Heat Transfer Correlation to Predict the Evaporation of a Water Droplet in Superheated Steam during Reflood Phase of a LOCA

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Abstract—A heat transfer correlation to predict the vaporization of a water droplet in highly superheated steam during a loss-of-coolant accident (LOCA) of a nuclear power plant is provided. Vaporization of liquid fuel or water droplets in superheated air or steam and subsequent interface heat transfer between a liquid droplet and superheated gas is typically correlated by way of a Nusselt number as a function of Reynolds number, Prandtl number, and in some cases including mass transfer number. Presently available correlations and experimental data of the evaporation of liquid droplets in air or steam are analyzed and a new Nusselt number correlation is proposed taking Schmidt number into consideration in order to account for binary diffusion of the vapor as well. \[ \text{Nu} = 2 + 0.53 \text{Sc}^{1/3} \text{Re}_{\text{B}}^{1/3} \text{Pr}_{\text{T}}^{1/3} \] for which properties are evaluated at film condition except the density of Reynolds number evaluated at ambient condition. Diverse correlations for various combinations of liquid and gas species are put into single equation. The blowing correction factor of \((1+B)^{3/2}\) is confirmed appropriate, and a criterion to distinguish so-called high- and low-temperature condition of ambient gas is set forth.

1. Introduction

The usual reflooding situations of a LOCA are characterized by low pressure and low mass fluxes. Under such conditions, the presence of dispersed flow film boiling has been widely recognized, where the void fraction is high and the liquid is in the form of dispersed droplets. Heat transfer in this regime is usually calculated in two steps: the heat flux from the wall superheats the vapor in contact with it; the vapor then transfers some of its heat to the liquid. Although there is largely no direct liquid contact with the wall, it is still the presence of liquid in the channel that is decisive for heat transfer in this regime. Thus correct prediction of water evaporation in superheated steam is crucial for the analysis of the reflood phase of a LOCA. The vapor-to-droplet heat transfer is usually modeled by a correlation for heat transfer to droplets in superheated steam, and this is typically Lee and Ryley\(^{[1]}\) correlation which is, however, based on a very low range of vapor superheats. Its use in high vapor temperature environments is reckless because of the effect of the vapor properties and, moreover, it predicts higher heat transfer rates than other correlations. Even though Yadigaroglu et al.\(^{[2]}\) recommended Renksizbulut and Yuen\(^{[3]}\) correlation as a safer alternative, their correlation, however, needs confirmation of the applicability to superheated steam instead of air. The importance of liquid droplet evaporation in superheated gas stream is studied more extensively in the field of fuel-spray combustion rather than in nuclear safety. Fortunately, the studies of liquid fuel spray include the evaporation of water droplets as well even if the gas is usually air instead of steam. In order to maintain generality of liquid droplet evaporation, various studies of liquid fuel evaporation are analyzed first and a new correlation applicable to both fields of combustion and LOCA is eventually provided in this paper.

Vaporization of liquid fuel droplets determines fuel consumption rate and performance of a combustor. Such droplets are often constitutes a dispersed flow regime where liquid droplets are distributed in gas species. Significant portions of such droplet flows can have Reynolds numbers on the order of 100. Experimental investigations and analytical models for such dispersed flow are typically based on the behavior of individual droplets, which are assumed independent
of each other. The external flow and heat transfer from a gas to single liquid droplet often behave as quasi-steady laminar flow forced convection heat transfer to a solid sphere. According to Harpole\textsuperscript{39}, internal circulation, free stream turbulence, accelerations, and oscillations are known by Galloway and Sage\textsuperscript{29} to typically have only small effects on the external flow and heat transfer to droplets when the Reynolds number is on the order of 100. For example, a water droplet surface is typically an isotherm, because liquid side Peclet numbers are usually so large that liquid near the surface circulates from the front of the droplet to the back much faster than heat can diffuse into the droplet.

