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Conditional Least Squares Estimators of the Parameters of the NLAR(p) Time Series Model

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Abstract

Conditional least square estimators for the parameters of the NLAR(p) time series models are obtained. It is also shown that these estimators are consistent and asymptotically normal.

Keywords : New Laplace Autoregressive process; Random coefficient autoregressive process; Conditional least squares estimator; Consistency; Asymptotic normal distribution

1. Introduction

It is uaually assumed in standard time series analysis that the marginal distributions of $\{X_t\}$ are Gaussian. However, there is number of real data such that Gaussian distribution is not appropriate, for example, highly skewed and long-tailed data. Recently a number of non-Gausian time series models have been developed. One class of the models is the class of Laplacian time series models characterized by the fact that the marginal distribution of the observation follows Laplace distribution.

A new Laplace autoregressive time series model of order 2 - NLAR(2) was introduced by Dewald and Lewis(1985). In their paper, correlation structure and distributional properties have been studied extensively so that we have a good understanding of the underlying mechanism. Necessary and sufficient condition for existence of stationary ergodic NLAR(p) time series model has been obtained by Kim and Billard(1997). However, very little has been done on estimation : only the conditional least square estimators for the NALR(2) model have been obtained by Karleen and Tjosteim(1988).

It is pointed out by Lawrence and Lewis(1985) and Dewald and Lewis(1985) that the NLAR(2) model is a special case of the so-called random coefficient autoregressive (RCA) models which are treated by Nicholls and Quinn(1982). In this paper, we will show that the NLAR(p) model is a special case of the RCA models in section 2 and obtain the conditional least square estimators for the parameters of the NLAR(p) models by using the estimation

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techniques for the RCA models in section 3. Furthermore, it will be shown that the conditional least square estimators are strongly consistent and asymptotically normal in section 4.

2. Preliminary

The NLAR(p) process $\{X_t\}$ is of the form

$$X_t = e_t + \begin{cases} \beta_1 X_{t-1} & w.p.\ \alpha_1 \\ \beta_2 X_{t-2} & w.p.\ \alpha_2 \\ \vdots & \\ \beta_p X_{t-p} & w.p.\ \alpha_p \\ 0 & w.p.\ \alpha_0 = 1-\alpha_1-\alpha_2-\cdots-\alpha_p \end{cases} \quad (2.1)$$

where the distribution of the iid innovation squence $\{e_t\}$ is chosen so that the stationary sequence $\{X_t\}$ is standard Laplace, *i.e.*,

$$f(x) = \frac{1}{2}\exp(-|x|), \qquad -\infty < x < \infty \quad (2.2)$$

As shown in Kim and Billard(1997), there exists a strictly stationary ergodic process $\{X_t\}$ satisfying the equation (2.1) if either of the following conditions holds:

(1) $P(\beta_{J(n)} = 0) > 0$

or (2.3)

(2) $P(|\beta_{J(n)}| > 0) = 1$, $E|e_t| < \infty$, and $E|\beta_{J(n)}| < 1$

where $\beta_{J(n)}$ is a random variable of the following form:

$$\beta_{J(n)} = \begin{cases} \beta_1 & w.p.\ \alpha_1 \\ \beta_2 & w.p.\ \alpha_1 \\ \cdots & \\ \beta_p & w.p.\ \alpha_p \\ 0 & w.p.\ \alpha_0 = 1-\alpha_1-\cdots-\alpha_p \end{cases}$$

The NLAR(p) models can be formulated as a pth-order RCA process which is treated in Nicholls and Quiin(1982). The univariate pth-order RCA models are given by

$$X_t = \sum_{i=1}^{p} \{\gamma_i + B_i(t)\} X_{t-i} + e_t \quad (2.4)$$

where γ_i , $i = 1, 2, \cdots, p$ are constants and $\boldsymbol{B}_t = (B_1(t), B_2(t), \cdots, B_p(t))$ is iid random vector and also independent of $\{e_t\}$ which is iid innovation sequence of mean zero and variance $\sigma_e^2 < \infty$.

