

# The Phase Estimation Algorithm of Arrival Phase Differences in Space-Time Coded Communication

Ji Won Jung, Xinping Huang, Mario Caron, Min Hyuk Kim, Ki Man Kim, and Young Yun

*ABSTRACT*—One objective in developing the next generation of wireless communication systems is to increase data rates and reliability. A promising way to achieve this is to combine multiple-input and multiple-output signal processing with a space-time coding scheme, which offers higher coding and diversity gains and improves the spectrum efficiency and reliability of a wireless communication system. It is noted, however, that time delay differences and phase differences among different channels increase symbol interference and degrade system performance. In this letter, we investigate phase differences and their effects on multiple-input and multiple-output systems, and propose a compensation algorithm for the Rayleigh fading model to minimize their effects.

*Keywords*—Multiple-input and multiple-output, space-time code, Rayleigh fading, phase compensation algorithm.

## I. Introduction

One objective in developing next generation wireless communication systems is to increase the data rates and thus improve reliability to satisfy the rapidly growing demand for high quality, multi-media services. However, such systems suffer from multi-path propagation effects, which exhibit rapidly time-varying channel characteristics. Space-time coding techniques have been widely proposed to combat these adverse effects. In [1]-[3], the authors proposed several space-time coding schemes, which offer higher coding and diversity gains and are suitable to improve the spectrum efficiency and reliability of broadband applications. In their studies, it is assumed that there are no delay

or phase errors among the multiple transmitter and receiver chains. This assumption is difficult to warrant in practice. Since transmit chains inevitably have different characteristics due to component mismatches, the transmitted signals usually have different phase shifts and time delays. Therefore, phase differences increase symbol interference and degrade system performance. In this letter, we attempt to model phase differences, and investigate their effects on MIMO systems. We also propose a compensation algorithm for the Rayleigh fading model to minimize the effects of phase differences.

## II. System Model

Figure 1 illustrates the operation of a space-time coded communication system that consists of two transmit and two receive antennas. To focus on the phase error issue in this letter, we assume that the channel is a slow flat-fading Rayleigh channel. We also assume that the carrier phase/frequency and symbol timing synchronization is perfect. At the  $k$ -th receive antenna, the received signal can be represented as

$$r_k(t) = \sqrt{2}e^{j\theta_1(t)}e^{j\tilde{\theta}_{1k}}\alpha_{1k}(t) + \sqrt{2}e^{j\theta_2(t)}e^{j\tilde{\theta}_{2k}}e^{j\tilde{\theta}}\alpha_{2k}(t) + \eta_k(t), \quad (1)$$

where  $\eta_k(t)$  is a zero-mean complex white Gaussian process with the two-sided power spectral density  $N_f/2$  per dimension. If we generalize it to a system with  $N$  transmit antennas and  $M$  receive antennas, it is assumed that  $\eta_j(t)$  and  $\eta_k(t)$  are independent for  $j \neq k$ . The coefficient  $\alpha_j(t)$  is the complex path gain from the  $i$ -th transmit antenna to the  $j$ -th receive antenna and is modeled by a complex Gaussian random process of zero mean and a variance of 0.5 per dimension.

It is also assumed that  $\alpha_j(t)$  and  $\alpha_{qk}(t)$  are independent for  $i \neq q$ ,  $j \neq k$ ,  $1 \leq i, q \leq N$ ,  $1 \leq j, k \leq M$ , and that these path gains are