The vaporization of liquid droplets and subsequent interface heat transfer between a liquid droplet and a gas is typically correlated by way of a Nusselt number as a function of Reynolds number and Prandtl number. Many correlations are available for the prediction of the vaporization of liquid droplets and one of the prominent and most commonly referred ones is

\begin{equation}
\text{Nu} = 2 \times 0.6 \text{Re}_{\text{m}}^{0.8} \text{Pr}^{0.3}
\end{equation}

which is proposed by Ranz and Marshall\textsuperscript{31}. They obtained equation (1) by evaporating water, benzene and aniline droplets in laminar air jets up to 220°C in temperature. The correlation is widely used for the prediction of convective heat transfer where the temperature difference between droplet and gas is small. Froessling\textsuperscript{40} and Lee and Ryley\textsuperscript{41} performed similar investigations and suggested 0.552 and 0.74 respectively, instead of the constant 0.6. Froessling evaporated water, aniline and nitrobenzene in air at room temperature, and correlated Sherwood number as a function of Reynolds number and Schmidt number. From the similarity of mass and heat transfer correlations, the Nusselt number can be expressed as a function of Re and Pr, which gives 0.552 as a coefficient of non-dimensional number term. Lee and Ryley evaporated water droplets in superheated steam, which characterized their data compared with others done in air stream. Above 3 correlations have the same formula but different coefficients of non-dimensional number term and 0.6 is widely used for the prediction of forced convection heat transfer between liquid droplets and gas species. Special attention is given here that the correlation of Ranz and Marshall type equation is used for low gas temperature, or low degree of superheating. If the temperature or the degree of superheating of gas species is considerably high, then the correlation is usually modified applying a correction factor in a function of mass transfer number. Interface heat transfer between a liquid droplet and a gas substantially reduces in high temperature gas flow because significant mass transport occurs at the interface by convection as well as by diffusion, and the Nusselt number cannot be adequately correlated with Reynolds number and Prandtl number alone. The effect of vapor concentration at the vicinity of the interface cannot be neglected, and high blowing rates normal to the surface due to evaporation, strongly variable fluid properties, and binary diffusion with non-equal heat capacities should be taken into consideration. A function of blowing mass transfer number is directly implemented into the correlation for low temperature gas species in the form of a multiplier, and the variable fluid properties are considered by way of reference state method. The Nusselt number can be shown to be a function of Schmidt number as well from dimensional analysis

\begin{equation}
\text{Nu} = \nu(\text{Re}, \text{Pr}, \text{Sc}, B, T/T_{\text{h}})
\end{equation}

as already mentioned by Yuen and Chen\textsuperscript{31}, no correlations, however, took the Schmidt number into account yet.

Downing\textsuperscript{27} used a multiplier to the correlation of Ranz and Marshall for acetone, benzene, n-hexane, and water droplets in hot air with a reference temperature $T_{\text{h}} = 0.6T + 0.4T_{\text{E}}$. He used \( \frac{\ln(1+B)}{B} \left[ 1 - 0.4(1 - \ln(1+B) \right] \) as a blowing correction factor and multiplied it to the right hand side of the equation (1). Blowing parameter B was defined as

\begin{equation}
B = \frac{C_{\text{r}}(T_{\text{h}} - T_{\text{E}}) + \frac{q_{\text{h}}}{q_{\text{E}}}}{L_{\text{h}}}
\end{equation}

Yuen and Chen\textsuperscript{31} used evaluating properties at film temperature except the density in the Reynolds number evaluated at the free stream temperature. Radiation heat transfer was attentively excluded in the estimation of the amount of heat transfer and in the calculation of B. When the temperature difference is small between a liquid droplet and ambient gas, esti-
mations of properties at film temperature or at free stream temperature give negligible differences. Renkiszubut and Yuen\textsuperscript{18} used evaluating properties in the same way as Yuen and Chen\textsuperscript{16} did, and using the same definition of B as Downing\textsuperscript{19}. Kuo\textsuperscript{20} and Glassman\textsuperscript{21} suggested as a correction factor. This correction factor, however, is a Nusselt number itself for quiescent flow with Lewis, Prandtl and Schmidt numbers of unity. As the quiescent flow condition is conceptually already considered by way of the constant term in equation (1) and it is firmly confirmed by most of the investigators, this correction factor confronts with illogicality.

