus many researchers have tried to express the distribution function as the mathematical formulation in order to reduce the experimental efforts. Of those formulations, the Gauss function [3] and the Lorentz function [4] are used frequently.

However those methods have some problems. First, the distribution function along the coercivity axis on the Preisach plane is not usually Gaussian. Second, the method using the Lorentz function does not make use of the initial magnetization curve data and is also not enough to express the variation of the hysteresis. And most of them ignore the reversible magnetization component [5, 6] and seem to be insufficient to express the real state of the hysteresis phenomena.

In this paper, we compare the Everett function obtained from the experimental data with that computed by the mathematical formulations. From the comparison, it is found that the previous mathematical methods are insufficient to simulate the hysteresis phenomena. Thus, a new technique for mathematical formulation using the initial magnetization curve for the coercivity axis and the Gaussian distribution function for the interaction axis is proposed. The initial magnetization curve is used for the distribution of the Everett function for the coercivity axis. A major, minor loop and the initial curve are used to get the Everett function for the interaction axis using the Gaussian distribution function because there are three known points on each curve and we can determine the parameters for the Gaussian distribution. The proposed method needs just the initial magnetization curve, major and a few minor loops and it is shown that the method can describe the hysteresis characteristics accurately.

II. Distribution function

In the Preisach model, the magnetization is calculated by the following equation

$$M(t) = \iint_{\alpha \geq \beta} \rho(\alpha, \beta)\, \gamma_{\alpha\beta}\, H(t)\, d\alpha\, d\beta \tag{1}$$

where $M(t)$: magnetization, $\rho(\alpha, \beta)$: distribution function, $\gamma_{\alpha\beta}$: elementary hysteresis operator, $H(t)$: input field

The distribution function denotes the density of the hysteresis operator with respect to the switching field α and β. To calculate the hysteresis phenomena with the Preisach model, it is needed to obtain the distribution function and many first order transition curves must be measured. Typically, more than 15 curves are necessary to accurately describe the hysteresis loop [2]. However, it is difficult to measure the curves, especially for the hard magnetic materials because the shape of the loop is very sharp and with a little variation of the applied field, the change of the flux density becomes too large. Once the distribution function is obtained, the magnetization is calculated by the double integral of the function using equation (1).

Ⅲ. Everett function

To get the distribution function from experimental data, the changes of the magnetization caused by the variation of the applied field should be measured. Everett [6] proposed a function which can be used to calculate the magnetization easily without the double integral of the distribution function. The function is expressed as follows

$$E(\alpha, \beta) = \int\int_{T(\alpha,\beta)} \rho(x, y)\, dx\, dy = \int_{\beta}^{\alpha} \left(\int_{\beta}^{y} \rho(x, y)\, dx\right) dy \tag{2}$$

where $T(\alpha, \beta)$ is the triangle formed by the intersection of the lines $x = \alpha$, $y = \beta$ in the Preisach plane as shown in Fig. 1. Therefore, the Everett function can be obtained directly from the first order transition curves, that is, the Everett function is obtained from following equation

$$E(\alpha, \beta) = \frac{1}{2}\,(M_\alpha - M_{\alpha,\beta}) \tag{3}$$

where M_α is the magnetization when the magnetic field intensity is increased from the negative saturation field to the field intensity α, and $M_{\alpha,\beta}$ is the magnetization when the field is decreased from α to β as shown in Fig. 2. For the convenience of the calculation, Everett function is usually tabulated from the equation (3), and the magnetization is computed directly from the Everett function [6].

Ⅳ. Formulation of the distribution function

A. Distribution function

In formulating the distribution function, instead of measuring the many first order transition curves, the distribution function is assumed to be Gaussian [3] for the interaction field H_i and the coercivity field H_c as shown in Fig. 3. H_i and H_c are given by

$$H_i = \frac{H_\alpha + H_\beta}{2}$$

$$H_c = \frac{H_\alpha - H_\beta}{2} \tag{4}$$

where H_α : upper switching field of the hysteresis operator, H_β : lower switching field of the hysteresis operator

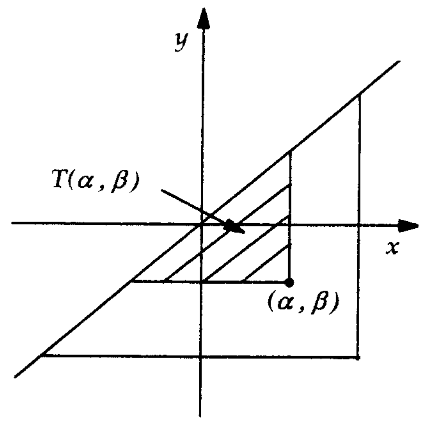

Fig. 1. Triangular region on the Preisach diagram.