It is easily seen that the equation (2.1) can be written as the equivalent form to the equation (2.4), *i.e.*,

$$X_t = \sum_{i=1}^{p} K_{t_i} X_{t-i} + e_t \tag{2.5}$$

where the joint distribution of $(K_{t_1}, K_{t_2}, \cdots, K_{t_p})$ is given by

$$\begin{cases} P(K_{t_1}=\beta_1,\ K_{t_2}=0,\ \cdots,\ K_{t_p}=0\ )=\alpha_1 \\ P(K_{t_1}=0,\ K_{t_2}=\beta_2,\ \cdots,\ K_{t_p}=0\ )=\alpha_2 \\ \qquad\vdots \\ P(K_{t_1}=0,\ K_{t_2}=0,\ \cdots,\ K_{t_p}=\beta_p\ )=\alpha_p \\ P(K_{t_1}=0,\ K_{t_2}=0,\ \cdots,\ K_{t_p}=0\ )=1-\alpha_1-\alpha_2-\cdots-\alpha_p \end{cases} \tag{2.6}$$

Hence, if we set

$$\gamma_i = \alpha_i\beta_i \quad \text{and} \quad B_i(t) = (K_{t_i}-\alpha_i\beta_i),\ i=1,\ 2,\ \cdots,\ p, \tag{2.7}$$

then the NLAR(p) model becomes a special case of the RCA(p) model and can be simply expressed as the following matrix form:

$$X_t = \boldsymbol{\gamma}' \boldsymbol{X}_{t-1} + u_t \tag{2.8}$$

where $\boldsymbol{\gamma}' = (\gamma_1, \gamma_2, \cdots, \gamma_p)$, $\boldsymbol{X}_{t-1}' = (X_{t-1}, X_{t-2}, \cdots, X_{t-p})$, and $u_t = \boldsymbol{B}_t\boldsymbol{X}_{t-1} + e_t$

It is noted that letting

$$E(K_{t_i}) = \gamma_i,\ \ Var(K_{t_i}) = \sigma_{ii},\ \text{and}\ \ Cov(K_{t_i}, K_{t_j}) = \sigma_{ij},\ \ i,\ j=1,2,\cdots,p,$$

we have

$$\gamma_i = \alpha_i\beta_i\ ,\ \ E[\boldsymbol{B}_t] = \boldsymbol{0},\ \text{and}\ \ E[\boldsymbol{B}_t'\boldsymbol{B}_t] = \boldsymbol{\Sigma} \tag{2.9}$$

where $\boldsymbol{0}$ is the (1×p) zero vector and $\boldsymbol{\Sigma}$ is the (p×p) symmetric matrix whose $(i,\ j)$th element is

$$\sigma_{ii} = \beta_i^2\alpha_i(1-\alpha_i)\ \ \text{and}\ \ \sigma_{ij} = -\alpha_i\alpha_j\beta_i\beta_j,\ \ i,\ j=1,2,\cdots,p,\ \ i\neq j \tag{2.10}$$

3. Conditional Least Squares Estimation

The parameters α_i and β_i of the NLAR(p) model in the equation (2.1) can be expressed as a function of γ_i and σ_i from the equations (2.9) and (2.10). Solving these equations in terms of α_i, β_i, $i=1,2,\cdots,p$ and letting $\widehat{\gamma_i}$ and $\widehat{\sigma_{ii}}$ be estimators of γ_i and σ_{ii} respectively, we obtain estimators $\widehat{\alpha_i}$ and $\widehat{\beta_i}$ of the parameters of the NLAR(p) model as follow :

$$\widehat{\alpha}_i = \frac{\widehat{\gamma}_i}{\widehat{\beta}_i} \qquad i = 1, 2, \cdots, p \qquad (3.1)$$
$$\widehat{\beta}_i = \frac{\widehat{\sigma}_{ii}}{\widehat{\gamma}_i} + \widehat{\gamma}_i$$

Thus we only need to find out the estimators $\widehat{\gamma}_i$ and $\widehat{\sigma}_{ii}$ to estimate α_i and β_i, $i = 1, 2, \cdots, p$.

