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ON THE STABILITY OF INVOLUTIVE A-QUADRATIC MAPPINGS

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ABSTRACT. In this paper, we will investigate the Hyers-Ulam stability of an involutive A-quadratic mapping.

1. Introduction

In 1940, S. M. Ulam gave a wide ranging talk before the Mathematics Club of the University of Wisconsin in which he discussed a number of important unsolved problems ([16]). Among those was the question concerning the stability of homomorphisms:

Let G_1 be a group and let G_2 be a metric group with the metric $d(\cdot,\cdot)$. Given $\varepsilon > 0$, does there exist a $\delta > 0$ such that if a function $h : G_1 \to G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$ for all $x, y \in G_1$ then there is a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \varepsilon$ for all $x \in G_1$?

The case of approximately additive mappings was solved by D. H. Hyers [6] under the assumption that G_1 and G_2 are Banach spaces. In 1978, Th. M. Rassias [14] gave a generalization of the Hyers's result. Recently, Găvruta [5] also obtained a further generalization of the Hyers-Ulam stability.

The quadratic functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \tag{1.1}$$

clearly has $f(x) = cx^2$ as a solution with c an arbitrary constant when f is a real valued function of a real variable. We define any solution of (1.1) to be a *quadratic mapping*. The Hyers-Ulam stability for the equation (1.1) was proved by F. Skof for functions $f : V \to X$ where V is a normed space and X a Banach space ([15]). In the paper [4], S. Czerwik proved the Hyers-Ulam stability of the quadratic functional

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equation (1.1). Since then, the stability problems of functional equations have been extensively investigated by a number of mathematicians ([1, 2, 7, 8, 9, 11, 12, 13]).

Throughout this paper, let A be a unital Banach *-algebra, $A_1 = \{a \in A \mid \|a\| = 1\}$, and let X a normed left A-module.

DEFINITION 1.1. A quadratic mapping $f : X \to A$ is called *involutive A-quadratic* if $f(ax) = af(x)a^*$ for all $a \in A$ and all $x \in X$.

EXAMPLE 1.1. Consider the normed left $M_2(\mathbb{C})$-module $\mathbb{C}^2$ equipped with the module multiplication $M_2(\mathbb{C}) \times \mathbb{C}^2 \to \mathbb{C}^2$ given by $(a, x) \longmapsto ax$ as the product of matrices. For $\lambda \in \mathbb{C}$, the mapping $f : \mathbb{C}^2 \to M_2(\mathbb{C})$ given by $f(x) := \lambda x x^*$ is involutive $M_2(\mathbb{C})$-quadratic.

EXAMPLE 1.2. Consider the normed left $M_2(\mathbb{C})$-module $M_2(\mathbb{C})$ equipped with the module multiplication $M_2(\mathbb{C}) \times M_2(\mathbb{C}) \to M_2(\mathbb{C})$ given by $(a, x) \longmapsto ax$ as the product of matrices. For $c \in M_2(\mathbb{C})$, the mapping $f : M_2(\mathbb{C}) \to M_2(\mathbb{C})$ given by $f(x) := xcx^*$ is involutive $M_2(\mathbb{C})$-quadratic.

The solution of an involutive A-quadratic mapping was solved by J. Vukman [17].

In Sections 2 and 3 of this paper, we extend the Hyers-Ulam stability of an involutive A-quadratic mapping.

2. Stability of an involutive A-quadratic mapping

Let $\varphi : X \times X \to [0, \infty)$ be a function satisfying

$$\tilde{\varphi}(x, y) := \sum_{m=0}^{\infty} \frac{1}{4^{m+1}} \varphi(2^m x, 2^m y) < \infty \tag{2.1}$$

for all $x, y \in X$, or

$$\tilde{\varphi}(x, y) := \sum_{m=0}^{\infty} 4^m \varphi\left(\frac{x}{2^{m+1}}, \frac{y}{2^{m+1}}\right) < \infty \tag{2.2}$$

for all $x, y \in X$.