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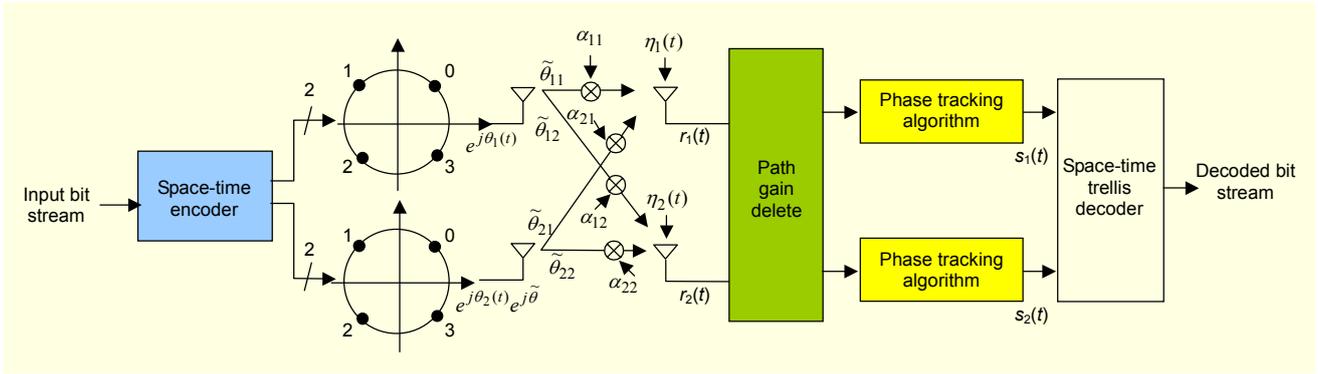


Fig. 1. System model.

constant during a frame and vary from one frame to another (aquasi-static flat fading). This is equivalent to an assumption that each spatial channel (the link between one transmit antenna and one receive antenna) experiences a statistically independent slow Rayleigh fading. This condition is satisfied if the transmit antennas are well separated (by more than  $\lambda/2$ ) or if antennas with different polarization are used. The information phase at the  $i$ -th transmit antenna is represented by  $\theta_i(t)$ . The phase offsets between the  $i$ -th transmit antenna and the  $k$ -th receive antenna are represented by  $\tilde{\theta}_{ik}$ . Since transmit devices inevitably have different characteristics due to component mismatches, the two transmitted signals usually have different phases. Therefore,  $\tilde{\theta}$  is introduced to represent the phase error of the 2nd transmit antenna relative to the first one. The decision is made on the basis of the received signal. Assuming that path gain  $\alpha_{ij}(t)$  is deleted after some manipulation, we can express  $r_k(t)$  as

$$r_k(t) = \tilde{\alpha}(t)e^{j\theta_0(t)}e^{j\phi_k} + \eta_k(t), \quad (2)$$

where  $\theta_0(t) = (\theta_1(t) + \theta_2(t))/2$  is a function of the transmitted information,  $\phi_k = (\tilde{\theta}_{1k} + \tilde{\theta}_{2k} + \tilde{\theta})/2$  is the total phase error in the  $k$ -th receive channel, and the amplitude of the received signal is represented as

$$\tilde{\alpha}(t) = 2\sqrt{2} \cos\left(\frac{\theta_1(t) + \tilde{\theta}_{11} - \theta_2(t) - \theta_{21} - \tilde{\theta}}{2}\right). \quad (3)$$

In the next section, we describe a decision-directed phase error estimation and compensation algorithm, and use it to solve the phase error problem in a space-time coded communication system. Based on the received signal model of (2), a decision-directed approach can be used to estimate and

track the total phase error in each receive channel.

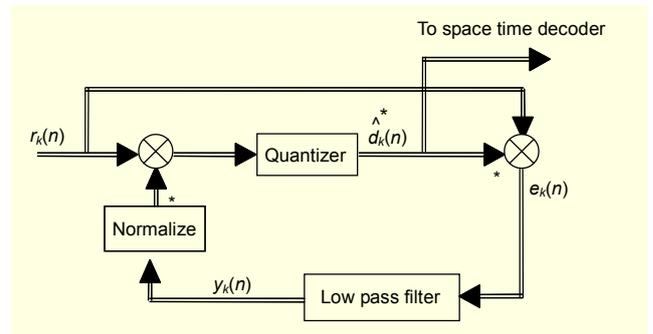


Fig. 2. Block diagram of the decision-directed phase estimation and compensation.

### III. Phase Estimation and Compensation

The covariance approach [9] can only estimate the phase offset, and it does not correct it. We will now describe a decision-directed phase error estimation and compensation algorithm, and use this algorithm in a space-time coded communication system.