Let $\mathcal{F}_t$ be the σ-algebra generated by $\{X_t, X_{t-1}, X_{t-2}, \cdots\}$ which satisfies the equation (2.8). The conditional least squares(CLS) estimation has a two-step procedure. The first step is to estimate the parameters γ_i, $i = 1, 2, \cdots, p$. Given the sample $X_1, X_2, \cdots, X_n$, we can obtain the conditional least squares estimate $\widehat{\gamma}_i$ of γ_i by minimizing $\sum_{t=p+1}^{n} u_t^2$ where $u_t = X_t - E[X_t \mid \mathcal{F}_{t-1}] = X_t - \boldsymbol{\gamma}' \boldsymbol{X}_{t-1}$.

Hence, $\widehat{\boldsymbol{\gamma}}$ is given by

$$\widehat{\boldsymbol{\gamma}} = \Big(\sum_{p+1}^{n} \boldsymbol{X}_{t-1} \boldsymbol{X}_{t-1}'\Big)^{-1} \sum_{t=p+1}^{n} \boldsymbol{X}_{t-1} X_t \qquad (3.2)$$

The above equation can be rewritten in terms of $\widehat{\gamma}_i$ as follows :

$$\sum_{t=p+1}^{n} \sum_{j=1}^{p} X_t X_{t-j} = \sum_{t=p+1}^{n} \sum_{j=1}^{p} \widehat{\gamma}_i X_{t-i} X_{t-j}, \quad i = 1, 2, \cdots, p. \qquad (3.3)$$

The second step in the estimation procedure begins by developing $E[u_t^2 \mid \mathcal{F}_{t-1}]$. From the equation (2.6), we have

$$\begin{aligned} E[u_t^2 \mid \mathcal{F}_{t-1}] &= E[e_t^2] + 2E[e_t \boldsymbol{B}_t \boldsymbol{X}_{t-1} \mid \mathcal{F}_{t-1}] + E[(\boldsymbol{B}_t \boldsymbol{X}_{t-1})^2 \mid \mathcal{F}_{t-1}] \\ &= \sigma_e^2 + \boldsymbol{X}_{t-1}' E[\boldsymbol{B}_t' \boldsymbol{B}_t] \boldsymbol{X}_{t-1} \\ &= \sigma_e^2 + \boldsymbol{X}_{t-1}' \boldsymbol{\Sigma} \boldsymbol{X}_{t-1} \end{aligned} \qquad (3.4)$$

In order to solve the above eqation for σ_{ij}, we need the Kronecker product and a component vector which are defined in Nicholls and Quinn(1982).

Definition 1. Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be $(m \times n)$ and $(p \times q)$ matrices respectively. Then the Kronecker product $\boldsymbol{A} \otimes \boldsymbol{B}$ of $\boldsymbol{B}$ with $\boldsymbol{A}$ is the $(mp \times nq)$ matrix whose *(i, j)*th block is the $(p \times q)$ matrix $A_{ij}\boldsymbol{B}$ where A_{ij} is the *(i, j)*th element of $\boldsymbol{A}$.

Definition 2. Let $\boldsymbol{A}$ be $(m \times n)$ matrix. Then the mn-component vector $vec\boldsymbol{A}$ is obtained from $\boldsymbol{A}$ by stacking the columns of $\boldsymbol{A}$, one on the top of the other in order from left to right.

Definition 3. Let $\boldsymbol{A}$ be an $(n \times n)$ symmetric matrix. The $n(n+1)/2$ -component vector $vech\boldsymbol{A}$(the vector-half of $\boldsymbol{A}$) is obtained from $\boldsymbol{A}$ by stacking those parts of column of $\boldsymbol{A}$, on and below the main diagonal, one on the top of the other in order from left to right.

Two properties from the above definitions hold for any matrix products which are defined.

Property 1. $vec(\boldsymbol{ABC}) = (\boldsymbol{C}'\otimes\boldsymbol{A})vec\boldsymbol{B}$

Property 2. There exist constant $(n(n+1)/2\times n^2)$ matrix $\boldsymbol{H}$ such that $vec\boldsymbol{A} = \boldsymbol{H}'vech\boldsymbol{A}$ for any $(n\times n)$ symmetric matrix $\boldsymbol{A}$.

