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Partial Diallel Cross Block Designs For GCA Effect[1)]

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Abstract

Partially balanced diallel cross designs with m-associate classes are defined and two general methods of construction are presented. Two-associate class designs based upon group divisible, triangular, and extended group divisible association schemes obtained using the general methods are also given. Tables of designs for $p \leq 24$ are also provided.

keywords : m-associate, partially balanced diallel cross, PBDCB

1. Introduction

Diallel crosses are commonly used to study the genetic properties of inbred lines in plant and animal breeding experiments. Suppose there are p inbred lines and let a cross between lines i and j be denoted by (i, j) with $i < j = 1, 2, \cdots, p$. Let n_c denote the total number of distinct crosses in the experiment. Our interest lies in comparing the lines with respect to their general combining ability (gca) parameters. The complete diallel cross (CDC) involves all possible crosses among p parental lines with $n_c = p(p-1)/2$, as discussed in detail by Griffing (1956) who referred to it as type Ⅳ mating design. Gupta and Kageyama (1994) gave a method of constructing balanced block designs for CDC using the nested balanced incomplete block (BIB) designs of Preece (1967). Subsequently, Dey and Midha (1996), Das, Dey and Dean (1998), Das and Ghosh (1999), Prasad, Gupta and Srivastava (1999), and Choi and Gupta (2000), among others, gave further methods of constructing balanced diallel cross block designs.

Complete diallel crosses involve equal numbers of occurrences of each of the $p(p-1)/2$ distinct crosses. If r_c denotes the number of times that each cross appears in a complete

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diallel, then the experiment requires $r_c = p(p-1)/2$ crosses. When p is large, sometimes it becomes impractical to carry out a balanced or even a partially balanced complete diallel cross. In such situations ,only a subset of all possible $p(p-1)/2$ crosses is used in the experiment, which is called a partial diallel cross (PDC). Das, Dean and Gupta (1998) and Mukerejee (1997) gave some PDC block designs. Ghosh and Divecha (1997) obtained partially balanced PDC and CDC block designs by forming all pairs of crosses between the treatment labels within each block of a conventional incomplete block design. The purpose of this paper is to define partially balanced partial diallel cross block (PBDCB) designs in a unified way and to give some new general methods of constructing them. The PBDCB block designs are defined in Section 2. Two general methods of construction and some classes of designs based on group divisible, triangular and extended group divisible association schemes are given in Section 3. Finally, tables of designs for $p \leq 24$ are provided in Section 4.

2. Preliminaries

Consider a block design D_b for a diallel cross experiment involving $n_c = pr/2$ distinct crosses laid out in b block of k crosses each, each cross replicated r_c times, with each line contributing to r crosses. Let r_{ij} be the number of replications of cross (i, j), $i< =1, 2, \cdots, p$, where

$$r_{ij} = \begin{cases} r_c & \text{if } \textit{the cross } (i, j) \textit{ occurs in } D_b \\ 0 & \textit{otherwise} \end{cases}$$

Then, the total number of crosses n in D_b is given by

$$n = \sum_i^p \sum_j^p r_{ij} = r_c n_c = bk$$

Following Gupta and Kageyama (1994), the model for the data is assumed to be

$$Y = \mu 1_n + \Delta_1 g + \Delta_2 \beta + \varepsilon \tag{2.1}$$

where Y is the $n \times 1$ vector of responses, μ is the overall mean, 1_t is the $t \times 1$ vector of 1's and $g = (g_1, g_2, \cdots, g_p)'$ and $\beta = (\beta_1, \beta_2, \cdots, \beta_b)'$ are the vectors of p gca effects and b block effects respectively; the rectangular matrices Δ_1, Δ_2 are the corresponding designs matrices, and ε is the $n \times 1$ vector os independent random errors with zero expectations and constant variance σ^2. The information matrix C for estimating all pairwise comparisons among the gca parameters is then given by

$$C = G - \frac{1}{k} N_b N_b' \tag{2.2}$$

where $G = (g_{ij})$ is a symmetric matrix with $g_{ii} = r$, $g_{ij} = r_{ij}$ for $i < j = 1, 2, \cdots, p$, and N_b

is the $p \times b$ line versus block incidence matrix of the design. The matrix N_b is the usual incidence matrix ; it the present context, it is obtained by ignoring the crosses, and thus by ignoring the crosses, and thus by considering $2k$ lines as the contents of a block. Note that $N_b 1_b = r 1_p$, $N_b N_b' = 2k 1_b$.

