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EXISTENCE OF EQUILIBRIA IN LOCALLY CONVEX SPACES

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ABSTRACT. The purpose of this paper is to prove new equilibrium existence theorems of social systems with coordination under general conditions on the preference correspondence in locally convex spaces, and we also give an example which the previous existence results on SSC do not work but our theorem can be applied.

1. Introduction

In mathematical economics, showing the existence of equilibrium is the main problem of investigating various kind of economic models, and till now, a number of equilibrium existence results in general economic models have been investigated by several authors, e.g., Debreu [3], Nash [10], Gale and Mas-Colell [6], Ding-Kim-Tan [4], Kim-Yuan [9] and others. Recently, social systems with coordination (simply, SSC) were introduced by Vind [12] as a general model of economic institutions containing most of the known types of institutions (trade market, bilateral exchanges etc.) as special cases. Vind [12] gave a proof of the existence of equlibrium in an SSC; however, the assumptions on agent's preferences are rather restrictive. Thus, in one of the prominent applications, the case of usual competitive equilibrium, the assumptions will not be fulfilled. Next, Keiding [7] gave a general existence theorem of equlibrium in a social systems with coordination under weaker assumptions on preferences - the same as those which are made to prove existence of competitive equlibrium , e.g. Gale and Mas-Colell [6]. Using the Eilenberg-Montgomery fixed point theorem, he proved the existence theorem under the usual assumptions (convexity, open graph) on preferences; but the preferences are well-behaved on the boundary of the feasible set X.

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In this paper, we shall give new existence theorems of equilibrium in an SSC in locally convex settings under general condition on the preference correspondence by using the Fan-Glicksberg fixed point theorem, and also give some of its corollaries containing the equilibrium existence results on abstract economies. Finally, we give a simple example which the previous existence results on SSC do not work but our theorem can be applied.

2. Preliminaries

Let A be a subset of a topological space X. We shall denote by 2^A the family of all subsets of A, and by $int_X A$ the interior of A in X. Let X, Y be topological spaces and $T : X \to 2^Y$ be a correspondence. A correspondence T is said to be *closed* or have *closed graph* (*open* or have *open graph*) if the graph of T ($Graph(T) = \{(x, y) \in X \times Y : x \in X, y \in T(x)\}$) is closed (open) in $X \times Y$. If T has open graph, then it is easy to see that $T(x)$ is open for each $x \in X$. A correspondence $T : X \to 2^Y$ is said to be (1) *upper semicontinuous* if for each $x \in X$ and each open set V in Y with $T(x) \subset V$, there exists an open neighborhood U of x in X such that $T(y) \subset V$ for each $y \in U$ and (2) *lower semicontinuous* if for each $x \in X$ and each open set V in Y with $T(x) \cap V \neq \emptyset$, there exists an open neighborhood U of x in X such that $T(y) \cap V \neq \emptyset$ for each $y \in U$, and (3) *continuous* if T is both upper semicontinuous and lower semicontinuous.

It should be noted that any closed correspondence mapping into a compact space is upper semicontinuous, e.g., see Proposition 3 in Aubin [1, p. 72].

Let Φ denote either the real field or the complex field. Denote $\mathbb{O}$ the zero vector, $\mathbb{R}$ the set of real numbers, and $\mathbb{Q}$ the set of rational numbers. Let E and F be vector spaces over Φ. Recall that $f : E \to \mathbb{R}$ is *quasi-concave* if for each $\lambda \in \mathbb{R}$, the set $\{x \in E \mid f(x) \geq \lambda\}$ is convex. Let $\langle\, ,\, \rangle : E \times F \to \Phi$ be a bilinear functional, and $f : F \to F$ be a given map. We now define some quasi-concavity for the map f. The map f is said to be *quasi*-concave* if for each $u \in E$, the set $\{x \in F \mid Re\langle u, f(x)\rangle \geq \lambda\}$ is convex for every $\lambda \in \mathbb{R}$. In fact, the quasi*-concavity of f means that the map $x \mapsto Re\langle u, f(x)\rangle$ is quasi-concave in the standard definition for each $u \in E$. If f is an affine map, then f is clearly quasi*-concave. Note also that if $f, g : F \to F$ are quasi*-concave, then the map $f + g$ is not quasi*-concave in general. In fact, we know that for every $\lambda \in \mathbb{R}$, $\{x \in F \mid Re\langle u, (f+g)(x)\rangle \geq \lambda\} = \cup_{\mu \in \mathbb{Q}} (\{x \in F \mid Re\langle u, f(x)\rangle \geq \mu\} \cap \{x \in F \mid Re\langle u, g(x)\rangle \geq \lambda - \mu\})$, and the union of convex sets is not convex in general.