By using these properties, the equation (3.4) can be expressed as

$$\begin{aligned} E[u_t^2 \mid \mathcal{F}_{t-1}] &= \sigma_e^2 + (\boldsymbol{X}_{t-1}'\otimes\boldsymbol{X}_{t-1})vec\boldsymbol{\Sigma} \\ &= \sigma_e^2 + (vec(\boldsymbol{X}_{t-1}'\otimes\boldsymbol{X}_{t-1}))'\boldsymbol{H}'vech\boldsymbol{\Sigma} \\ &= \sigma_e^2 + \boldsymbol{Z}_t'\boldsymbol{\delta} \\ &= \sigma_e^2 + \boldsymbol{\delta}'\boldsymbol{Z}_t \end{aligned} \tag{3.5}$$

where $\boldsymbol{\delta} = vech\boldsymbol{\Sigma}$ and $\boldsymbol{Z}_t = \boldsymbol{H}vec(\boldsymbol{X}_{t-1}'\otimes\boldsymbol{X}_{t-1}')$ with $(p(p+1)/2\times p^2)$ matrix $\boldsymbol{H}$ whose (i, j)th block H_{ij} is $((p-i+1)\times p)$ zero matrix below diagonal, $(\boldsymbol{0}\ \ \boldsymbol{I})$ on diagonal where $\boldsymbol{0}$ is $((p-i+1)\times(i-1)$ zero matrix and $\boldsymbol{I}$ is $((p-i+1)\times(p-i+1))$ identity matrix, and (j, i)th element of H_{ij} is 1 and 0 elsewhere above diagonal.

Let $\eta_t = u_t^2 - E[u_t^2 \mid \mathcal{F}_{t-1}]$. Then the CLS estimator $\widehat{\boldsymbol{\delta}}$ of $\boldsymbol{\delta}$ can be obtained by minimizing $\sum_{t=p+1}^{n}\eta_t^2$. Hence, we have

$$\widehat{\boldsymbol{\delta}} = \{\sum_{t=p+1}^{n}(\boldsymbol{Z}_t-\overline{\boldsymbol{Z}})(\boldsymbol{Z}_t-\overline{\boldsymbol{Z}})'\}^{-1}\sum_{t=p+1}^{n}\widehat{u}_t^2(\boldsymbol{Z}_t-\overline{\boldsymbol{Z}}) \tag{3.6}$$

where $\overline{\boldsymbol{Z}} = \frac{1}{n}\sum_{t=p+1}^{n}\boldsymbol{Z}_t$ in which the elements $\frac{1}{n}\sum_t X_{t-i}^2$, $i=1,2,\cdots,p$ of $\frac{1}{n}\sum_t \boldsymbol{Z}_t$ are equal to 2, since $\{X_t\}$ follows the standard Laplace distribution.

The equation (3.7) can be rewritten in terms of $\widehat{\sigma}_{jj}$ as follows:

$$\sum_{t=p+1}^{n}\widehat{G}_t(X_{t-i}^2-2) = \sum_{j=1}^{p}\widehat{\sigma}_{jj}\sum_{t=p+1}^{n}(X_{t-i}^2-2)(X_{t-j}^2-2),\ i=1,2,\cdots,p \tag{3.7}$$

where $\widehat{G}_t = (X_t - \sum_{i=1}^{p}\widehat{\gamma}_i X_{t-1})^2 - 2\sum_{i<j}\sum\widehat{\sigma}_{ij}(X_{t-i}X_{t-j}-\overline{Z}_{ij})$ and

$$\overline{Z}_{ij} = \frac{1}{n}\sum_{t=p+1}^{n}X_{t-i}X_{t-j}$$

4. Strong consistency and Asymptotic normality

In this section, the CLS estimators $\widehat{\boldsymbol{\gamma}}$ and $\widehat{\boldsymbol{\delta}}$ will be shown to be strong consistent and have asymptotic normal distribution. These facts imply that the CLS estimators $\widehat{\alpha}_i$ and

$\widehat{\beta}_i$, $i=1,2,\cdots,p$ will have the same results.