THEOREM 2.1. *Let $f : X \to A$ be a mapping such that*

$$\|f(ax + ay) + f(ax - ay) - 2af(x)a^* - 2af(y)a^*\| \le \varphi(x, y) \tag{2.3}$$

for all $a \in A_1$ and all $x, y \in X$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each $x \in X$, then there exists a unique involutive A-quadratic mapping $Q : X \to A$ such that

$$\|f(x) - f(0) - Q(x)\| \le \tilde{\varphi}(x, x) + \tilde{\varphi}(0, 0) \tag{2.4}$$

for all $x \in X$. *The mapping* $Q : X \to A$ *is given by*

$$Q(x) = \begin{cases} \lim_{m\to\infty} \frac{f(2^m x)}{4^m} & \text{if } \varphi \text{ satisfies (2.1)} \\ \lim_{m\to\infty} 4^m\Big(f(\frac{x}{2^m}) - f(0)\Big) & \text{if } \varphi \text{ satisfies (2.2)} \end{cases}$$

for all $x \in X$.

Proof. If φ satisfies the condition (2.2), we easily know the fact that

$$\tilde{\varphi}(0,0) = 0.$$

Suppose that φ satisfies the condition (2.1). Let x be an arbitrary fixed element of X. Putting $a = 1$ in (2.3), we obtain

$$\left\|\frac{f(0)}{2}\right\| \le \frac{1}{4}\varphi(0,0), \tag{2.5}$$

$$\left\|\frac{1}{4}\Big(f(2x) + f(0)\Big) - f(x)\right\| \le \frac{1}{4}\varphi(x,x). \tag{2.6}$$

From (2.5) and (2.6), we get

$$\begin{aligned} &\left\|f(x) - f(0) - \frac{1}{4}\Big(f(2x) - f(0)\Big)\right\| \\ \le\ &\left\|f(x) - \frac{1}{4}\Big(f(2x) + f(0)\Big)\right\| + \left\| - \frac{1}{2}f(0)\right\| \\ \le\ &\frac{1}{4}\varphi(x,x) + \frac{1}{4}\varphi(0,0). \end{aligned}$$

Induction argument implies

$$\begin{aligned} \left\|f(x) - f(0) - \frac{1}{4^m}\Big(f(2^m x) - f(0)\Big)\right\| &\le \sum_{i=0}^{m-1} \frac{\varphi(2^i x, 2^i x) + \varphi(0,0)}{4^{i+1}} \\ &\le \tilde{\varphi}(x,x) + \tilde{\varphi}(0,0). \end{aligned} \tag{2.7}$$

From (2.7), we can easily show that $\{\frac{f(2^m x) - f(0)}{4^m}\}$ is a Cauchy sequence and thus converges. From this, we can define $Q : X \to A$ such that

$$Q(x) = \lim_{m\to\infty} \frac{f(2^m x) - f(0)}{4^m} = \lim_{m\to\infty} \frac{f(2^m x)}{4^m}.$$

For $a = 1$ in (2.3), by the definition of Q, we get

$$Q(x+y) + Q(x-y) = 2Q(x) + 2Q(y). \tag{2.8}$$

From (2.3), (2.8) and the definition of Q, we have

$$Q(ax) = aQ(x)a^* + aQ(0)a^* = aQ(x)a^* \tag{2.9}$$

for all $a \in A_1$. Under the assumption that $f(tx)$ is continuous in $t \in \mathbb{R}$ for each $x \in X$, by a similar method to the proof of ([14]), we obtain that

$$Q(tx) = t^2 Q(x)$$

for all $t \in \mathbb{R}$ and all $x \in X$. By (2.9) and the above equality, we get

$$\begin{aligned} Q(ax) &= Q\left(\|a\|\frac{a}{\|a\|}x\right) = \|a\|^2 Q\left(\frac{a}{\|a\|}x\right) \\ &= \|a\|^2 \frac{a}{\|a\|} Q(x) \frac{a^*}{\|a\|} = aQ(x)a^* \end{aligned}$$

for all $a \in A \setminus \{0\}$ and all $x \in X$. And $Q(0x) = 0Q(x)0^*$ for all $x \in X$. So Q is involutive A-quadratic. Taking the limit in (2.7) as $m \to \infty$, we obtain the inequality (2.4). If Q' is another involutive A-quadratic mapping satisfying (2.4), then

$$\begin{aligned} &\|Q(x) - Q'(x)\| \\ \leq\ & \left\|\frac{Q(2^n x)}{4^n} - \frac{f(2^n x)}{4^n} + \frac{f(0)}{4^n}\right\| + \left\|\frac{f(2^n x)}{4^n} - \frac{f(0)}{4^n} - \frac{Q'(2^n x)}{4^n}\right\| \\ \leq\ & \frac{2}{4^n}\left(\tilde{\varphi}(2^n x, 2^n x) + \tilde{\varphi}(0,0)\right) \end{aligned}$$

for all $x \in X$ and all $n \in \mathbb{N}$. Taking the limit in the above inequality, we have

$$Q(x) = Q'(x)$$

for all $x \in X$. For the case that φ satisfies the condition (2.2), the proof is analogous to the case (2.1). □