Following Schaefer [11], we now introduce some duality facts in locally convex spaces. Let $E,\ F$ be locally convex spaces and E^* be the dual space of E. And the dual pairing $\langle\, ,\, \rangle$ is a bilinear functional on $F \times E$. We recall that the *weak topology* $\sigma(E, F)$ is the coarsest topology on E for which the linear functional $x \mapsto \langle y, x \rangle$ are continuous for each $y \in F$. Then we can obtain the following :

$f : E \to \Phi$ is a $\sigma(E, F)$–continuous linear functional on E if and only if it is of the form $f(x) = \langle y, x \rangle$ for a (unique) $y \in F$.

In particular, if $E = F$ is a locally convex space equipped with $\sigma(E, E)$-topology, and $\langle\, ,\, \rangle$ is a continuous dual pairing on $E \times E$, then for each $f \in (E(\sigma(E, E))^*$, we can find a unique $u \in E$ such that $f(x) = \langle u, x \rangle$ for all $x \in E$.

Next we introduce the following economic systems, which is slightly different from SSC due to Vind [12]. Let I be a (possibly uncountable) set of agents. A *social system with coordination* (simply *SSC*) $\Gamma = (X_i, A_i, P_i, e_i)_{i \in I}$ is defined as a family of ordered quadruples (X_i, A_i, P_i, e_i) such that for each $i \in I$,

(1) X_i is a non-empty set of actions available to the agent i in a topological vector space E_i (a choice set), and $X = \Pi_{j \in I} X_j$ is the set of states of the system Γ,

(2) $A_i : X \to 2^{X_i}$ is a state correspondence such that $A_i(x)$ is the state attainable for the agent i, and we denote $A(x) = \Pi_{j \in I} A_j(x)$ for each $x = (x_i)_{i \in I} \in X$,

(3) $P_i : X \to 2^{X_i}$ is a preference correspondence such that $P_i(x)$ is the state preferred by the agent i to x, and we denote $P(x) = \Pi_{j \in I} P_j(x)$ for each $x \in X$,

(4) $e_i : X \times X \to E_i$ is an expectation map such that for each $(x, y) \in X \times X$, $e_i(x, y)$ can be interpreted as the state in X_i by the agent i to obtain when in state x the state y should be chosen.

An *equilibrium* for SSC Γ is a point $\hat{x} \in X = \Pi_{i \in I} X_i$ such that for each $i \in I$,

(i) $\hat{x}_i \in A_i(\hat{x})$,

(ii) there is no state $y \in X$ such that the set $I_o = \{\, i \in I \mid e_i(\hat{x}, y) \neq \hat{x}_i\}$ is non-empty, and $e_i(\hat{x}, y) \in P_i(\hat{x}) \cap A_i(\hat{x})$ for all $i \in I_o$.

Actually, the equilibrium point $\hat{x}$ for SCC Γ means a fixed state of the given state correspondence $A = \Pi_{i \in I} A_i$ such that $\hat{x}_i$ is the optimal social state for every agent i. In fact, the condition (ii) means there can not exist better expected state $e_i(\hat{x}, y)$ different from the state $\hat{x}_i$ for some i under given constraints. And it should be noted that in real applications, the preference correspondence P_i satisfies the irreflexivity, i.e., $x_i \notin P_i(x)$ for each $x \in X$ since $P_i(x)$ has better evaluation than x_i in X_i at the given state $x \in X$.

As we mentioned, Keiding [7] use the interior condition for the state correspondence A, i.e. $A(x) \subset int\, X$ for each $x \in X$; but this condition is unappealing in some applications, and so in our proof of the main theorem, we will use the separation property for open convex and compact convex sets in locally convex spaces.

Throughout this paper, we assume that every locally convex space E having a dual pairing on $E \times E$ is equipped with $\sigma(E, E)$-topology, and any product space is equipped with the product topology.

3. Equilibrium existence for SSC in locally convex spaces

We begin with the following new existence theorem of equilibrium in an SSC under general assumptions on the preference correspondences without assuming the strong interior condition of the state correspondence in locally convex spaces.