Now consider two lines in each of the n crosses as the block contents of d designs D_c with block size $k=2$, and let N_c denote the $p \times n$ incidence matrix of the block designs thus obtained. Then $G = N_c N_c'$. Thus, the information matrix C of equation can be written as

$$C = 2(C_b - C_c) \qquad (2.3)$$

where, taking lines as p treatments, C_b and C_c are the usual information matrices for designs with constant block size $2k$ and 2 respectively.

Following Das and Ghosh (1999), we now present the definition of a balanced CDC block design.

Definition 2.1. A diallel cross deigns D_b will be called a balanced CDC block design with parameters $\{p, n_c, b, r_c, k, \lambda\}$ if kC takes the form

$$kN_c N_c' - N_b N_b' = a\left(I_p - \frac{1}{p} J_p\right)$$

for some positive constant a, where I_p is the identity matrix of order p and $J_p = 1_p 1_p'$.

Now we define partially balanced diallel cross block designs. The definition requires the concept of an $m-$association scheme for which a reference may be made to Raghavarao (1971).

Definition 2.2. A PDC block design D_b will be called an $m-$association class partially balanced PDC block (PBDCB) design with parameters $\{p, n_c, b, r_c, k, \alpha_1, \alpha_2, \cdots, \alpha_m\}$ if the following holds for a given pair of lines β and γ that are ith associates,

$$k\lambda_{c(\beta,\gamma)} - \lambda_{b(\beta,\gamma)} = \alpha_i$$

where $\lambda_{b(\beta,\gamma)}$ and $\lambda_{c(\beta,\gamma)}$ are the numbers of concurrences of the lines β and γ in design D_b and D_C respectively, and α_i is a constant independent of the pair of ith associates chosen, $i = 1, 2, \cdots, m$. For CDC, D_b will be called a partially balanced CDC block (PBCDCB) design.

Note that for finding the number of within-block concurrences of two lines, the lines are taken as the contents of a block. Also, since each of the n_c distinct crosses is replicated r_c times, $\lambda_{c(\beta,\gamma)}$ equals r_c if the cross (β, γ) appears in the design and it is zero otherwise.

For a $m-$association class PBDCB design, we can write down the following spectral decomposition

$$kN_cN_c' - N_bN_b' = \sum_{i=1}^{m} \theta_i L_i$$

where the matrices L_i are idempotent, with respective nonzero eigen values θ_i, $i=1,2,\cdots,m$. These idempotent matrices depend only on the association scheme. The eigen values θ_i can be obtained using the approach of John (1980, Section 9.5), and the idempotent matrices L_i can be obtained as described by Gupta and Singh (1989). The Moore-Penrose generalized inverse of the matrix C of equation (2.2) is then given by

$$C^+ = k\sum_{i=1}^{m} \frac{1}{\theta_i} L_i$$

3. Two general methods of construction

Two widely applicable methods of constructing PBDCB designs are presented in this section. The methods are given first in Theorems 3.1 and 3.2. PBDCB designs obtained using the two methods based on specific association schemes are then presented in Theorems 3.3 - 3.7.

Let D_1 be an $m-$association class PBIB design with parameters $v=p$, $b=b_1$, $r=r_1$, $k=2, \lambda_1, \lambda_2, \cdots, \lambda_m$ such that $\lambda_i=0$ for $i(\neq s)=1,2,\cdots,m$, and $\lambda_s=1$, where $s\in\{1,2,\cdots,m\}$. For any association scheme, these designs can be obtained by taking all possible distinct pairs of lines that are sth associates. Although D_1 is based upon an $m-$association class scheme, it has only tow distinct values of the λ parameters. In this sense D_1 is equivalent to a two-associate class PBIB designs with a suitable defined association scheme. Though D_1 need not be connected, N_cN_c' is assumed to be of full-rank so that all pairwise comparisons among gca parameters are estimable. We then have the following result.