COROLLARY 2.1. *Let δ be a real number, and $f : X \to A$ a mapping such that*

$$\|f(ax + ay) + f(ax - ay) - 2af(x)a^* - 2af(y)a^*\| \leq \delta \tag{2.10}$$

for all $a \in A_1$ and all $x, y \in X$. Then there exists a unique involutive A-quadratic mapping $Q : X \to A$ such that

$$\|f(x) - f(0) - Q(x)\| \leq \frac{2}{3}\delta$$

for all $x \in X$.

EXAMPLE 2.1. For $\lambda \in \mathbb{C}$ and $\varepsilon \in \mathbb{C}^2$, consider a mapping $f : \mathbb{C}^2 \to M_2(\mathbb{C})$ given by $f(x) = \lambda(x + \varepsilon)(x + \varepsilon)^*$. If f satisfies (2.10) for all $a \in M_2(\mathbb{C})_1$ and all $x, y \in \mathbb{C}^2$, then

$$\|\lambda(x + \varepsilon)(x + \varepsilon)^* - \lambda\varepsilon\varepsilon^* - \lambda x x^*\| = |\lambda|\ \|x\varepsilon^* + \varepsilon x^*\| \leq \frac{2}{3}\delta$$

for all $x \in \mathbb{C}^2$.

EXAMPLE 2.2. For $c, \varepsilon \in M_2(\mathbb{C})$, consider a mapping $f : M_2(\mathbb{C}) \to M_2(\mathbb{C})$ given by $f(x) = (x + \varepsilon)c(x + \varepsilon)^*$. If f satisfies (2.10) for all $a \in M_2(\mathbb{C})_1$ and all $x, y \in M_2(\mathbb{C})$, then

$$\|(x + \varepsilon)c(x + \varepsilon)^* - \varepsilon c\varepsilon^* - xcx^*\| = \|xc\varepsilon^* + \varepsilon cx^*\| \leq \frac{2}{3}\delta$$

for all $x \in M_2(\mathbb{C})$.

COROLLARY 2.2. *Let E be a complex normed space and $f : E \to \mathbb{C}$ a function such that*

$$\left\|f(\lambda x + \lambda y) + f(\lambda x - \lambda y) - 2\lambda f(x)\overline{\lambda} - 2\lambda f(y)\overline{\lambda}\ \right\| \leq \varphi(x, y)$$

for all $\lambda \in \mathbb{T}^1$ and all $x, y \in E$, where $\mathbb{T}^1 := \{\lambda \in \mathbb{C} \mid |\lambda| = 1\}$. If (tx) is continuous in $t \in \mathbb{R}$ for each fixed $x \in E$, then there exists a unique involutive $\mathbb{C}$-quadratic mapping $Q : E \to \mathbb{C}$ such that

$$\|f(x) - f(0) - Q(x)\| \leq \tilde{\varphi}(x, x) + \tilde{\varphi}(0, 0)$$

for all $x \in E$.

THEOREM 2.2. *Let $f : X \to A$ be a mapping such that*

$$\|af(x + y)a^* + af(x - y)a^* - 2f(ax) - 2f(ay)\| \leq \varphi(x, y)$$

for all $a \in A_1$ and all $x, y \in X$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in X$, then there exists a unique involutive A-quadratic mapping $Q : X \to A$ such that (2.4). The mapping $Q : X \to A$ is given by the same formula as the statement of Theorem 2.1.

Proof. By a similar method to the proof of Theorem 2.1, one can obtain the result. □

3. Results in Banach modules over a C^*-algebra

Throughout this section, let A be a unital C^*-algebra. Put $\mathbb{U}(A) := \{a \in A \mid aa^* = a^*a = 1\}$, $A_{in} := \{a \in A \mid a \text{ is invertible}\}$, $A_{sa} := \{a \in A \mid a = a^*\}$ and $A^+ := \{a \in A \mid a \text{ is positive}\}$. Let φ be nonnegative real valued functions defined on $X \times X$ satisfying (2.1) or (2.2) on X.

Kadison and Pedersen [10] showed the following.