THEOREM 1. *Let $\Gamma = (X_i, A_i, P_i, e_i)_{i \in I}$ be an SSC where I is a finite set of agents such that for each $i \in I$,*

(1) X_i is a compact convex subset of a locally convex Hausdorff topological vector space E_i, $X := \Pi_{i \in I} X_i$, and $\langle \cdot, \cdot \rangle$ is a continuous dual pairing on $E_i \times E_i$,

(2) the correspondence $A_i : X \to 2^{X_i}$ is continuous such that $A_i(x)$ is non-empty closed convex for each $x = (x_i)_{i \in I} \in X$,

(3) the correspondence $P_i : X \to 2^{X_i}$ has open graph such that $P_i(x)$ is (possibly empty) convex and $x_i \notin P_i(x)$ for each $x \in X$,

(4) $e_i : X \times X \to E_i$ is a continuous expectation map such that $e_i(x, \cdot)$ is quasi-concave and $e_i(x, x) = x_i$ for each $x \in X$,*

(5) if $e_i(x, y) \in A_i(x)$, then $y_i \in A_i(x)$.

Then there exists an equilibrium $\hat{x} \in X$ for an SCC Γ, i.e. for each $i \in I$,

(i) $\hat{x}_i \in A_i(\hat{x})$,

(ii) there is no state $y \in X$ such that the set $I_o = \{i \in I \mid e_i(\hat{x}, y) \neq \hat{x}_i\}$ is non-empty, and $e_i(\hat{x}, y) \in P_i(\hat{x}) \cap A_i(\hat{x})$ for all $i \in I_o$.

Proof. For each $i \in I$, we first assume that $P_i(x) = \emptyset$ for all $x \in X$. Since each A_i is upper semicontinuous, and hence $A = \Pi_{i \in I} A_i : X \to 2^X$ is upper semicontinuous and has non-empty compact convex value. Then, by the Fan-Glicksberg fixed point theorem, there exists a fixed point $\hat{x} \in X$

such that $\hat{x} \in \Pi_{i\in I} A_i(\hat{x})$, so that $\hat{x}_i \in A_i(\hat{x})$, and the conclusion (ii) is automatically satisfied. Therefore we have an equilibrium $\hat{x}$ for Γ.

For each $i \in I$, we may assume that there exists a point $x \in X$ such that $P_i(x)$ is non-empty open, and so X_i has non-empty interior.

We first define a map $f : X \times X \times \Pi_{i\in I} E_i \to \mathbb{R}$ by

$$f(x, y, p_1, \cdots, p_n) := \sum_{i\in I} Re\langle p_i, e_i(x, y)\rangle,$$
$$\text{for each } x, y \in X,\ (p_1, \cdots, p_n) \in \Pi_{i\in I} E_i.$$

Then, by assumptions (1) and (4), f is continuous, and $f(x, \cdot, p_1, \cdots, p_n)$ is quasi-concave. In fact, for each $i \in I$, the map $y \mapsto Re\langle p_i, e_i(x, y)\rangle$ is quasi-concave and hence $f(x, \cdot, p_1, \cdots, p_n)$ is quasi-concave.

Define the correspondence $\phi_o : X \times \Pi_{i\in I} E_i \to 2^X$ by

$$\phi_o(x, p_1, \cdots, p_n) := \{y \in A(x) \mid f(x, y, p_1, \cdots, p_n) \geq \sup_{y' \in A(x)} f(x, y', p_1, \cdots, p_n)\},$$

for each $x \in X, (p_1, \cdots, p_n) \in \Pi_{i\in I} E_i$.

Since A is continuous and has non-empty compact values and $\langle\cdot,\cdot\rangle$ is a continuous bilinear functional, by Theorem 3 in Aubin [1, p. 70], ϕ_o is upper semicontinuous. Also, since $f(x, \cdot, p_1, \cdots, p_n)$ is quasi-concave and each $A(x)$ is non-empty closed convex, each $\phi_o(x, p_1, \cdots, p_n)$ is a non-empty closed convex subset of $A(x)$ in X.