Theorem 3.1. The existence of an $m-$association class PBIB designs D_1 with parameters $p, b_1, r_1, k=2, \lambda_s=1, \lambda_i=0, i(\neq s)=1,2,\cdots,m$, and the existence of a BIB design D_2 with parameters $v=b_1, b_2, r_2, k_2, \lambda$ implies the existence of an $m-$association class PBDCB design with parameters $\{p,\ n_c=b_1,\ b=b_2,\ r_c=r_2,\ k=k_2,\ a_s=\lambda(b_1-r_1^2),\ a_i=-r_1^2\lambda,$ $i(\neq s)=1,2,\cdots,m\}$

Theorem 3.2. The existence of an $\alpha-$resolvable $m-$associate class PBIB design D_1 with parameters $p, b_1, r_1, k=2, \lambda_i=0$ or $1, i=1,2,\cdots,m$ implies the existence of an $m-$associate class PBDCB design with parameters $\{p,\ n_c=b_1,\ b=r_1/\alpha,\ r_c=1,\ k=\alpha p/2,$

$a_i = k\lambda_i - ar_1,\ i = 1, 2, \cdots, m\}$

Example 3.1. Let D_1 be a resolvable group divisible (GD) design having parameter $p=6$, $b_1=12$, $r_1=4$, $k=2$, $\lambda_1=0$, $\lambda_2=1$, with the following replication sets:

1st replication set : (1,3), (2,5), (4,6)
2nd replication set : (1,4), (2,6), (3,5)
3rd replication set : (1,5), (2,4), (3,6)
4th replication set : (1,6), (2,3), (4,5)

Then by taking each replication set as one block, Theorem 3.2 yields a PBDCB design with parameters $p=6$, $n_c=12$, $b=4$, $r_c=1$, $k=3$, $\alpha_1=-4$, $\alpha_2=-1$.

We now present some GD, triangular, and extended group divisible (EGD) PBDCB designs.

3.1 GD designs

For GD designs, $p=mn$ lines are arranged in m groups of size n each, where m, n are positive integers. Then, a GD designs D_1 with parameters $p=mn$, $b_1=mn(n-1)/2$, $r_1=n-1$, $k=2$, $\lambda_1=1$, $\lambda_2=0$ can always be constructed. Thus, we have the following from Theorem 3.1.

Theorem 3.3. The existence of a BIB design D_2 with parameters $v=mn(n-1)/2, b_2, r_2, k_2, \lambda$, where $n \geq 2$, implies the existence of a GD PBDCB design with parameters $\{p,\ n_c=mn(n-1)/2,\ b=b_2,\ r_c=r_2,\ k=k_2,\ \alpha_1=\lambda(n-1)\{n(m-2)+2\}/2,\ \alpha_2=-(n-1)^2\lambda\}$

Example 3.2. For $m=2, n=3$, take D_1 as the GD design with parameter $p=b_1=6$, $r_1=k=2$, $\lambda_1=1$, $\lambda_2=0$ and D_2 as the BIB design with parameters $v=6$, $b_2=10, r_2=5$, $k_2=3$, $\lambda=2$. Theorem 3.3. we have the following

Corollary 3.1 There exists a GD PBDCB design with parameter $\{p=4(t+1), n_c=6(t+1)$, $b=2(t+1)(6t+5)$, $r_c=6t+5$, $\alpha_1=6(2t-1)$, $\alpha_2=-18$ }.

Similarly, using D_1 as the GD design with parameters $p=mn$, $b_1=n^2m(m-1)/2$, $r_1=n(m-1)$, $k=2, \lambda_1=0, \lambda_2=1$ in Theorem 3.1, we have the following.

Theorem 3.4. The existence of a BIB design D_2 with parameters $v=mn^2(m-1)/2$, b_2, r_2

k_2, λ implies the existence of a GD PBDCB design with parameters $\{p,\ n_c = mn^2(m-1)/2$ $b = b_2, r_c = r_2, k = k_2, a_1 = -\lambda n^2(m-1)(m-2)/2,\ a_2 = -\lambda n^2(m-1)^2\}$.

3.2 Triangular designs

Triangular designs have $p = n(n-1)/2$ lines, where n is an integer greater than 2. Then for $n \geq 3$, by taking all distinct pairs of lines that are first associates yields a triangular design D_1 with parameters $v = p = n(n-1)/2$, $b_1 = n(n-1)(n-2)/2$, $r_1 = 2(n-2)$, $k = 2$, $\lambda_1 = 1$, $\lambda_2 = 0$. Using this triangular design in Theorem 3.1, we have the following.