LEMMA 3.1. *Let $a \in A$ and $\|a\| < 1 - \frac{2}{m}$ for some integer m greater than 2. Then there are m unitary elements $u_1, \ldots, u_m \in A$ such that $ma = u_1 + \cdots + u_m$.*

We prove the generalized Hyers-Ulam stability of the functional equation (1.1) in Banach modules over a unital C^*-algebra.

THEOREM 3.1. *Let A be of real rank 0. Let $f : X \to A$ be a mapping satisfying (2.3) for all $a \in A_1 \cap A_{in}$ and all $x, y \in X$. For each fixed $x \in X$, let the sequence $\left\{\frac{f(2^m x)}{4^m}\right\}$ converge uniformly on A_1 for the case (2.1), and the sequence $\left\{4^m f(\frac{x}{2^m})\right\}$ converge uniformly on A_1 for the case (2.2). If $f(ax)$ is continuous in $a \in A_1 \cup \mathbb{R}$ for each fixed $x \in X$, then there exists a unique involutive A-quadratic mapping $Q : X \to A$ satisfying (2.4) for all $x \in X$.*

Proof. By the same reasoning as in the proof of Theorem 2.1, there exists a unique quadratic mapping $Q : X \to A$ satisfying (2.4) for all $x \in X$, and

$$Q(ax) = aQ(x)a^* \tag{3.1}$$

for all $a \in A_1 \cap A_{in}$ and all $x \in X$. By the continuity and the uniform convergence, one can show that $Q(ax)$ is continuous in $a \in A_1$ for each $x \in X$.

Let $b \in A_1 \setminus A_{in}$. Since $A_{in} \cap A_{sa}$ is dense in A_{sa}, there exists a sequence $\{b_m\}$ in $A_{in} \cap A_{sa}$ such that $b_m \to b$ as $m \to \infty$. Put $a_m := \frac{1}{|b_m|} b_m$. Then $a_m \to b$ as $m \to \infty$ and $a_m \in A_1 \cap A_{in}$. By the continuity of Q

$$\lim_{m\to\infty} Q(a_m x) = Q(\lim_{m\to\infty} a_m x) = Q(bx) \tag{3.2}$$

for all $x \in X$. By (3.1),

$$\begin{aligned} \|Q(a_m x) - bQ(x)b^*\| &= \|a_m Q(x) a_m^* - bQ(x)b^*\| \\ &\longrightarrow \|bQ(x)b^* - bQ(x)b^*\| = 0 \end{aligned} \tag{3.3}$$

as $m \to \infty$. By (3.2) and (3.3),

$$\begin{aligned} \|Q(bx) - bQ(x)b^*\| &\le \|Q(bx) - Q(a_m x)\| + \|Q(a_m x) - bQ(x)b^*\| \\ &\to 0 \text{ as } m \to \infty \end{aligned} \tag{3.4}$$

for all $x \in X$. By (3.1) and (3.4), $Q(ax) = aQ(x)a^*$ for all $a \in A_1$ and all $x \in X$.

The rest of the proof is similar to the proof of Theorem 2.1. □

THEOREM 3.2. *Let A be of real rank 0 and commutative and let $f : X \to A$ a mapping satisfying (2.3) for all $a \in \exp(\mathbb{U}(A)) \cup \{1\}$ and* *all $x, y \in X$. For each fixed $x \in X$, let the sequence $\left\{\frac{f(2^m x)}{4^m}\right\}$ converge uniformly on A_1 for the case (2.1), and the sequence $\{4^m f(\frac{x}{2^m})\}$ converge uniformly on A_1 for the case (2.2). If $f(ax)$ is continuous in $a \in A_1 \cup \mathbb{R}$ for each fixed $x \in X$, then there exists a unique involutive A-quadratic mapping $Q : X \to A$ satisfying (2.4) for all $x \in X$.*

Proof. By the same reasoning as in the proof of Theorem 2.1, there exists a unique quadratic mapping $Q : X \to A$ satisfying (2.4) for all $x \in X$. By a similar method to the proof of Theorem 2.1, the quadratic mapping Q satisfies $Q(ax) = aQ(x)a^*$ for all $a \in \exp(\mathbb{U}(A)) \cup \{1\}$ and all $x \in X$.