Even though the correlations are usually grouped into two for low and high gas temperatures, the criterion is so ambiguous and sometimes wrong. Latent heat of vaporization of benzene and water at atmospheric pressure are 391 and 2257 kJ/kg respectively. Assuming the liquid droplets are of room temperatures and ambient air is at 100°C, it is estimated that the mass transfer numbers of water and benzene are 0.07 and 0.24 respectively excluding radiation heat transfer. Benzene has an order of magnitude higher mass transfer number than the water. Air temperature of 100°C is high enough for the vaporization of benzene whose saturation temperature at atmospheric pressure is 80°C and the mass transfer number of benzene-air combination needs to be explicitly considered. Another example is from Renkiszubut and Yuen\textsuperscript{18}, their evaluations of mass transfer numbers for water and heptane at atmospheric pressure and 100°C of ambient air are 0.028 and 0.201 respectively. Usually 100°C can be treated as low temperature and the mass transfer number of water can be successfully neglected but the mass transfer number of heptane is not the case. Henceforth, “high-temperature” or “low-temperature” condition of a gas can not be determined by the temperature itself. The ambiguous grouping can be overcome if the mass transfer number is used instead of the gas temperature.

The purpose of present paper is to provide a new correlation for forced convection heat transfer between a liquid droplet and a gas taking the Schmidt number into consideration, modifying the coefficient of non-dimensional number terms in equation (1) or the similar, and finding a blowing correction factor. The new correlation can be used to predict evaporation of entrained emergency core cooling water droplets in superheated steam during the reflood phase of a LOCA, as well as liquid fuel evaporation in superheated air. The necessity of Schmidt number to be included in the correlation can also be shown from the evaporation of water droplets in air and steam. Even though the experimental conditions of Lee and Ryley\textsuperscript{11} and Ranz and Marshall\textsuperscript{20} are not much different except the species of gas stream, the coefficients of non-dimensional number terms are different each other by more than 23%. The difference is presumed caused by the different diffusion behavior of evaporated steam into ambient air and steam.

2. Blowing Correction Factor

Blowing of evaporated vapor thickens the boundary layer and substantially reduces the heat transfer to a droplet from a gas across the boundary layer. As previously discussed, reduction of the heat transfer is accounted by multiplying a function of mass transfer number to a reference Nusselt number such as Ranz and Marshall’s\textsuperscript{21}. Such correction factors empirically determined to correlate the experimental data show still some controversies and have little analytical basis. Comparison of the correction factors as a function of the mass transfer number is shown in Fig. 1. (1/B) ln(1+B) gives the highest value and 1/(1+B)\textsuperscript{0.7} and Downing’s\textsuperscript{20} correction factor nearly coincide each other and 1/(1+B)\textsuperscript{0} is the lowest. The difference between 1/(1+B)\textsuperscript{0.7} and Downing’s complicated correction factor is less than 0.02 over the range of B, henceforth the simpler 1/(1+B)\textsuperscript{0.7} and 1/(1+B)\textsuperscript{0} are discussed in the following, excluding (1/B) ln(1+B) because of its illogicality as mentioned before.

