Hewitt Realcompactification and Basically Disconnected Cover*

Abstract

We show that if the Stone–Čech compactification of $\Lambda X$ and the minimal basically disconnected cover of $\beta X$ are homeomorphic and every real $\sigma Z(X)^*$-ultrafilter on $X$ has the countable intersection property, then there is a covering map from $\nu(\Lambda X)$ to $\nu X$ and every real $\sigma Z(X)^*$-ultrafilter on $X$ has the countable intersection property if and only if there is a homeomorphism from the Hewitt realcompactification of $\Lambda X$ to the minimal basically disconnected space of $\nu X$.

0. Introduction

All spaces in this paper are assume to be Tychonoff and for a space $X$, let $(\beta X, \beta_X)((\nu X, \nu_X))$, resp.) denotes the Stone–Čech compactification (Hewitt realcompactification, resp.) of $X$. For any regular space $X$, there is the absolute $(EX, k_X)$ of $X$ and if $X$ is Tychonoff, then there is a homeomorphism $k : \beta(EX) \to E(\beta X)$. Moreover, for any space $X$, the following are equivalent:

(i) there is a homeomorphism $\nu(\beta X) \to E(\beta X)$,

(ii) if $\{A_n : n \in N\}$ is a decreasing sequence in $R(X)$ and $\cap \{A_n : n \in N\} = \phi$, then $\cap\{\cl{\omega X}(A_n) : n \in N\} = \phi$,

(iii) if $\{A_n : n \in N\}$ is a decreasing sequence in $R(X)$, then $\cl{\omega X}(\cap \{A_n : n \in N\}) = \cap\{\cl{\omega X}(A_n) : n \in N\}$, and

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(iv) every stable $R(X)$-ultrafilter has the countable intersection property [4].

For any Tychonoff space $X$, there is a minimal basically disconnected cover $(\Lambda X, A_X)$ [5] and if $X$ is locally weakly Lindelöf, then $\Lambda X$ are given by a filter space [2] and [4].

In this paper, we show that if the Stone–Čech compactification of $\Lambda X$ and the minimal basically disconnected cover of $\beta X$ are homeomorphic, then $\Lambda X$ is a filter space and that if every real $\sigma Z(X)^u$-ultrafilter on $X$ has the countable intersection property, then there is a covering map from $\nu(\Lambda X)$ to $\nu X$. Using this, we will show that if the Stone–Čech compactification of $\Lambda X$ and the minimal basically disconnected cover of $\beta X$ are homeomorphic, then every real $\sigma Z(X)^u$-ultrafilter on $X$ has the countable intersection property and that if there is a homeomorphism from the Hewitt realcompactification of $\Lambda X$ to the minimal basically disconnected space of $\nu X$. For the terminology, we refer to [1] and [4].

1. Fixed $\sigma Z(X)^u$-ultrafilter space

Recall that a subspace $Y$ of a space $X$ is said to be $C^*$-embedded in $X$ if for any bounded real-valued continuous map $f : Y \to R$, there is a bounded real-valued continuous map $g : X \to R$ with $g|_Y = f$ and that a space $X$ is called basically disconnected if every cozero-set in $X$ is $C^*$-embedded in $X$.

Definition 1.1. Let $X$ be a space. Then a pair $(Y, f)$ is called

1. a cover of $X$ if $f : Y \to X$ is a covering map,

2. a basically disconnected cover of $X$ if $(Y, f)$ is a cover of $X$ and $Y$ is a basically disconnected space and,

3. a minimal basically disconnected cover of $X$ if $(Y, f)$ is a cover of $X$ and it is a basically disconnected cover of $X$ and for any basically disconnected cover $(Z, g)$ of $X$, there is a covering map $h : Z \to Y$ with $f \circ h = g$.

For any space $X$, the collection $R(X)$ of all regular closed sets in $X$, when partially ordered by inclusion, becomes a complete Boolean algebra, in which the join, meet, and complementation operations are defined as follows:
If \( A \subseteq R(X) \) and \( \{ A_i : i \in I \} \subseteq R(X) \), then
\[
\bigvee \{ A_i : i \in I \} = \text{cl}_X \left( \bigcup \{ A_i : i \in I \} \right),
\]
\[
\bigwedge \{ A_i : i \in I \} = \text{cl}_X \left( \text{int}_X (\bigcap \{ A_i : i \in I \}) \right),
\]
and
\[
\Lambda = \text{cl}_X (X - A)
\]
and a sublattice of \( R(X) \) is a subset of \( R(X) \) that contains \( \phi \), \( X \) and is closed under finite joins and meets [4].