Theorem 3.5. The existence of a BIB designs D_2 with parameters $v = n(n-1)(n-2)/2$, b_2, r_2, k_2, λ, where $n \geq 3$, implies the existence of a triangular PBDCB design with parameter.
$\{p,\ n_c = n(n-1)(n-2)/2,\ b = b_2,\ r_c = r_2,\ k = k_2\ \ a_1 = \lambda(n-2)(n^2-9n+16)/2,\ a_2 = -4(n-2)^2\lambda\}$

Similarly for $n \geq 4$, there also exists a triangular design D_1 with parameters $v = p = n(n-1)/2$, $b_1 = n(n-1)(n-2)(n-3)/8$, $r_1 = (n-2)(n-3)/2$, $k = 2$, $\lambda_1 = 0$, $\lambda_2 = 1$ obtained by interchanging the roles of the first and the second associates. Thus, we have the following.

Theorem 3.6. The existence of a BIB design D_2 with parameters $v = n(n-1)(n-2)(n-3)/8$, b_2, r_2, k_2, λ, where $n \geq 4$, implies the existence of a triangular PBDCB designs with parameters

$$\{p, n_c = n(n-1)(n-2)(n-3)/8, b = b_2, r_c = r_2, a_1 = \lambda(n-2)(n-3)(9n-n^2-12)/8,$$
$$k = k_2, a_2 = -\lambda(n-2)^2(n-3)^2/4\}.$$

3.3 Extended group divisible (EGD) designs

Hinkelmann and kempthorne (1963) defined the EGD association scheme as a generalization of the GD association scheme. In an EGD design, $p = \Pi_{i=1}^{f} m_i$, where m_i, $i = 1, 2, \cdots, f$, and f are positive integers. Further, the lines are labeled using $f-$digit numbers $a_1 a_2 \cdots a_f$, where
$a_i = 0, 1, \cdots, m_i - 1$, $i = 1, \cdots, f$. Let $x = (x_1, x_2, \cdots, x_f)$, $x_i = 0$ or 1 $i = 1, 2, \cdots, f$. Then, two treatments in the EGD scheme are $x-$associates where $x_i = 1$ if the ith factor occurs at the same level in both the treatments and $x_i = 0$ otherwise. Let $\lambda(x)$ denote the number of times two treatments which are $x-$associates occur together in the design. Note that $\lambda(x)$ depends only on x and is independent of the specific pair of the $x-$associates chosen. The EGD scheme was earlier considered by Nair and Rao (1948) and Shah (1959), and has been referred

to as the binary number association scheme by Paik and Federer (1977). Adetailed study of the EGD scheme is due to Hinkelmann (1964). Cleary, a total of 2^f-1 distinct values of $\lambda(x)$ are possible in an EGD design. An EGD design in which only one of these values is non-zero is a first-order design. see Gupta (1987). It is easy to verify that for an EGD design, the number of $x-$associates of any treatment is given by

$$n(x) = \Pi_{i=1}^{f}(m_i-1)^{1-x_i}$$

Let D_1 be a first-order EGD design with parameters $p=\Pi_{i=1}^{f} m_i$, $b_1=p\{n(x)\}/2$, $r_1=n(x), k=2$. A first-order EGD design D_1 can be constructed for each of the distinct values of $x_0=(x_{10}, x_{20}, \cdots, x_{f0})$, $x_{i0}=0$ or 1, giving a total of 2^f-1 such first-order designs. For each of these 2^f-1 designs, we have the following.

Theorem 3.7. The existence of a BIB designs D_2 with parameters $v=pn(x_0)/2, b_2, r_2, k_2, \lambda$ implies the existence of an EGD PBDCB design with parameters $\{p=\Pi_i^f m_i, n_c=pn(x_0)/2,$ $b=b_2, r_c=r_2, k=k_2, \alpha(x_0)=\lambda n(x_0)[p-2n(x_0)]/2,$ $\alpha(x)=-\{n(x_0)\}^2\lambda$ for $x\neq x_0\}$, where $x_0=(x_{10}, x_{20}, \cdots, x_{f0}), x_{i0}=0$ or 1.

Since the designs of Theorem 3.7 have only two distinct values of the α parameters, these designs are equivalent to two-associate class PBDCB designs.