Integral heat balance at the interface of a vaporizing droplet with no radiation from the gas can be written such that

\[ \pi d^2 h(T_\infty - T_0) = \left[ L_v + C_p(T_\infty - T_0) \right] \frac{dm}{dt} \tag{4} \]

With a small temperature difference or with low gas temperature, the second term of right hand side of equation (4) in the bracket can be neglected. Then the amount of vapor generation is
Fig. 1. Blowing correction factors.

\[
\frac{\text{dm}}{\text{dt}} = \frac{\pi d^2 h(T_r-T_d)}{L_d}
\]

If it cannot be neglected, the amount of vapor generation is

\[
\frac{\text{dm}}{\text{dt}} = \frac{\pi d^2 h(T_r-T_d)}{L_d + C_r(T_r-T_d)}
\]

And

\[
\frac{(\text{dm/dt})_{tr}}{(\text{dm/dt})_{r}} = \frac{1}{1 + C_r(T_r-T_d)} = \frac{1}{1+B}
\]

From equation (4), it is deduced that the Nusselt number in high temperature gas flow is

\[
\text{Nu}_{tr} = \text{Nu}_r \left( \frac{1}{1+B} \right)
\]

And the blowing correction factor is

\[
\frac{\text{Nu}_{tr}}{\text{Nu}_r} = \frac{1}{1+B}
\]

Equation (5) is exactly the correction factor of Yuen and Chen. Using the modification of the mass transfer number by Kays to account for radiation heat transfer and rearranging, B becomes

\[
B = \frac{C_r(T_r-T_d) - C_r(T_r-T_d)}{L_d} \left( \frac{q_k}{q_k} \right)
\]

where \(q = \pi d^2 \sigma \epsilon [T_r^4 - T_i^4]\) and the sum of \(q_k\) and \(q_c\) can be evaluated from the right-hand side of equation (4).

With the blowing correction factor of equation (5) to equation (1), Yuen and Chen obtained good correlation for the vaporization of water and methanol droplets in air. Even though Renksizbulut and Yuen also could obtain a reasonable correlation for their water and methanol droplets using the correction factor of Yuen and Chen, they needed further modification of the correction factor to \(1/(1+B)^{0.7}\) to correlate the evaporation of n-heptane droplets in air, from which they provided excellent Nusselt number correlation with 0.57 instead of original 0.6 of equation (1). N-heptane in 100°C air at atmospheric pressure has considerably higher mass transfer number of 0.201 which is evaluated at film temperature keeping the droplet at room temperature, while water and methanol have negligible 0.028 and 0.067 respectively at the same condition. Considering that the boiling temperature of n-heptane is 98.4°C at atmospheric pressure, the mass transfer number of n-heptane is found to increase explosively as soon as boiling begins because the latent heat of vaporization is smaller than those of water and methanol by an order of magnitude. As for this kind of very volatile liquid species, inclusion of the mass transfer number is inevitable only if the boiling occurs whatever the temperature difference might be.

With low mass transfer numbers up to 0.15, difference between \(1/(1+B)\) and \(1/(1+B)^{0.7}\) is less than 5% as shown in Fig. 1. From the findings of Renksizbulut and Yuen (1983) and the correction factor of equation (5), it is conjectured that the blowing of the
evaporated vapor has dual effects on the interfacial heat transfer. It deteriorates the interface heat transfer by thickening the vapor blanket surrounding the droplet and, on the other hand, enhances the heat transfer by agitating the boundary layer. Based on the results of Renksizbulut and Yuen, the enhancement is presumed to be a factor of \((1+B)^{\alpha}\). Exclusion of the enhancement factor from equation (1) or the similar can be justified by the fact that \((1+B)^{\alpha}\) increases the heat transfer by less than 5% up to \(B\) of 0.17. For very volatile liquid like n-heptane whose mass transfer numbers are evaluated as 1 to 5 between 300 and 800°C while those for water and methanol are less than 1, the enhancement needs to be considered and the blowing correction factor of \((1+B)^{\alpha}\) seems to be reasonable. With those very high mass transfer numbers of order of magnitude of unity, Nusselt numbers can not be adequately described by sole adjustment of the coefficient of the non-dimensional number term without explicitly taking the mass transfer number into consideration.