A lattice \( L \) is called \( \sigma \)-complete if every countable subset of \( L \) has join and meet. For a subset \( M \) of a complete Boolean algebra \( L \), \( \sigma M \) denotes the smallest \( \sigma \)-complete Boolean subalgebra of \( L \) containing \( M \). For any space \( X \), \( Z(X) \) denotes the set of all zero-sets and let \( Z(X)^* = \{ \text{cl}_X (\text{int}_X (A)) : A \subseteq Z(X) \} \). For a space \( X \) and a zero-set \( Z \) in \( X \), there is a zero-set \( A \) in \( \beta X \) with \( A \cap X = Z \). It is well-known that for any covering map \( f : Y \to X \), the map \( \phi : R(Y) \to R(X) \), defined by \( \phi(A) = f(A) \), is a Boolean isomorphism and that for any extension \( Y \) of a space \( X \), the map \( \phi : R(Y) \to R(X) \), defined by \( \phi(A) = A \cap X \), is a Boolean isomorphism. Hence, for any space \( X \), the isomorphism \( \phi : R(\beta X) \to R(X) \) induces Boolean isomorphisms \( \sigma Z(\beta X)^* \to \sigma Z(X)^* \) and \( \sigma Z(\nu X)^* \to \sigma Z(X)^* \).

For any space \( X \), \( (\Lambda X, \Lambda_X) \) \( ((\Lambda(\beta X), \Lambda_\beta), \text{resp.}) \) denotes the minimal basically disconnected cover of \( X(\beta X, \text{resp.}) \). Vermeer showed that for a compact space \( X \), \( \Lambda X \) is given by the Stone-space \( S(\sigma Z(X)^*) \) of \( \sigma Z(X)^* \) and \( \Lambda_X(a) = \cap a \) [5].

Recall that a space \( X \) is called weakly Lindelöf if every open cover of \( X \) has a countable subfamily that is dense in \( X \) and that a space \( X \) is called locally weakly Lindelöf if every element of \( X \) has a weakly Lindelöf neighborhood. In [2] and [4], it is shown that for any locally weakly Lindelöf space \( X \), \( \Lambda X \) is given by the filter space \( \{ a : a \) is a fixed \( \sigma Z(X)^* \)-ultrafilter \( \} \) and \( \Lambda_X(a) = \cap a \).

For a space \( X \), there is the Stone extension \( \Lambda^\beta : \beta(\Lambda X) \to \beta X \) of \( \beta_X \). Since \( \beta(\Lambda X) \) and \( \beta X \) are compact, \( \Lambda^\beta \) is a covering map and since \( \beta(\Lambda X) \) is basically disconnected [5], there is a covering map \( h_X : \beta(\Lambda X) \to \Lambda(\beta X) \).

Since \( \Lambda^\beta = \Lambda_\beta \cdot h_X \). If \( h_X \) is a homeomorphism, then we write \( \beta(\Lambda X) = \Lambda(\beta X) \) and in case, we will identify \( (\beta(\Lambda X), \Lambda^\beta) \) and \( (\Lambda(\beta X), \Lambda_\beta) \). In [2], it is shown that if \( X \) is a weakly Lindelöf space, then \( \beta(\Lambda X) = \Lambda(\beta X) \).
Proposition 1.2. Suppose that $X$ is a space and $\beta(\Lambda X) = \Lambda(\beta X)$. Then $\Lambda X$ is given by the filter space \( \{ a : a \text{ is a fixed } \sigma\mathcal{Z}(X)^* - \text{ulfilter} \} \).