Theorem 3.1 For the NLAR process $\{X_t\}$ satisfying the equation (2.8) under the assumption (2.3) and the CLS estimator $\widehat{\boldsymbol{\gamma}}$ given by (3.2), $\widehat{\boldsymbol{\gamma}}$ converges almost surely to $\boldsymbol{\gamma}$. furthermore, $\sqrt{n}(\widehat{\boldsymbol{\gamma}}-\boldsymbol{\gamma})$ converges in distribution to the normal distribution with mean $\mathbf{0}$ vecter and covariance matrix $\sigma_e^2 \boldsymbol{V}^{-1} + \boldsymbol{V}^{-1}E[\boldsymbol{X}_{t-1}\boldsymbol{X}_{t-1}{}'\boldsymbol{\gamma}'\boldsymbol{Z}_t]\boldsymbol{V}^{-1}$ where $\boldsymbol{V}=E[\boldsymbol{X}_{t-1}\boldsymbol{X}_{t-1}{}']$

Proof From the equation (3.2), $\widehat{\boldsymbol{\gamma}}-\boldsymbol{\gamma}$ is givn by

$$\begin{aligned}\widehat{\boldsymbol{\gamma}}-\boldsymbol{\gamma} &= \{\frac{1}{n}\sum_t \boldsymbol{X}_{t-1}\boldsymbol{X}_{t-1}{}'\}^{-1}\{\frac{1}{n}\sum_t \boldsymbol{X}_{t-1}X_t\} - \boldsymbol{\gamma} \\ &= \{\frac{1}{n}\sum_t \boldsymbol{X}_{t-1}\boldsymbol{X}_{t-1}{}'\}^{-1}\{\frac{1}{n}\sum_t (\boldsymbol{X}_{t-1}X_t - \boldsymbol{X}_{t-1}\boldsymbol{X}_{t-1}{}'\boldsymbol{\gamma})\} \\ &= \{\frac{1}{n}\sum_t \boldsymbol{X}_{t-1}\boldsymbol{X}_{t-1}{}'\}^{-1}\{\frac{1}{n}\sum_t \boldsymbol{X}_{t-1}u_t\}\end{aligned} \qquad (4.1)$$

Since $\{X_t\}$ is a strictly stationary and ergodic under the assumption (2.3), so are $\{\boldsymbol{X}_{t-1}\boldsymbol{X}_{t-1}{}'\}$ and $\{\boldsymbol{X}_{t-1}u_t\}$. Furthermore, $\boldsymbol{V}$ is finite and

$$\begin{aligned}E[\boldsymbol{X}_{t-1}u_t] &= E[E[\boldsymbol{X}_{t-1}u_t \mid \mathcal{F}_{t-1}]] \\ &= E[\boldsymbol{X}_{t-1}E[u_t \mid \mathcal{F}_{t-1}]] \\ &= \mathbf{0}\end{aligned} \qquad (4.2)$$

Thus, $\frac{1}{n}\sum_t \boldsymbol{X}_{t-1}\boldsymbol{X}_{t-1}{}'$ and $\frac{1}{n}\sum_t \boldsymbol{X}_{t-1}u_t$ converge almost surely to V and $\mathbf{0}$ respectively so that $\widehat{\boldsymbol{\gamma}}-\boldsymbol{\gamma}$ converges almost surely to $\mathbf{0}$.

Now if $\boldsymbol{c}$ is any p-component vector, then we have

$$E[\boldsymbol{c}'\boldsymbol{X}_{t-1}u_t \mid \mathcal{F}_{t-1}] = 0 \qquad (4.4)$$

and

$$\begin{aligned}E[(\boldsymbol{c}'\boldsymbol{X}_{t-1}u_t)^2 \mid \mathcal{F}_{t-1}] &= E[E[(\boldsymbol{c}'\boldsymbol{X}_{t-1}u_t)^2 \mid \mathcal{F}_{t-1}]] \\ &= E[(\boldsymbol{c}'\boldsymbol{X}_{t-1})^2 E[u_t^2 \mid \mathcal{F}_{t-1}]] \\ &= E[(\boldsymbol{c}'\boldsymbol{X}_{t-1})^2(\sigma_e^2 + \boldsymbol{\gamma}'\boldsymbol{Z}_t)] \\ &< \infty\end{aligned} \qquad (4.5)$$

since $E[X_t^4] < \infty$.