The criterion to distinguish “high-temperature” and “low-temperature” gases can be set forth as the mass transfer number of 0.07 when \((1+B)^{\alpha}\) is used as a blowing correction factor, restricting the difference range of predicted to measured Nusselt numbers less than 5%. As the blowing correction factor fundamentally differentiate the heat transfer correlation, the criterion should be based on the mass transfer number not on the temperature itself. Mass transfer number reaches 0.07 when the ambient temperatures are 200 and 100°C for water and methanol respectively. Above these temperatures mass transfer numbers need to be considered into the Nusselt number and the temperatures should be classified as “high temperatures” if water and methanol droplets are concerned. Because the evaporation or boiling properties such as boiling temperature, latent heat of evaporation and specific heat at a certain temperature are different each other depending on the species of the liquid, the temperature criterion is not appropriate for the prediction of liquid evaporation. Mass transfer number of equation (6) assembles the boiling properties and can account for specific characteristics of the evaporation of liquid species. Henceforth it is more reasonable to set forth the mass transfer number as a criterion.

3. Effect of Binary Diffusion on Interfacial Heat Transfer

Diffusion of evaporated vapor into ambient gas is a critical parameter characterizing the flow behavior. Not only the mass but energy also is transferred by the diffusion because the diffusing mass has different enthalpy level from ambient gas. Inclusion of the diffusivity, or its non-dimensional Schmidt number into Nusselt number invokes the coupling of mass transfer and energy transfer resulting in solution of the problem more complicated. But this is the case in the convection heat transfer with evaporation of liquid phase. Without consideration of the diffusion of the vapor into ambient gas stream, discrepancies in the coefficients of non-dimensional number terms of equation (1) like correlations can not be adequately described.

In present paper, the diffusion is taken into consideration by way of non-dimensional Schmidt number. As the Schmidt number is also a property like Prandtl or Reynolds numbers, it is just included into the second term of equation (1) considering the blowing correction factor as well. The new correlation form is

\[
Nu_x(1+B)^{\alpha} = 2 + CSc_c Re^{\alpha}_w Pr^{\alpha}_f
\]

To determine coefficient \(C\) and exponent \(\alpha\), experimental data of Narasimhan and Gauvain\(^{14}\), Yuen and Chen\(^{15}\), Renksizbulut and Yuen\(^{13}\) were analyzed evaluating the properties at film condition except the density of Reynolds number evaluated at free stream temperature. With the degree of superheating of 3~34°C in the experiment of Lee and Ryley\(^{13}\), the film condition and ambient gas condition show negligible difference and ambient condition averaged over the range of experiment was used to evaluate relevant properties. Original correlation of Narasimhan and Gauvain\(^{14}\) for water droplets in aiding steam flow of 250~650°C is

\[
Nu_x(1+B)^{\alpha} = 0.68Re^{1/2}_w
\]

According to Renksizbulut and Yuen\(^{13}\), the equation is transformed with reasonable accuracy (±5%) into

\[
Nu_x(1+B)^{\alpha} = 2 + 0.57Re^{1/2}_w Pr^{\alpha}_f
\]

Due to the scarcity of the data of the evaporation of water droplet in superheated steam instead of air,
works of Narasimhan and Gauvin\cite{4} and Lee and Ryley\cite{10} are paid special attention. Diffusion coefficients for binary gas systems at low pressures were evaluated using the equation of Fuller et al. provided by Reid et al.\cite{18}. Self-diffusion of steam into ambient steam was also evaluated using the same equation. Equating "CSc" of equation (7) to the coefficients of non-dimensional number term of equation (1) or the similar with an average Schmidt number for each combination of liquid and gas species, average "a" was obtained as -0.218. However, -1/5 was substituted for -0.218 in order to make it concise and to maintain consistency with the exponents of Reynolds and Prandtl numbers, which are presented as a fraction of integer numbers. Substituting relevant values of all the data points in equation (7), "C" was found to be 0.53 with standard deviation of 0.049. Now the new correlation becomes

\[ \text{Nu}_l(1+B)^2 = 2 + 0.53 \text{Sc}_l \text{Re}_l^{1/5} \text{Pr}_l^{1/5} \]  