Proof. Since the diagram
\[
\begin{array}{ccc}
\Lambda_{\beta}^{-1}(X) & \xrightarrow{\Lambda_{\beta_x}} & X \\
\downarrow j & & \downarrow \beta_X \\
\beta(\Lambda X) & \xrightarrow{\Lambda_{\beta}} & \beta X
\end{array}
\]
is a pullback in the category Top, there is a continuous map $h_X : \Lambda X \to \Lambda_{\beta}^{-1}(X)$ such that $\Lambda_{\beta X} \circ h_X = \Lambda_X$ and $j \circ h_X = h_X \circ \beta_{AX}$. where $j$ is the inclusion map and $\Lambda_{\beta X}$ is the restriction and corestriction of $\Lambda_{\beta}$ with respect to $\Lambda_{\beta}^{-1}(X)$ and $X$, respectively. Take any $x \in \Lambda_{\beta}^{-1}(X)$. Then there is $y \in \beta(\Lambda X)$ with $h_X(y) = x$ and $\Lambda_{\beta}(x) = \Lambda_{\beta X}(x) \in X$. Since $\Lambda_X$ is a covering maps, $y \in \Lambda X$. Hence $h_X$ is onto. Since $\Lambda_{\beta X} \circ h_X = \Lambda_X$ and $\Lambda_X$ is perfect, $h_X$ is a perfect map [4]. Since $h_X$ is $1 - 1$, $h_X$ is a homeomorphism. Hence $(\Lambda_{\beta}^{-1}(X), \Lambda_{\beta X})$ is the minimal basically disconnected cover of $X$. Thus $\Lambda_{\beta}^{-1}(X)$ is the fixed $\sigma\mathcal{Z}(X)^* - \text{ulfilter}$ \( \{ a : a \text{ is } a \text{ fixed } \sigma\mathcal{Z}(X)^* - \text{ulfilter} \} \).

Proposition 1.3. Let $X$ be a space. Suppose that $\Lambda X$ is given by the fixed $\sigma\mathcal{Z}(X)^*$ -ulfilter space. Then for any decreasing sequence $(A_n)_n$ in $\sigma\mathcal{Z}(X)^*$,
\[\Lambda_X(\cap \{ A_n : n \in \mathbb{N} \}) = \cap(\Lambda_n : n \in \mathbb{N}), \text{ where } A_n^* = \{ a : a \text{ is a fixed } \sigma\mathcal{Z}(X)^* - \text{ulfilter and } A_n \subseteq a \} \).

Proof. Take any $A \in \sigma\mathcal{Z}(X)^*$ and $a \in A^*_n$. Then $\Lambda_X(A^*) \subseteq A$.

Take any $x \in A$. Let $A_x = \{ B \in \sigma\mathcal{Z}(X)^* : x \in \text{int}_X(B) \}$. Then $A_x \cup \{ A \}$ has the finite meet property and hence there is a $\sigma\mathcal{Z}(X)^*$-ulfilter $a$ containing $A_x \cup \{ A \}$. 

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Since \( a_\ast \) is a local base at \( x \) in \( X \), \( \Lambda_X(a) = \bigcap a = x \) and so \( A \subseteq \Lambda_X(A) \). Thus \( \Lambda_X(\bigcap \{ A_n^* : n \in N \}) \subseteq \bigcap \{ A_n : n \in N \} \). Take any \( y \in \bigcap \{ A_n : n \in N \} \), then \( a_y \cup \{ A_n : n \in N \} \) has the finite meet property and hence it is contained in a \( \alpha Z(X)^* \)-ultrafilter \( \eta \) and so \( \eta \in \bigcap \{ A_n^* : n \in N \} \) and \( \Lambda_X(\eta) = y \).

2. Hewitt realcompactification and minimal basically disconnected cover

In the following, we may assume that every space has the property \( \Lambda(\beta X) = \beta(\Lambda X) \).

For any space \( X \), let \( \nu : \Lambda X \to \nu(\Lambda X) \) be the Hewitt realcompactification of \( \Lambda X \) and \((\Lambda(\nu X), \Lambda_\rho)\) the minimal basically disconnected cover of \( \nu X \). Since \( \nu X \) is realcompact, there is a continuous map \( r_X : \nu(\Lambda X) \to \nu X \) such that \( \nu_X \cdot A = r_X \cdot \nu \Lambda \) \([4] \). If there is a homeomorphism \( k : \nu(\Lambda X) \to \Lambda(\nu X) \) such that \( \Lambda_\rho \cdot k = r_X \), then we write \( \Lambda(\nu X) = \nu(\Lambda X) \) and in case, we will identify \((\nu(\Lambda X), r_X)\) and \((\Lambda(\nu X), \Lambda_\rho)\). Recall that a covering map \( f : Y \to X \) is called \( \alpha Z^* - \text{irreducible} \) if \( \{ f(A) : A \in \alpha Z(Y)^* \} = \alpha Z(X)^* \) and that a subspace \( D \) of a space \( X \) is \( \alpha Z^* - \text{embedded} \) if for any \( B \in \alpha Z(D)^* \), there is \( S \subseteq \alpha Z(X)^* \) such that \( S \cap D = B \). For any compact space \( X \), \( \Lambda X \) is \( \alpha Z^* - \text{irreducible} \) \([3]\) and every dense \( C^* - \text{embedded} \) subspace of a space is \( \alpha Z^* - \text{embedded} \).