4. Table of designs

We now give GD, triangular, and EGD PBDCB designs for $p\leq 24$ obtained using Theorems 3.2 - 3.7. The designs are presented in Tables 1-4. As noted earlier, since the parameters α_i of a PBDCB design have two distinct values, the designs are equivalent to two-associate class PBDCB designs. For a two-associate class PBDCB design we have

$$var(\widehat{g}_i-\widehat{g}_j)=\begin{cases}\theta_1\sigma^2, & \text{if } \textit{lines } i \text{ and } j \textit{ are sth associates}\\ \theta_2\sigma^2, & \textit{otherwise}\end{cases}$$

where s is as in Theorem 3.1, and θ_1, θ_2 are constants. Further,

$$eff(\widehat{g}_i-\widehat{g}_j)=\begin{cases}e_1, & \text{if } \textit{lines } i \text{ and } j \textit{ are sth associates}\\ e_2, & \textit{otherwise}\end{cases}$$

where $eff(\widehat{g}_i-\widehat{g}_j)$ denotes efficiency of the designs for estimating the elementary contrast g_i-g_j relative to an appropriate randomized complete block design. The efficiencies e_1 and e_2 were computed using equation (16) of Singh and Hinkelmann (1998). Theses two efficiencies of PBDCB designs are also presented in the tables.

The parameters of the BIB designs D_2 used in constructing the PBDCB designs of Tables 1-3 are given by $v=n_c, b, r=r_c, k, \lambda$ with $\lambda=r_c(k-1)/(n_c-1)$. In Tables 1 and 2, the column labeled as $D_1(m,n)$ gives the values of m and n for GD designs D_1 used in Theorems 3.3 and 3.4. For $p\leq 24$. the EGD designs obtained using Theorem 3.7 were found to be equivalent to GD designs. Thus EGD PBDCB designs are not being listed separately as these designs are included in Table 1. GD designs D_1 used in constructing the designs of Table 4 also have $\lambda_s=1$ and $\lambda_i(i\neq s)=0$, $i=1,2$, and the values of m, n and s are given in the column labeled as $D_1(m,n,s)$

Table 1. GD PBDCB designs obtained using Theorem 3.3

p	n_c	b	r_c	k	a_1	a_2	e_1	e_2	$D_1(m,n)$
6	6	15	5	2	-2	-4	0.375	0.500	2,3
6	6	10	5	3	4	-8	0.500	0.667	2,3
6	6	6	5	5	8	-16	0.600	0.800	2,3
6	6	15	10	4	12	-24	0.563	0.750	2,3
8	12	44	11	3	6	-18	0.566	0.679	2,4
8	12	22	11	6	15	-45	0.707	0.849	2,4
9	9	12	4	3	5	-4	0.429	0.571	3,3
9	9	36	8	2	5	-4	0.321	0.429	3,3
9	9	18	8	4	15	-12	0.482	0.643	3,3
9	9	12	8	6	25	-20	0.536	0.714	3,3
9	9	9	8	8	35	-28	0.563	0.750	3,3
9	9	18	10	5	25	-20	0.514	0.686	3,3
10	20	38	19	10	36	-144	0.799	0.914	2,5
12	12	44	11	3	16	-8	0.400	0.533	4,3
12	12	33	11	3	24	-12	0.450	0.600	4,3
12	12	22	11	6	40	-20	0.500	0.667	4,3
12	18	102	17	3	18	-18	0.518	0.621	3,4
12	18	34	17	9	72	-72	0.690	0.828	3,4
12	30	58	29	15	70	-350	0.850	0.944	2,6
14	42	82	41	21	120	-720	0.881	0.961	2,7
15	15	35	7	3	11	-4	0.385	0.513	5,3
15	15	15	7	7	33	-12	0.495	0.659	5,3
15	15	15	8	8	44	-16	0.505	0.673	5,3
15	15	35	14	6	55	-20	0.481	0.641	5,3
15	30	58	29	15	196	-224	0.780	0.891	3,5
16	24	184	23	3	30	-18	0.497	0.596	4,4
16	24	46	23	12	165	-99	0.683	0.820	4,4
16	56	56	11	11	14	-98	0.850	0.915	2,8
16	56	70	15	12	21	-147	0.857	0.923	2,8
18	45	99	11	5	20	-25	0.696	0.773	3,6
18	45	55	11	9	40	-50	0.773	0.859	3,6
18	45	45	12	12	60	-75	0.797	0.885	3,6

Table 1. GD PBDCB designs obtained using Theorem 3.3<계속>

20	40	40	13	13	96	-64	0.750	0.857	4,5
20	30	290	29	3	42	-17	0.485	0.582	5,4
21	21	30	10	7	51	-12	0.474	0.632	7,3
21	21	42	12	6	51	-12	0.461	0.614	7,3
21	21	35	15	9	102	-24	0.491	0.655	7,3
24	36	420	35	3	54	-18	0.478	0.574	6,4