Thus, $\frac{1}{\sqrt{n}}\sum_t \boldsymbol{c}'\boldsymbol{X}_{t-1}u_t$ converges in distribution to the normal distribution with mean 0 and variance $E[(\boldsymbol{c}'\boldsymbol{X}_{t-1})^2(\sigma_e^2 + \boldsymbol{\gamma}'\boldsymbol{Z}_t)]$ by the Martingale central limit theorem.

This implies that $\sqrt{n}(\widehat{\boldsymbol{\gamma}}-\boldsymbol{\gamma})$ converges in distribution to the normal distribution with mean

$\mathbf{0}$ vecter and covariance matrix $\sigma_e^2 V^{-1} + V^{-1}E[X_{t-1}X_{t-1}'\gamma' Z_t]\, V^{-1}$ where

$V = E[X_{t-1}X_{t-1}']$.

Theorem 3.2 For the NLAR process $\{X_t\}$ satisfying the equation (2.8) under the assumption (2.3) with $E[X_t^{\,8}] < \infty$ and the CLS estimator $\hat{\delta}$ given by (3.6), $\hat{\delta} - \delta$ converges almost surely to $\mathbf{0}$. Furthermore, $\sqrt{n}(\hat{\delta} - \delta)$ converges in distribution to the normal distribution with mean $\mathbf{0}$ vector and covariance matrix

$$R^{-1}E[(Z_t - E(Z_t))(Z_t - E(Z_t))'(u_t^2 - \sigma_e^2 - \gamma' Z_t)^2]R^{-1} \qquad (4.6)$$

where $R = E[(Z_t - E(Z_t))((Z_t - E(Z_t))']$.

Proof Let $\tilde{\delta}$ be defined by

$$\tilde{\delta} = \{\sum_t (Z_t - \bar{Z})(Z_t - \bar{Z})'\}^{-1}\sum_t u_t^2 (Z_t - \bar{Z}) \qquad (4.7)$$

It was shown in Nichools and Quinn (1982) that $\tilde{\delta} - \hat{\delta}$ converges almost surely to $\mathbf{0}$, while $\sqrt{n}(\tilde{\delta} - \hat{\delta})$ converges in probability to $\mathbf{0}$. Hence if $\tilde{\delta} - \delta$ is shown to converge almost surely to $\mathbf{0}$, and $\sqrt{n}(\tilde{\delta} - \delta)$ converges in distribution to a normal, then $\hat{\delta} - \delta$ converges almost surely to $\mathbf{0}$ and $\sqrt{n}(\hat{\delta} - \delta)$ converges in distribution in the same way as $\sqrt{n}(\tilde{\delta} - \delta)$. Thus we need only prove the result for $\tilde{\delta}$.

From the equation (4.6), we have

$$\begin{aligned}\tilde{\delta} - \delta &= \{\frac{1}{n}\sum_t (Z_t - \bar{Z})(Z_t - \bar{Z})'\}^{-1}\frac{1}{n}\sum_t (Z_t - \bar{Z})u_t^2 - \delta \\ &= \{\frac{1}{n}\sum_t (Z_t - \bar{Z})(Z_t - \bar{Z})'\}^{-1}\frac{1}{n}\sum_t (Z_t - \bar{Z})(u_t^2 - (Z_t - \bar{Z})'\delta) \qquad (4.8) \\ &= \{\frac{1}{n}\sum_t (Z_t - \bar{Z})(Z_t - \bar{Z})'\}^{-1}\frac{1}{n}\sum_t (Z_t - \bar{Z})\xi_t\end{aligned}$$

where $\xi_t = u_t^2 - \sigma_e^2 - \gamma' Z_t$

It is easily seen that $\frac{1}{n}\sum_t \xi_t$ converges almost surely to 0 by the ergodic theorem, since $\{\xi_t\}$ is ergodic and $E[\xi_t \mid \mathcal{F}_{t-1}] = 0$. It is also seen that $\frac{1}{n}\sum_t Z_t\xi_t$ converges almost surely to $\mathbf{0}$, since $\{Z_t\xi_t\}$ is ergodic and $E[Z_t\xi_t] = E[Z_t E(\xi_t \mid \mathcal{F}_{t-1})] = \mathbf{0}$. Moreover, $\frac{1}{n}\sum_t (Z_t - \bar{Z})(Z_t - \bar{Z})'$ converges almost surely to R. Thus $\tilde{\delta} - \delta$ converges almost surely to $\mathbf{0}$.