We will give some characterizations of a space \( X \) for which \( \Lambda(\nu X) = \nu(\Lambda X) \).

**Definition 2.1.** Let \( X \) be a space. A \( \alpha Z(X)^* \)-ultrafilter \( \alpha \) is called **real** if \( \bigcap \{ \text{cl}_{\beta X}(A) : A \in \alpha \} \subseteq \nu X \).

**Theorem 2.2.** Let \( X \) be a space. Then we have the following:

(a) Suppose that every \( \alpha Z(X)^* \)-ultrafilter has the countable intersection property and \( \Lambda(\beta X) = \beta(\Lambda X) \). Then \( r_X \) is a covering map.

(b) The following are equivalent:

1. \( \Lambda(\nu X) = \nu(\Lambda X) \),
2. if \( \{ A_n : n \in N \} \) is a decreasing sequence in \( \alpha Z(X)^* \) with \( \bigcap \{ A_n : n \in N \} = \phi \),
then \( \cap \{ \text{cl}_{X}(A_n) : n \in N \} = \phi \),

(3) if \( \{ A_n : n \in N \} \) is a decreasing sequence in \( \sigma Z(X)^* \), then
\[
\text{cl}_{X}(\cap \{ A_n : n \in N \}) = \cap \{ \text{cl}_{X}(A_n) : n \in N \},
\]
and

(4) every real \( \sigma Z(X)^* \)-ultrafilter has the countable intersection property.

Proof. (a) Let \( j_1 : v(AX) \to \beta(AX) \) and \( j_2 : vX \to \beta X \) be inclusion maps.

The following diagram commutes.

\[
\begin{array}{ccc}
v(AX) & \xrightarrow{r_X} & vX \\
\downarrow{j_1} & & \downarrow{j_2} \\
\beta(AX) & \xrightarrow{\Lambda_{X} \ast h_{X}} & \beta X
\end{array}
\]

Since \( j_2 \circ r_X \circ v_A = j_2 \circ v_X \circ \Lambda_X = \Lambda_X \ast h_{X} \ast j_1 \circ v_A \) and \( v_A \) is dense, \( j_2 \circ m_X = \Lambda_X \ast h_X \ast j_1 \). Let \( \rho \in vX \) and \( a \in \Lambda^{-1}_{X}(\rho) \). Suppose that \( a \notin v(AX) \).

Then there is a sequence \( \{ Z_n : n \in N \} \in \sigma Z(\beta(AX))^* \) such that for any \( n \in N \), \( a \in \text{int}_{\beta(AX)}(Z_n) \) and \( (\cap \{ Z_n : n \in N \}) \cap \Lambda X = \phi \) [4].

Since \( \Lambda_X \) is \( \sigma Z^* \)-irreducible, \( \Lambda_X(Z_n) \in \sigma Z(\beta X)^* \). Hence \( \alpha_X = \{ U \cap X : U \in \alpha \} \) is a \( \sigma Z(X)^* \)-ultrafilter.

Let \( n \in N \). Since \( a \in \text{int}_{\beta(AX)}(Z_n) \) and \( \{ A^* : A \in \sigma Z(\beta X)^* \} \) is a base for \( \beta(AX) \), there is \( A \in \sigma Z(\beta X)^* \) with \( a \in A^* \subseteq Z_n \) and hence \( \Lambda_X(a) \in \Lambda_X(A^*) = \Lambda_X \Lambda_X(Z_n) \). So \( \Lambda_X(Z_n) \subseteq a \).