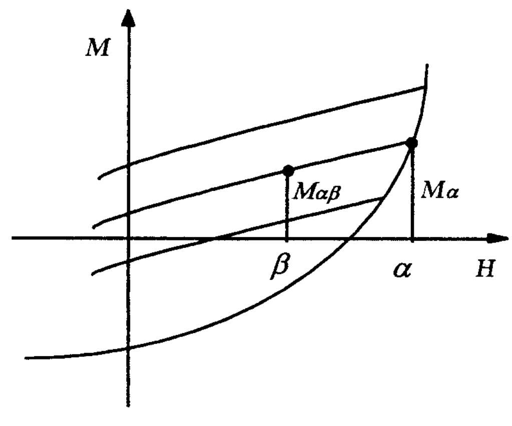

Fig. 2 First order transition curves.

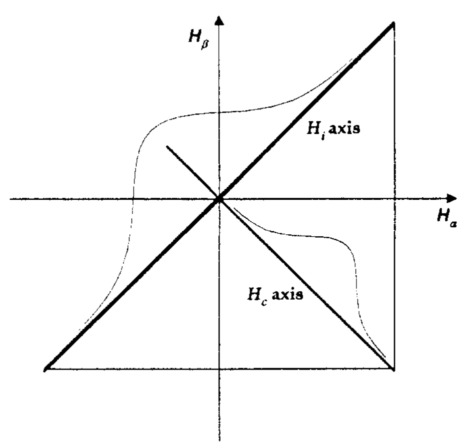

Fig. 3 Gaussian distribution of H_c and H_i in the Preisach plane.

It is generally known that the distribution function for the interaction field is Gaussian [3] and experimental results show a good agreement with this assumption. Fig. 4. shows that Gaussian function can represent the distribution for the interaction field very well. The specification of the sample material is that the coercive force is 7,500 [A/m] and the residual flux density is 1.4 [T].

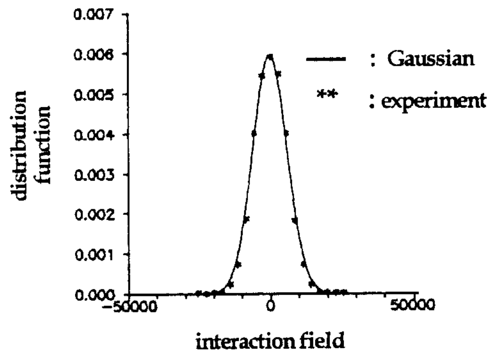

Fig. 4. Distribution function for the interaction field.

However, the distribution for the coercivity field is not Gaussian as shown in Fig. 5 and the distribution for the zero coercivity can not be ignored because there exists the reversible magnetization component which may occur in the Rayleigh region [5].

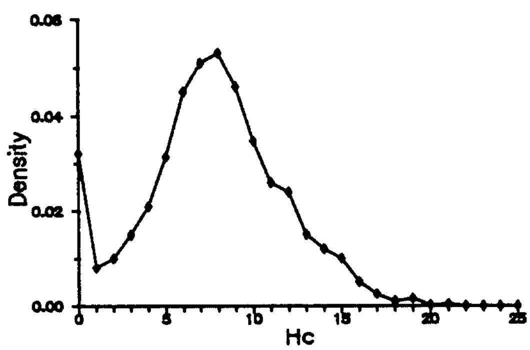

Fig. 5. Distribution function for the coercive field.

B. Proposed method

As mentioned above, the assumption that the distribution function for the interaction filed is Gaussian is a good approximation. Therefore it is reasonable to formulate the distribution function for the interaction field using the Gaussian function. That is, the function is expressed as

$$P_i(H_i) = \frac{1}{\sqrt{2\pi}\,\sigma_i} \exp\left[-\frac{(H_i - \mu_i)^2}{2\,\sigma_i^2}\right] \tag{5}$$

where μ_i is the mean value of the interaction field, which becomes 0 and σ_i^2 is the variance of the Gaussian function.

In many cases, using the Everett function is more convenient than the distribution function [6]. So, in this paper, the Everett function is formulated. Fig. 6 and 7 show the relationship between hysteresis loop and Everett function plane. The line 1, 2 and 3 in Fig. 6 correspond to the hysteresis curve 1, 2, and 3 in Fig. 7. Therefore, the line 1 in Fig. 6 indicates the Everett values along the initial magnetization curve and the line 2 and 3 in Fig. 6 correspond to the saturation curve.

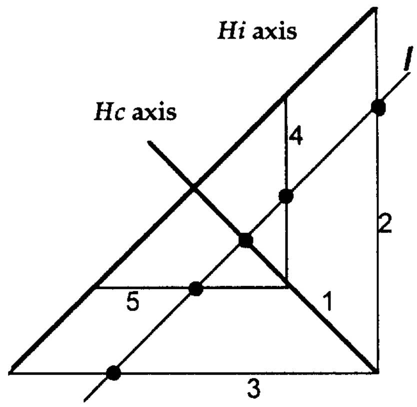

Fig. 6. Everett function plane.