Since X_i has non-empty interior and compact convex, we can find a non-empty compact convex absorbing neighborhood Y_i of $\mathbb{O}$ in E_i. Then, for each $i \in I$, we define the correspondence $\phi_i : X \times \Pi_{i\in I} Y_i \to 2^{Y_i}$ by

$$\phi_i(x, p_1, \cdots, p_n) := \begin{cases} \{p' \in Y_i \mid p' \neq \mathbb{O},\ \mathrm{Re}\langle p', x'\rangle \geq \mathrm{Re}\langle p', x_i\rangle, \\ \quad \text{for all } x' \in P_i(x)\}, & \text{if } P_i(x) \neq \emptyset, \\ \quad Y_i, & \text{if } P_i(x) = \emptyset. \end{cases}$$

Note that each ϕ_i is independent of the variables $(p_1, \cdots, p_n)$. Since P_i is irreflexive and $P_i(x)$ is open convex, if $P_i(x) \neq \emptyset$, then by the separation theorem for convex sets, there exists a non-zero functional $g \in E_i^*$ which separates the sets $P_i(x)$ and $\{x_i\}$. Therefore we can find $p' \in Y_i$ with $p' \neq \mathbb{O}$ such that $Re\langle p', x'\rangle \geq Re\langle p', x_i\rangle$ for all $x' \in P_i(x)$. In fact, by Theorem 8 of Dunford-Schwartz [3, p. 417] and the duality fact as we mentioned, there exists $\mathbb{O} \neq \hat{p} \in E_i$ such that $Re\langle \hat{p}, x'\rangle \geq Re\langle \hat{p}, x_i\rangle$ for all $x' \in P_i(x)$. Since Y_i is an absorbing neighborhood of $\mathbb{O}$, there exists $k > 0$ such that

$p' := k{\cdot}\hat{p} \in Y_i$, and p' clearly satisfies the inequality. Thus ϕ_i has non-empty value, and it is easy to show that ϕ_i has closed convex value.

Next we shall show that ϕ_i is upper semicontinuous. If $P_i(x) = \emptyset$, then ϕ_i is trivially upper semicontinuous at $(x, p_1, \cdots, p_n)$. If $P_i(x) \neq \emptyset$, since X and X_i are compact sets, it suffices to show that the graph of ϕ_i is closed. Since P_i has open graph and $\langle \cdot, \cdot \rangle$ is a continuous dual pairing on $E_i \times E_i$, it is easy to show that ϕ_i has closed graph, and hence ϕ_i is upper semicontinuous.

Finally we define $\Phi : X \times \Pi_{i \in I} Y_i \to 2^{X \times \Pi_{i \in I} Y_i}$ by

$$\Phi(x, p_1, \cdots, p_n) := \phi_o(x, p_1, \cdots, p_n) \times \Pi_{i \in I}\, \phi_i(x, p_1, \cdots, p_n),$$
$$\text{for each } (x, p_1, \cdots, p_n) \in X \times \Pi_{i \in I} Y_i.$$

Then, Φ is upper semicontinuous and has non-empty closed convex value, and $X \times \Pi_{i \in I} Y_i$ is compact convex. Therefore, by the Fan-Glicksberg fixed point theorem, there exists a point $(\bar{x}, \bar{p}_1, \cdots, \bar{p}_n) \in X \times \Pi_{i \in I} Y_i$ such that $(\bar{x}, \bar{p}_1, \cdots, \bar{p}_n) \in \Phi(\bar{x}, \bar{p}_1, \cdots, \bar{p}_n)$. Therefore $\bar{x} \in \phi_o(\bar{x}, \bar{p}_1, \cdots, \bar{p}_n)$ and hence $\bar{x}_i \in A_i(\bar{x})$ for each $i \in I$, and

$$(*) \qquad f(\bar{x}, \bar{x}, \bar{p}_1, \cdots, \bar{p}_n) \geq f(\bar{x}, y, \bar{p}_1, \cdots, \bar{p}_n) \quad \text{for all} \quad y \in A(\bar{x}).$$