Let $D := \{a \in A | Sp(a) \subset \mathbb{C} \setminus [0, \infty)\}$. For each element $a \in A_1 \cap D$, there exists a positive integer m satisfying $3 + \|\log a\| < m$. By Lemma 3.1, there are unitary elements $u_1, \ldots, u_m \in \mathbb{U}(A)$ such that $1 + \log a = u_1 + \cdots + u_m$. Put $b := \frac{1+\log a}{m}$. Then

$$\begin{aligned} Q(\exp(1)ax) &= Q(\exp(mb)x) \\ &= Q(\exp(u_1)\cdots\exp(u_m)x) \\ &= \exp(u_1)\cdots\exp(u_m)Q(x)\exp(u_m)^*\cdots\exp(u_1)^* \\ &= \exp(mb)Q(x)\exp(mb)^* \\ &= \exp(1)aQ(x)a^*\exp(1)^* \end{aligned}$$

for all $a \in A_1 \cap D$ and all $x \in X$. But $Q(\exp(1)ax) = \exp(1)Q(ax)\exp(1)^*$ for all $a \in A$ and all $x \in X$. So $Q(ax) = aQ(x)a^*$ for all $a \in A_1 \cap D$ and all $x \in X$.

Let $E := \{a \in A | Sp(a) \subset \mathbb{C} \setminus (-\infty, 0]\}$. By the same process as the above argument, one can obtain that $Q(ax) = aQ(x)a^*$ for all $a \in A_1 \cap E$ and all $x \in X$. Hence $Q(ax) = aQ(x)a^*$ for all $a \in A_1 \cap (D \cup E) = A_1 \cap A_{in}$ and all $x \in X$.

The rest of the proof is the same as in the proof of Theorem 3.1. □

THEOREM 3.3. *Let A be of real rank 0 and commutative and let $f : X \to A$ a mapping satisfying (2.3) for all $a \in \{\pm 1, i\}\exp(A^+) := \{\pm d, id | d \in \exp(A^+)\}$ and all $x, y \in X$. For each fixed $x \in X$, let the sequence $\left\{\frac{f(2^m x)}{4^m}\right\}$ converge uniformly on A_1 for the case (2.1), and the sequence $\{4^m f(\frac{x}{2^m})\}$ converge uniformly on A_1 for the case (2.2). If $f(ax)$ is continuous in $a \in A_1 \cup \mathbb{R}$ for each fixed $x \in X$, then there exists a unique involutive A-quadratic mapping $Q : X \to A$ satisfying (2.4) for all $x \in X$.*

Proof. By the same reasoning as in the proof of Theorem 2.1, there exists a unique quadratic mapping $Q : X \to A$ satisfying (2.4) for all $x \in X$. By a similar method to the proof of Theorem 2.1, the quadratic mapping Q satisfies $Q(ax) = aQ(x)a^*$ for all $a \in \{\pm 1, i\} \exp(A^+)$ and all $x \in X$.

Let $D := \{a \in A | Sp(a) \subset \mathbb{C} \setminus [0, \infty)\}$. For each element $a \in A_1 \cap D$, put $b = 1 + \log a$. Note $b = b_1 + ib_2$, where $b_1 = \frac{b+b^*}{2}$ and $b_2 = \frac{b-b^*}{2i}$ are self-adjoint elements, furthermore, $b = {b_1}^+ - {b_1}^- + i{b_2}^+ - i{b_2}^-$, where ${b_1}^+$, ${b_1}^-$, ${b_2}^+$, and ${b_2}^-$ are positive elements (see Lemma 38.8 in [3]). Thus

$$\begin{aligned} Q(\exp(1)ax) &= Q(\exp(b_1^+)\exp(-b_1^-)\exp(ib_2^+)\exp(-ib_2^-)x) \\ &= \exp(b_1^+)\exp(-b_1^-)\exp(ib_2^+)\exp(-ib_2^-)Q(x) \\ &\quad \exp(-ib_2^-)^*\exp(ib_2^+)^*\exp(-b_1^-)^*\exp(b_1^+)^* \\ &= \exp(b)Q(x)\exp(b)^* \\ &= \exp(1)aQ(x)a^*\exp(1)* \end{aligned}$$

for all $a \in A_1 \cap D$ and all $x \in X$.

Let $E := \{a \in A | Sp(a) \subset \mathbb{C} \setminus (-\infty, 0]\}$. By the same process as the above argument, one can obtain that $Q(\exp(1)ax) = \exp(1)Q(ax)$ $\exp(1)^*$ for all $a \in A_1 \cap E$ and all $x \in X$.

The rest of the proof is the same as in the proof of Theorem 3.2. □

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