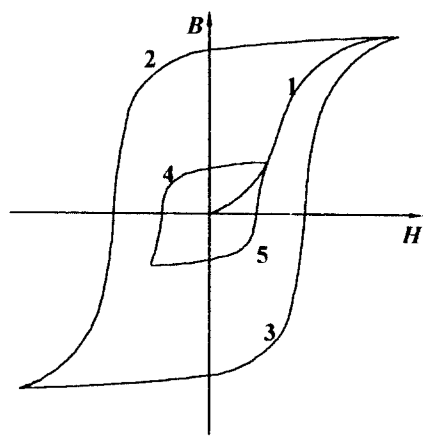

Fig. 7. Hysteresis loops.

Therefore, if the initial magnetization curve and the saturation loop are measured, the data of the Everett function corresponding to the line 1, 2, and 3 can be obtained. It is found that the variation of the Everett data along the line l in Fig. 6 is also Gaussian as shown in Fig. 8 because the distribution function along the interaction field is Gaussian and the double integral of this function also becomes Gaussian. Therefore it is natural that the Everett function for the interaction field has Gaussian distribution.

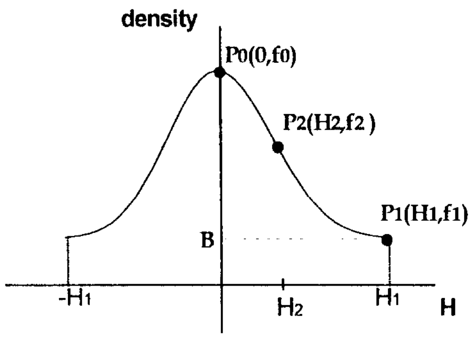

Fig. 8. Everett function for the interaction field.

To complete the Gaussian function, the variance σ_i^2 should be determined and one minor loop is used to determine it. That is, if the minor loop 4 and 5 are measured, the Everett values on the line 4 and 5 in Fig. 6 are found. Then three points P_0, P_1 and P_2 in Fig. 8 become known and from these points, the variance σ_i^2 can be computed. In this paper, the equation (6) is proposed to get the analytical formulation of the Everett function.

$$f(H_c) = k(H_c)\,G(H_i, \sigma_i) + B(H_c) \tag{6}$$

where G is the Gaussian function and the coefficient k, and B are the function of H_c. Because the number of variables is three and there exist three equations, the variable k, σ and B can be determined.

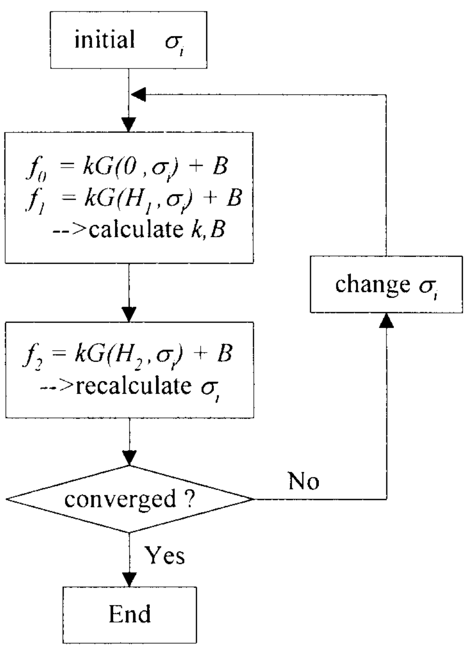

Fig. 9. Flow chart of the proposed calculation algorithm.

Fig. 9 shows the flow chart of the proposed calculation algorithm. In the first stage, the initial variance is assumed, then from the two points P_0 and P_1, the k and B are computed. Using the calculated k, B and another equation made from point P_2, new value of the variance is computed. This procedure is repeated until the convergence condition is satisfied.

V. Results

The sample is a semi-hard magnetic material that the coercivity is 7,500 [A/m] and the residual flux density is 1.4 [T]. Fig. 10 shows the measured initial magnetization curve for the total field which is the sum of the applied field and ζM where ζ is magnetization-dependent constant and M is magnetization [7]. Fig. 11 is the measured major and minor loops of the sample. Using the initial magnetization curve, saturation curve and one minor loop, the Everett function is composed by the equation (3).