Given the received signal model of (2), a decision-directed approach can be used to estimate and track the total phase error in each receive channel, as shown in Fig. 2, where  $*$  denotes the complex-conjugate operator, and  $\hat{d}_k^*(n)$  is the quantizer output after the phase error is compensated. Ideally, we have

$$\hat{d}_k^*(n) = e^{j\theta_0(n)}. \quad (4)$$

In the figure, the phase error is given by

$$e_k(n) = r_k(n) \cdot \hat{d}_k^*(n). \quad (5)$$

Correspondingly, the phase error estimate can be obtained by taking the expectation of (5) as

$$e_k(n) = E\left[e^{j\theta_0(n)}e^{j\phi_k}e^{-j\theta_0(n)} + \eta_k(n)e^{-j\theta_0(n)}\right] = e^{j\phi_k}. \quad (6)$$

The expectation operation is implemented as the low-pass filtering in Fig. 2. Both infinite impulse response (IIR) and finite impulse response (FIR) filters can be used for the low-pass filter [5]. In this letter, a first order IIR filter is used. It is defined as follows:

$$y_k(n) = \beta y_k(n-1) + (1-\beta)e_k(n), \quad (7)$$

where  $\beta$  ( $0 < \beta < 1$ ) means the loop bandwidth parameter of a low-pass filter. Its value controls the estimation accuracy and how fast the phase estimate reaches its steady-state value. The larger  $\beta$  is, the more accurate the estimate is, but the slower it is to converge. In the noiseless case, we can show that

$$\begin{aligned} y_k(n) &= \beta y_k(n-1) + (1-\beta)e^{-j\phi_k} \\ &= e^{-j\phi_k}(1-\beta^n). \end{aligned} \quad (8)$$

As the time  $n$  increases,  $\beta^n$  decreases. Eventually,  $y_k(n)$  will approach its steady-state value, that is,

$$y_k(n) \rightarrow e^{-j\phi_k}, \text{ as } n \rightarrow \infty. \quad (9)$$

Therefore, when  $y_k(n)$  reaches its steady-state value, we can remove the effect of the phase error by multiplying its complex-conjugate to the input signal  $r_k(n)$ .

#### IV. Simulation Results

Computer simulations were used to study the effect of phase errors on system performance. A space-time coded system with two transmit and two receive antennas was simulated. The source symbols were transmitted in frames of length 130. The code generator matrix for the quadrature phase shift keying (QPSK) space-time code with 32 states proposed in [1] was used. Additive white Gaussian noise was introduced as the channel noise in the simulation.

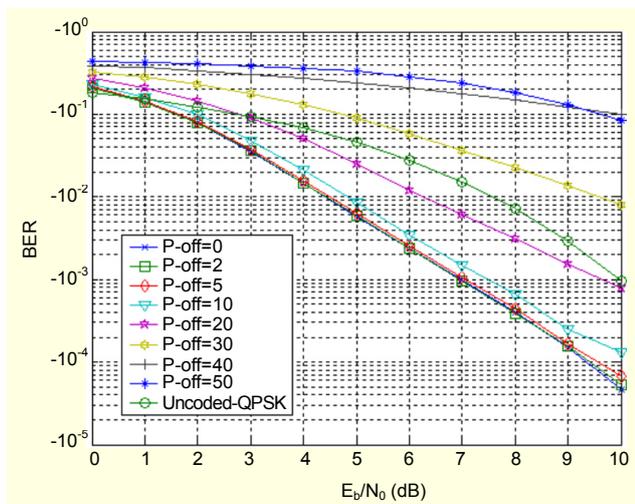


Fig. 3. Bit error rate for phase offset  $\tilde{\theta}$ .

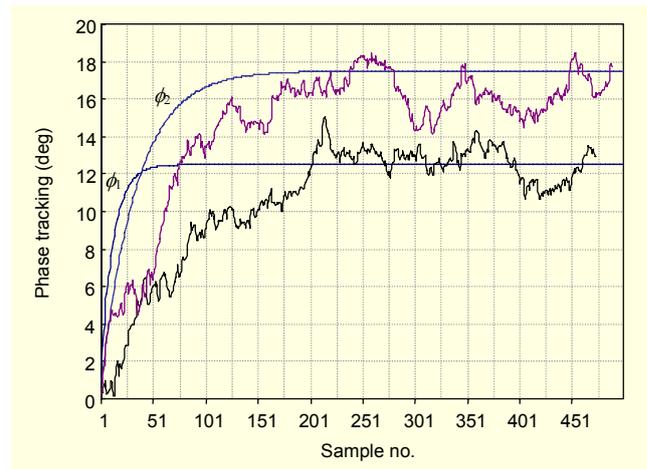


Fig. 4. Performance illustration of the proposed algorithm.