It remains to show the equilibrium condition (ii) for $\bar{x}$. Suppose the contrary. Then there exists $\bar{y} \in X$ and a non-empty subset $I_o \subset I$ such that $e_i(\bar{x}, \bar{y}) \in P_i(\bar{x}) \cap A_i(\bar{x})$ for all $i \in I_o$ and $e_i(\bar{x}, \bar{y}) = \bar{x}_i$ for each $i \in I \setminus I_o$. Then, by the assumption (5), $\bar{y}_i \in A_i(\bar{x})$ and $e_i(\bar{x}, \bar{y}) \in P_i(\bar{x})$ for each $i \in I_o$. Since $e_i(\bar{x}, \bar{y}) = \bar{x}_i \in A_i(\bar{x})$ for each $i \in I \setminus I_o$ and by the assumption (5) again, $\bar{y}_i \in A_i(\bar{x})$ for each $i \in I_o$; and hence $\bar{y} \in A(\bar{x})$. Since $P_i(\bar{x}) \neq \emptyset$ and $\bar{p}_i \neq \mathbb{O}$ for all $i \in I_o$, $\bar{p}_i \in \phi_i(\bar{x}, \bar{p}_1, \cdots, \bar{p}_n)$ implies that $Re\ \langle \bar{p}_i, e_i(\bar{x}, \bar{y}) \rangle \geq Re\langle \bar{p}_i, \bar{x}_i \rangle$ for all $i \in I_o$. Furthermore, we know that since $P_i(\bar{x})$ is non-empty open convex and $\bar{x}_i \notin P_i(\bar{x})$, $\bar{p}_i(\neq \mathbb{O})$ separates $P_i(\bar{x})$ and $\{\bar{x}_i\}$ in the Euclidean space $\mathbb{R}^n$, i.e., for each $i \in I_o$, $Re\langle \bar{p}_i, x' \rangle > Re\langle \bar{p}_i, \bar{x}_i \rangle$ for all $x' \in P_i(\bar{x})$ (e.g. see [11, p. 64]). Since $e_i(\bar{x}, \bar{y}) \in P_i(\bar{x})$ for all $i \in I_o$, we have Re $\langle \bar{p}_i, e_i(\bar{x}, \bar{y}) \rangle > Re\langle \bar{p}_i, \bar{x}_i \rangle$ for all $i \in I_o$. Therefore we have

$$\begin{aligned} f(\bar{x}, \bar{y}, \bar{p}_1, \cdots, \bar{p}_n) &= \sum_{i \in I} Re\langle \bar{p}_i, e_i(\bar{x}, \bar{y}) \rangle \\ &= \sum_{i \in I_o} Re\langle \bar{p}_i, e_i(\bar{x}, \bar{y}) \rangle + \sum_{i \in I \setminus I_o} Re\langle \bar{p}_i, \bar{x}_i \rangle \\ &> \sum_{i \in I} Re\langle \bar{p}_i, \bar{x}_i \rangle \\ &= f(\bar{x}, \bar{x}, \bar{p}_1, \cdots, \bar{p}_n), \end{aligned}$$

which contradicts $(*)$. This completes the proof. □

REMARKS. (1) In Theorem 1, we use general assumptions in the following aspects:

(i) the choice set X_i lies in the locally convex space;

(ii) the expectation map e_i satisfies the quasi*-concavity;

and hence Theorem 1 is a generalization of the previous equilibrium existence theorem of [9] to general locally convex spaces.

(2) Keiding [7] assumed that the image of the state correspondence A should be contained in the interior of the choice set, and Vind [12] assumed that the preference correspondence should be extendable to some open set containing X. However, in our theorem those restrictions do not need any more.

(3) It should be noted that in Theorem 2 in Keiding [7], the image of the state correspondence A must be convex contained in the interior of the state set X, and the expectation map e_i is a continuous affine map; and hence, modifying our proof, we can simplify his proof by using the Fan-Glicksberg fixed point theorem rather than the Eilenberg-Montgomery fixed point theorem.

For each $i \in I$, when $e_i(x, y) = y_i$ for each $x, y \in X$ in Theorem 1, then we obtain the following generalization of the Nash equilibrium existence theorem [10] for an abstract economy:

COROLLARY 1. *Let $\Gamma = (X_i, A_i, P_i)_{i \in I}$ be an abstract economy where I is a finite set of agents such that for each $i \in I$,*

(1) X_i is a compact convex subset of a locally convex Hausdorff topological vector space E_i, $X := \Pi_{i \in I} X_i$, and $\langle \cdot, \cdot \rangle$ is a continuous dual pairing on $E_i \times E_i$,

(2) the constraint correspondence $A_i : X \to 2^{X_i}$ is continuous such that $A_i(x)$ is non-empty closed convex for each $x = (x_i)_{i \in I} \in X$,

(3) the preference correspondence $P_i : X \to 2^{X_i}$ has open graph such that $P_i(x)$ is (possibly empty) convex and $x_i \notin P_i(x)$ for each $x \in X$.