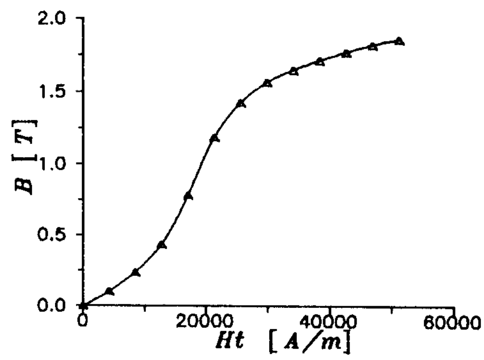

Fig. 10. Measured initial magnetization curve.

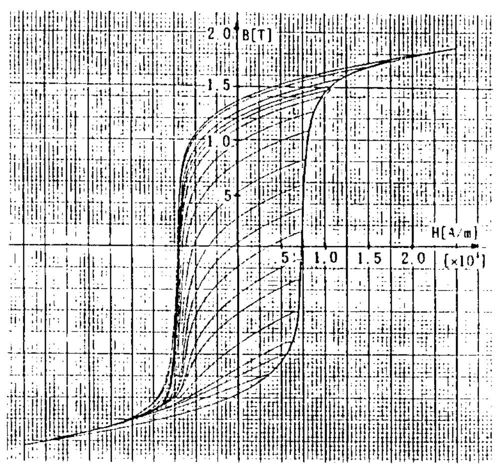

Fig. 11. Measured minor loops.

Fig. 12 shows the distribution of the Everett function for the interaction field obtained from the experiment and simulation. In the figure, solid lines are the calculated values of the Everett function and the dots are the measured values from (3). As shown in the figure, it is found that the simulation results give

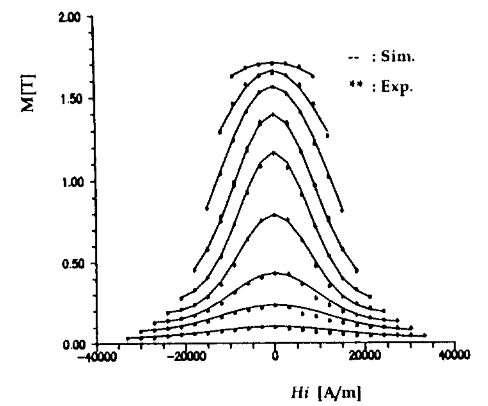

Fig. 12. Distribution of Everett function.

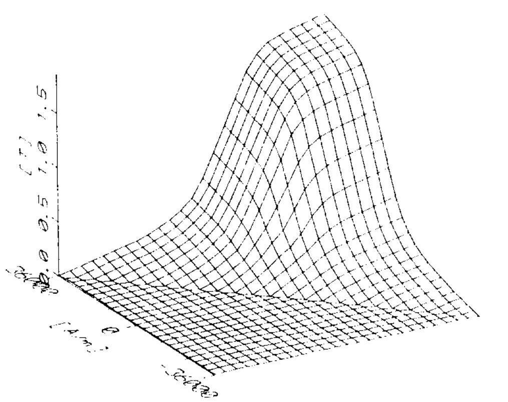

Fig. 13. Three dimensional plot of Everett function (simulation).

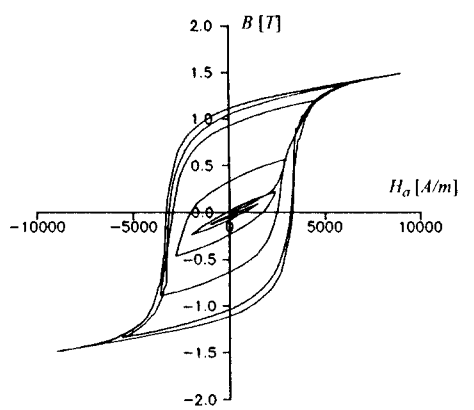

Fig. 14. Hysteresis curves (simulation).

good agreements with the experimental data. Fig. 13 is the three dimensional plot of the composed Everett function. Fig. 13 shows that a well-posed Everett function can be obtained by the proposed algorithm. From this function, the magnetization can be computed directly according to the variation of the magnetic field. Fig. 14 is the hysteresis curves simulated by the composed Everett function. It is remarkable that fairly acceptable hysteresis loops can be reproduced only using a few experimental data of hysteresis loops and initial magnetization curve.

Ⅵ. Conclusion

Gaussian distribution in magnetic materials is often assumed to represent the Preisach distribution because of the difficulties in experiments. However it is shown that the Gaussian function does not express the distribution function correctly for the coercivity axis. To overcome this problem, a simple method to determine an analytical expression for the Everett function is proposed. Although the proposed method uses only initial magnetization curve, saturation curve and one minor loop, the Everett function by the proposed method can be determined very easily and is very reliable. The reversible magnetization components can be considered in this procedure because the Everett function contains the reversible magnetization characteristics. Therefore the method is simple and useful in simulating the hysteresis characteristics.

Acknowledgment

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