Table 2. GD PBDCB designs obtained using Theorem 3.4

p	n_c	b	r_c	k	α_1	α_2	e_1	e_2	$D_1(m,n)$
6	12	44	11	3	-8	-32	0.214	0.606	3,2
6	12	33	11	4	-12	-48	0.307	0.682	3,2
6	12	22	11	6	-20	-80	0.437	0.758	3,2
8	24	46	23	12	-132	-396	0.630	0.893	4,2
9	27	39	13	9	-36	-144	0.437	0.791	3,3
9	27	27	13	13	-54	-216	0.533	0.822	3,3
10	40	40	13	13	-96	-256	0.624	0.913	5,2
12	48	94	47	24	-368	-1472	0.611	0.861	3,4
12	54	106	53	27	-702	-2106	0.716	0.925	4,3

Table 3. GD PBDCB designs obtained using Theorem 3.5 and 3.6

p	n_c	b	r_c	k	α_1	α_2	e_1	e_2
Theorem 3.5								
10	30	58	29	15	-84	-504	0.846	1.000
15	60	118	59	30	145	-2900	0.883	0.993
Theorem 3.6								
10	15	35	7	3	6	-9	0.536	0.357
10	15	15	7	7	18	-27	0.689	0.459
10	15	15	8	8	24	-36	0.703	0.469
10	15	35	14	6	30	-45	0.670	0.446
15	45	99	11	5	9	-36	0.771	0.617
15	45	55	11	9	18	-72	0.857	0.685
15	45	45	12	12	27	-108	0.883	0.707

Table 4. GD PBDCB designs obtained using Theorem 3.7

p	n_c	b	r_c	k	α_1	α_2	e_1	e_2	α	$D_1(m,n,s)$
6	12	4	1	3	-4	-1	1.000	0.833	1	3,2,2
6	12	2	1	6	-8	-2	1.000	0.833	2	3,2,2
8	12	3	1	4	2	-6	0.778	0.933	1	2,4,1
8	24	6	1	4	-6	-2	1.000	0.933	1	4,2,2
9	27	3	1	9	-12	-3	1.000	0.857	2	3,3,2
10	40	8	1	5	-8	-3	1.000	0.964	1	5,2,2
10	40	4	1	10	-16	-6	1.000	0.964	2	5,2,2
12	18	3	1	6	3	-3	0.733	0.880	1	3,4,1
12	30	5	1	6	1	-5	0.880	0.978	1	2,6,1
12	48	8	1	6	-8	-2	1.000	0.880	1	3,4,2
12	54	9	1	6	-9	-3	1.000	0.943	1	4,3,2
12	60	10	1	6	-10	-4	1.000	0.978	1	6,2,2
14	84	6	1	14	-24	-10	1.000	0.985	2	7,2,2
15	75	5	1	15	-20	-5	1.000	0.897	2	3,5,2
16	56	7	1	8	1	-7	0.918	0.989	1	2,8,1
16	112	14	1	16	-28	-12	1.000	0.989	2	8,2,2
18	45	5	1	9	4	-5	0.850	0.944	1	3,6,1
18	108	6	1	18	-24	-6	1.000	0.911	2	3,6,2
18	162	8	1	18	-32	-14	1.000	0.992	2	9,2,2
18	135	15	1	9	-15	-3	1.000	0.981	1	6,3,2
20	30	3	1	10	7	-3	0.704	0.844	1	5,4,1
20	150	15	1	10	-15	-5	1.000	0.960	1	4,5,2
20	180	18	1	10	-18	-8	1.000	0.993	1	10,2,2
20	160	8	1	20	-16	-6	1.000	0.974	2	5,4,2
22	220	10	1	22	-40	-18	1.000	0.995	2	11,2,2
24	36	3	1	12	9	-3	0.697	0.836	1	6,4,1
24	60	5	1	12	7	-5	0.836	0.929	1	4,6,1
24	84	7	1	12	5	-7	0.896	0.965	1	3,8,1
24	192	8	1	24	-32	-8	1.000	0.929	2	3,8,2
24	216	18	1	12	-18	-6	1.000	0.965	1	4,6,2
24	252	21	1	12	-21	-9	1.000	0.990	1	8,3,2

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