Now , if $\mathbf{c}$ is any $p(p+1)/2$ -component vector, we have

$$E[\ \mathbf{c}'(\mathbf{Z}_t-\overline{\mathbf{Z}})\xi_t \mid \mathcal{F}_{t-1}] = E[\ \mathbf{c}'(\mathbf{Z}_t-\overline{\mathbf{Z}})\{E(\xi_t)\mid \mathcal{F}_{t-1}\}] = 0$$

and

$$E[\{\ \mathbf{c}'(\mathbf{Z}_t-\overline{\mathbf{Z}})\xi_t\}^2 \mid \mathcal{F}_{t-1}] = E[\{\ \mathbf{c}'(\mathbf{Z}_t-\overline{\mathbf{Z}})\}^2\{E(\xi_t^{\ 2})\mid \mathcal{F}_{t-1}\}] < \infty$$

since $E[X_t^{\ 8}] < \infty$.

Thus $\frac{1}{\sqrt{n}}\sum_t \mathbf{c}'(Z_t-\overline{Z})\xi_t$ converges in distribution to the normal distribution with mean 0 and variance $\mathbf{c}'E[(\mathbf{Z}_t-E(\mathbf{Z}_t))(\mathbf{Z}_t-E(\mathbf{Z}_t))'(u_t^{\ 2}-\sigma_e^2-\boldsymbol{\gamma}'\mathbf{Z}_t)^2]\mathbf{c}$.

Hence, $\sqrt{n}(\tilde{\boldsymbol{\delta}}-\boldsymbol{\delta})$ converges in distribution to the normal distribution with mean $\mathbf{0}$ vector and covariance matrix given in (4.7).

Corollary 4.3 For the CLS estimators $\widehat{\sigma_{ii}}$ of σ_{ii}, $i=1,2,\cdots,p$, which are the elements in $\widehat{\boldsymbol{\delta}}' = (\sigma_{11}\ \sigma_{21}\cdots\sigma_{p1}:\sigma_{22}\ \sigma_{32}\cdots\sigma_{p2}:\cdots:\sigma_{pp})'$, $\widehat{\sigma_{ii}}$ converges almost surely to σ_{ii}. Furthermore, $\sqrt{n}(\widehat{\sigma_{ii}}-\sigma_{ii})$ converges in normal distribution with mean 0 and a appropriate variance.

Theorem 4.4 For the NLAR process $\{X_t\}$ satisfying the equation (2.1) under the assumption of (2.3), the CLS estimators $\widehat{\alpha_i}$ and $\widehat{\beta_i}$, $i=1,2,\cdots,p$ given in (3.1) are consistent. Furthermore, both of $\sqrt{n}(\widehat{\alpha_i}-\alpha)$ and $\sqrt{n}(\widehat{\beta_i}-\beta_i)$ converge in normal distribution with mean 0 and the respective appropriate variance.

Proof Since $\widehat{\gamma_i}\longrightarrow\gamma_i$ and $\widehat{\sigma_{ii}}\longrightarrow\sigma_{ii}$, we have $\widehat{\alpha_i}\longrightarrow\alpha_i$ and $\widehat{\beta_i}\longrightarrow\beta_i$. Convergency in distribution is clear from Slutsky's theorem.

5. Conclusion

The New Laplace autoregressive model of order p - NLAR(p) model is a special case of the random coefficient autoregressive models. In this paper, the conditional least square estimators for the parameters of the NLAR(p) time series models are obtained by using the estimation techqenic developed by Nicholls and Quinn (1982). It is also shown that these estimators are strongly consistent and asymptotically normal.

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