Hence for any \( n \in N \), \( \Lambda_X(Z_n) \cap X = \alpha_X \).

Since \( \rho \in vX \), \( \alpha_X \) is real and so \( \cap \{ (\Lambda_X(Z_n) \cap X : n \in N) \neq \phi \} \).

Pick \( x \in \cap \{ (\Lambda_X(Z_n) \cap X : n \in N) \} \).

Let \( n \in N \). Then \( \Lambda^{-1}_X(x) \cap Z_n \neq \phi \).

Since \( \Lambda^{-1}_X(x) = \Lambda^{-1}_X(x) \cap Z_n \) is a compact family of closed sets in \( \Lambda^{-1}_X(x) \) with the finite intersection property, \( \cap \{ \Lambda^{-1}_X(x) \cap Z_n : n \in N \} \neq \phi \) and hence \( (\cap \{ Z_n : n \in N \}) \cap \Lambda X \neq \phi \).

This is a contradiction.

Hence \( a \in v(AX) \). Thus \( r_X \) is onto. Since \( j_1 \) and \( j_2 \) are dense and \( \beta(AX) \) and \( \beta X \) are compact, \( r_X \) is a covering map [4].
(b) (1)⇒(2) Suppose that there is a sequence \( \{ A_n : n \in \mathbb{N} \} \) in \( \omega \mathcal{Z}(X)^* \) such that \[ \bigcap \{ \text{cl}_{\omega X}(A_n) : n \in \mathbb{N} \} \neq \emptyset. \] Since \( \beta(v(\Lambda X)) = \beta(\Lambda X) = \Lambda(\beta X) = \Lambda(\beta(vX)) \), \( \Lambda(\nu X) \) is given by the filter space \( \{ a : a \ is \ fixed \ \omega \mathcal{Z}(\nu X)^* \ - \ \text{ultrafilter} \} \) and \( \text{cl}_{\omega X}(A_n) \in \omega \mathcal{Z}(\nu X)^* \) for all \( n \in \mathbb{N} \), by Proposition 1.3, \( \Lambda \bigcap \{ \text{cl}_{\omega X}(A_n)^* : n \in \mathbb{N} \} \) = \( \bigcap \{ \text{cl}_{\omega X}(A_n) : n \in \mathbb{N} \} \neq \emptyset. \) Note that for any \( n \in \mathbb{N}, \) \( \text{cl}_{\omega X}(A_n)^* = \text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(\text{cl}_{\omega X}(A_n)))) \). Let \( t \in \bigcap \{ \text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(\text{cl}_{\omega X}(A_n)))) : n \in \mathbb{N} \}. \) Then there is the \( \omega \mathcal{Z}(X)^* \)-ultrafilter \( a \) such that \( t \in \bigcap \{ \text{cl}_{\Lambda(\omega X)}(A) \ : \ A \in a \} \) [4]. Since \( t \in \nu(\Lambda X) \), the \( \omega \mathcal{Z}(X)^* \)-ultrafilter \( a \) has the countable intersection property. Let \( n \in \mathbb{N}. \) Then there is \( B_n \in \omega \mathcal{Z}(\nu X)^* \) such that \( B_n \cap X = A_n. \) Since \( \Lambda X(\text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(B_n))) \cap \Lambda X) = \Lambda X(\text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(B_n)))) = \text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(\text{cl}_{\omega X}(A_n)))) \cap \Lambda X). \) Thus \( t \in \text{cl}_{\Lambda(\omega X)}(\text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(\text{cl}_{\omega X}(A_n)))))) \). Since \( \text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(B_n))) \subseteq \omega \mathcal{Z}(\nu X)^* \) and \( \Lambda(\nu X) \) is basically disconnected, \( \text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(B_n))) \subseteq \text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(\text{cl}_{\omega X}(A_n)))) \). Hence \( \text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(\text{cl}_{\omega X}(A_n)))) \subseteq a \) and so \( \bigcap \{ \text{cl}_{\Lambda(\omega X)}(A_n^{-1}(\text{int}_{\omega X}(\text{cl}_{\omega X}(A_n)))) : n \in \mathbb{N} \} = \bigcap \{ A_n : n \in \mathbb{N} \} \neq \emptyset. \)