Then there exists an equilibrium $\hat{x} \in X$ for Γ, i.e. for each $i \in I$,

$$\hat{x}_i \in A_i(\hat{x}) \quad \text{and} \quad P_i(\hat{x}) \cap A_i(\hat{x}) = \emptyset.$$

For each $i \in I$, when $A_i(x) = X_i$ and $e_i(x, y) = y_i$ for each $x, y \in X$ in Theorem 1, then we obtain the following maximal element theorem :

COROLLARY 2. *Let I be a finite set. For each $i \in I$, let X_i be a non-empty compact convex subset of a locally convex Hausdorff topological vector space E_i, and $X := \Pi_{i \in I} X_i$,. If $P_i : X \to 2^{X_i}$ has open graph such that $P_i(x)$ is (possibly empty) convex and $x_i \notin P_i(x)$ for each $x \in X$, then there exists an $\hat{x} \in X$ such that for each $i \in I$, $P_i(\hat{x}) = \emptyset$.*

For each $i \in I$, when $P_i(x) = \emptyset$ for each $x \in X$ in Theorem 1, then we obtain the following fixed point theorem as an easy consequence :

COROLLARY 3. *Let I be a finite set. For each $i \in I$, let X_i be a non-empty compact convex subset of a locally convex Hausdorff topological vector space E_i , $X := \Pi_{i \in I} X_i$, and $A_i : X \to 2^{X_i}$ be continuous such that $A_i(x)$ is non-empty closed convex for each $i \in I$. Then there exists an $\hat{x} \in X$ such that for each $i \in I$, $\hat{x}_i \in A_i(\hat{x})$.*

Finally we shall give a simple example which the previous existence results on SSC as in [7, 12] do not work, but our theorem can be applied even in the Euclidean space $\mathbb{R}^2$.

EXAMPLE. Let $I = \{1, 2\}$ be the two agents set, $X_i = [0, 1]$ be the compact convex choice set and $X = [0, 1] \times [0, 1]$ be the state set.

For each $i = 1, 2$, let the correspondences $A_i, P_i : X \to 2^{X_i}$ be defined as follows :

$$A_1(x_1, x_2) := [0, x_1], \qquad \text{for each} \quad x = (x_1, x_2) \in X,$$

$$A_2(x_1, x_2) := [x_2, 1], \qquad \text{for each} \quad x = (x_1, x_2) \in X,$$

$$P_1(x_1, x_2) := \begin{cases} (x_1, 1], & \text{for each } x_1 \in (0, 1), \\ \emptyset, & \text{for each } x_1 \in \{0, 1\}, \end{cases}$$

$$P_2(x_1, x_2) := \begin{cases} [0, x_2), & \text{for each } x_2 \in (0, 1], \\ \emptyset, & \text{for each } x_2 = 0. \end{cases}$$

And the expectation map $e_i((x_1, x_2), (y_1, y_2)) : X \to \mathbb{R}$ is given by
$e_i((x_1, x_2), (y_1, y_2)) := (1 + x_i)y_i - x_i^2 \quad$ for all $(x_1, x_2), (y_1, y_2) \in X$.

Then all hypotheses of Theorem 1 are satisfied; in fact, for each $i = 1, 2$, we can show

(4) $e_i(x, y)$ is an affine map on the variable y and hence is quasi*-concave; and $e_i(x, x) = x_i$ for all $x \in X$,

(5) if $e_i(x,y) = (1+x_i)y_i - x_i^2 \in A_i(x)$, then $y_i \in A_i(x)$.

Therefore by Theorem 1, we can obtain an equilibrium point $(0,0) \in X$ such that $0 \in A_i(0,0)$ for each $i \in I$, and there is no state y such that the set $I_o = \{i \in I \mid e_i(\hat{x},y) \neq 0\}$ is non-empty and $e_i(\hat{x},y) \in P_i(\hat{x}) \cap A_i(\hat{x})$ for all $i \in I_o$.

However, Theorem 2 in Keiding [7] or Theorem in Vind [12] can not be directly applied to this setting since the image $A_i(x_1,x_2)$ is not contained in the interior of X_i for each $i \in I$.

Finally, it should be noted that it is possible to generalize the equilibrium existence theorem in more general settings, e.g. SSC with infinite set of agents or generalized SSC, and for the real economic examples of SSC, see Vind [12].

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