(10)

Equation (10) correlates about 400 data points within 18\% at a probability level of 95\% as shown in Fig. 2. All the data of Lee and Ryley\cite{5}, Yuen and Chen\cite{11} and Renksizbulut and Yuen\cite{12} show excellent agreement with equation (10) with bias of -0.105 and standard deviation of 0.930. The data of Lee and Ryley\cite{5} which were inevitably under-predicted by the correlation of Renksizbulut and Yuen\cite{12} due to the coefficient of the non-dimensional number term shows also good agreement with equation (10). Comparison of equation (8),

\[ \text{Nu}_l = \left(2 + 0.53 \text{Sc}_l \text{Re}_l^{1/5} \text{Pr}_l^{1/5}\right) / (1 + B)^2 \]

Fig. 2. Correlation of improved Nusselt number incorporating Schmidt number.

\[ \text{LHS of eq. 8, 9 and 10} \]

\[ \text{Re} \]

Fig. 3. Comparison of equation 10 with modified Renksizbuluts and original Narasimhan' equations.
(9) and (10) with the exponent of 2/3 instead of 0.7 neglecting the difference is shown in Fig. 3 for which the Reynolds number is evaluated at ambient condition on purpose for the comparison with equation (8). No remarkable differences are found among three and the applicability of equation (10) is again confirmed for the evaporation of a water droplet in highly superheated steam.

4. Conclusions

When the ambient gas temperature is considerably high, high transfer rate of evaporated vapor across the interface between a liquid droplet and ambient gas needs to be adequately described in order to accurately predict the interface heat transfer coefficient. The heat transfer coefficient represented by Nusselt number is typically reduced in high temperature environment by a blowing correction factor which is a function of mass transfer number B. Controversies in the blowing correction factors were discussed and (1+B) of Renksizbulut and Yuen was confirmed reasonable. Using their correction factor, the Nusselt number was newly correlated taking the Schmidt number into consideration to reflect fluid-specific diffusion behaviors. Inclusion of the binary diffusion enabled excellent correlation of the evaporation of methanol and n-heptane droplets in air, and water droplets in air and steam, at various test conditions, into a single equation, equation (10), otherwise impossible. The new correlation is expected to predict the evaporation of water droplets in dispersed flow film boiling heat transfer regime of a LOCA as well as the evaporation of liquid fuel species other than those considered in this paper. In this way, various forms of Nusselt number correlations and diverse coefficients of the non-dimensional number term of equation (1) or the similar could be consolidated. In addition, the criterion to distinguish ambient gas condition from so-called high-temperature from low-temperature was proposed as a mass transfer number of 0.07 instead of temperature itself.

Nomenclature

\[ B = \left( \frac{1+q/q_b} {C_p} \right) C_p(T_w - T_a)/\Delta h, \] mass transfer number

C_p : heat capacity
D : diffusivity
d : droplet diameter
h : heat transfer coefficient
k : thermal conductivity
\( \Delta h \) : latent heat of vaporization
\( \frac{dm}{dt} \) : mass evaporation rate
Nu : \( \frac{h d}{k} \), Nusselt number
Pr : \( C_p \mu /k \), Prandtl number
q : heat transfer
Re : \( \frac{p u d}{\mu} \), Reynolds number
Re_m : \( \rho u d/\mu \), Reynolds number
Sc : \( \mu /\rho D \), Schmidt number
T : temperature
u : velocity

Greek Symbols
\( \varepsilon \) : emissivity
\( \sigma \) : Stephan-Boltzmann constant
\( \rho \) : density
\( \mu \) : viscosity

Subscripts
\( c \) : convection
\( d \) : droplet surface
\( e \) : free stream
\( f \) : film reference state
HT : high temperature
LT : low temperature
ref : reference state
r : radiation
w : wall

References