(2)⇒(3) Suppose that \( p \not\in \text{cl}_{\omega X}(\bigcap \{ A_n : n \in \mathbb{N} \}). \) Then there is \( B \in \omega \mathcal{Z}(\nu X)^* \) such that \( p \in \text{int}_{\omega X}(B) \) and \( B \cap (\bigcap \{ A_n : n \in \mathbb{N} \}) = \emptyset. \) Since \( \{ C \cap A_n : n \in \mathbb{N} \} \) is a decreasing sequence in \( \omega \mathcal{Z}(X)^* \) with empty intersection, \( \bigcap \{ \text{cl}_{\omega X}(C \cap A_n) : n \in \mathbb{N} \} = \emptyset. \) Suppose that \( p \in \bigcap \{ \text{cl}_{\omega X}(A_n) : n \in \mathbb{N} \}. \) Let \( W \) be a neighborhood of \( p \) in \( \omega X \) and \( n \in \mathbb{N}. \) Then \( \text{int}_{\omega X}(W) \cap \text{int}_{\omega X}(B) \cap A_n \neq \emptyset. \) Since \( C \cap A_n = \text{cl}_{\omega X}(\text{int}_{\omega X}(C \cap A_n)) = \text{cl}_{\omega X}(\text{int}_{\omega X}(C \cap A_n)) = \text{int}_{\omega X}(C \cap A_n) = \text{int}_{\omega X}(B \cap X) \cap A_n \supseteq \text{int}_{\omega X}(B \cap A_n) \cap W = \text{int}_{\omega X}(B) \cap A_n \cap W \neq \emptyset. \) Hence \( p \in \bigcap \{ \text{cl}_{\omega X}(C \cap A_n) : n \in \mathbb{N} \} \) and so \( p \in \bigcap \{ \text{cl}_{\omega X}(A_n) : n \in \mathbb{N} \}. \)

(3)⇒(4) Let \( a \) be a real \( \omega \mathcal{Z}(\nu X)^* \)-ultrafilter and \( \{ B_n : n \in \mathbb{N} \} \subseteq a. \) For any \( n \in \mathbb{N}, \) let \( A_n = \bigwedge \{ B_i : 1 \leq i \leq n \}. \) Then \( \{ A_n : n \in \mathbb{N} \} \) is a decreasing sequence in
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\(\alpha Z(X)^*\). Since \(a\) is real, there exist a \(p \in \nu X\) such that \(p \in \bigcap \{ \text{cl}_{\omega^*X}(A_n) : n \in N \}\). By the hypothesis, \(p \in \text{cl}_{\omega^*X}(\bigcap \{ A_n : n \in N \})\). Hence \(\bigcap \{ A_n : n \in N \} \neq \emptyset\) and so \(\bigcap \{ B_i : i \in N \} \neq \emptyset\). Thus \(a\) has the countable intersection property.

(4) \(\Rightarrow\) (1) By Proposition 1.2 and (a) in this theorem, \(r_X\) is covering and so there is a covering map \(t : \nu(\Lambda X) \to \Lambda(\nu X)\) with \(r_X = \Lambda_0 \cdot t\). Suppose that \(x \neq y\) in \(\nu(\Lambda X)\). Then there are \(A, B \in \alpha Z(\nu(\Lambda X))^*\) such that \(x \in A, y \in B\) and \(A \cap B = \emptyset\). Since \(\Lambda(\nu X)\) is dense \(C^*\)-embedded in \(\Lambda(\beta X) = \beta(\Lambda X)\), \(\Lambda(\nu X)\) is \(\alpha Z(X)^*\)-embedded and so \(t \cdot \Lambda_0\) is \(\alpha Z^*\)-irreducible [3]. Hence \(t\) is \(\alpha Z^*\)-irreducible.

Since \(A \cap B = \emptyset\) and \(t\) is a covering map, \(\tau(A) \cap \tau(B) = \emptyset\). Since \(t\) is \(\alpha Z^*\)-irreducible, \(\tau(A)\), \(\tau(B) \in \alpha Z(\Lambda(\nu X))^* = B(\Lambda(\nu X))\) and so \(\tau(A) \cap \tau(B) = \emptyset\). Hence \(\tau(x) \neq \tau(y)\) and so \(t\) is 1 \(-\) 1. Thus \(t\) is a homeomorphism.

References