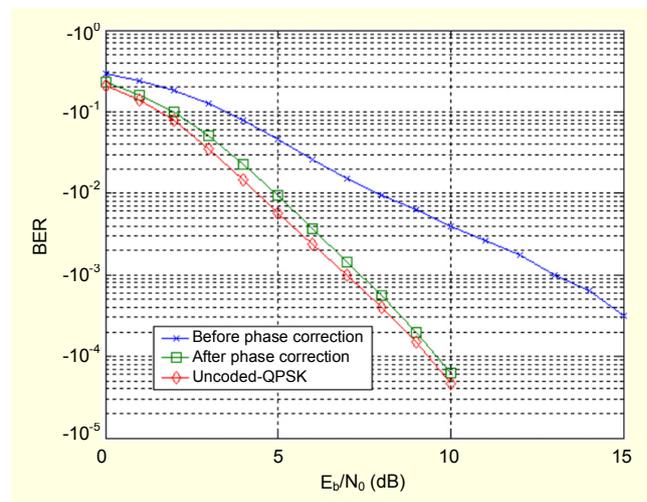


Fig. 5. Bit error rate comparison.

Figure 3 shows the bit error rate (BER) for different values of the transmitter phase offset  $\tilde{\theta}$  without any phase compensation. In this example, the channel phase errors were set to zero. The dotted line represents the BER of an uncoded QPSK system with one transmit and one receive antenna. We observe that when  $\tilde{\theta}$  is small ( $\tilde{\theta} \leq 5^\circ$ ), the BER performance degradation is not very significant. However, as  $\tilde{\theta}$  becomes bigger, the BER performance deteriorates rapidly. When  $\tilde{\theta} > 20^\circ$  the performance is worse than that of the uncoded system. The results in Fig. 3 clearly show that phase error compensation is necessary to preserve both the coding and diversity gains of the space-time coded system. In the following, we examine the performance of the proposed phase estimation and compensation algorithm and the BER improvement it can achieve.

The following phase errors are used:  $\tilde{\theta}_{11} = 10^\circ$ ,  $\tilde{\theta}_{12} = 5^\circ$ ,  $\tilde{\theta}_{21} = 5^\circ$ ,  $\tilde{\theta}_{22} = 20^\circ$ , and  $\tilde{\theta} = 10^\circ$ . Given these

values, we have  $\phi_1 = 12.5^\circ$  and  $\phi_2 = 17.5^\circ$ . Figure 4 shows the phase error estimate in two cases: a noise-free case which is represented by two smooth curves, and the other case with  $E_b/N_o = 7$  dB and  $\beta = 0.9$  which is represented by noisy curves. We notice that the proposed algorithm estimates the phase errors correctly and they converge within about 200 samples.

Figure 4 shows the BER performance of the space-time coded system with and without the phase estimation and compensation algorithm for the same set of phase error values used in Fig. 3. It is clear that the proposed algorithm significantly improves the BER performance. For example, at a bit error rate of  $10^{-3}$ , about 5.5 dB improvement is achieved.

## V. Conclusion

In a space-time coded system, it is often assumed that there are no phase errors among the multiple transmitter and receiver chains. This assumption is difficult to warrant in practice. The phase differences between the propagation paths increase the symbol interference and degrade system performance. In this letter, we have studied the effect of the phase errors between different transmit antennas and different propagation paths in a space-time coded communication system, and have shown through computer simulations of BER performance that the BER performance can be severely degraded. A decision-directed estimation and compensation algorithm has been proposed to minimize the effects on system performance. The described decision-directed approach works well under the assumption of constant phase over a frame. If phase is variable over a frame, the circuit needs a long transmit packet size to estimate the phase or we choose low-pass filter parameter trade-offs between the estimated time and performance. Therefore, this circuit may be useful as a phase estimator before transmitting data.

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