Secant Method for Economic Dispatch with Generator Constraints and Transmission Losses

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Abstract – This paper describes the secant method for solving the economic dispatch (ED) problem with generator constraints and transmission losses. The ED problem is an important optimization problem in the economic operation of a power system. The proposed algorithm involves selection of minimum and maximum incremental costs (lambda values) and then the evaluation of optimal lambda at required power demand is done by secant method. The proposed algorithm has been tested on a power system having 6, 15, and 40 generating units. Studies have been made on the proposed method to solve the ED problem by taking 120 and 200 units with generator constraints. Simulation results of the proposed approach were compared in terms of solution quality, convergence characteristics, and computation efficiency with conventional methods such as lambda iterative method, heuristic methods such as genetic algorithm, and meta-heuristic methods like particle swarm optimization. It is observed from different case studies that the proposed method provides qualitative solutions with less computational time compared to various methods available in the literature.

Keywords: Economic dispatch problem, Ramp rate limits, Secant method, Transmission losses and Quadratic fuel cost function

1. Introduction

The main objective of economic dispatch (ED) problems is to determine the optimal schedule of online generating units so as to meet the power demand at minimum operating cost under various system and operating constraints such as ramp rate limits [1] and prohibited zones [2]. The fuel cost function of each generating unit is approximately represented by a quadratic function. It is necessary to consider incremental transmission losses for optimal economic dispatch. In real time operation of a power system, some of the generating units may have prohibited zones lying between minimum and maximum output powers. These zones cause vibrations in the shaft bearing. Therefore, the prohibited zones should be avoided in real time operation. When the generating unit has prohibited zones, the fuel cost function of the generating unit will be broken into isolated sub regions.

Earlier efforts of solving the ED problems have employed many conventional optimization techniques such as lambda iteration method, lambda projection method, base point method, participation factors method, and gradient method [11]. In order to get the qualitative solution for ED problems, artificial neural network techniques such as back propagation algorithm (BPA) based neural network and Hopfield neural network (HNN) [3, 9] has been successfully employed for thermal units with piecewise quadratic function and prohibited zone constraints. The BPA takes a greater number of iterations due to improper selection of learning and momentum rates. Similarly, the Hopfield model suffers from excessive iterations due to an unsuitable sigmoid function as a result of which it takes more time to give optimal solution at required power demand.

In the past decade, a global optimization technique known as a genetic algorithm [4] has been used to solve the ED problem with a quadratic, piecewise quadratic fuel cost function and valve point loading. It is a parallel search technique, which imitates natural genetic operation. Due to its high potential for global optimization, the GA has received great attention in solving ED problems with quadratic and piecewise quadratic cost function and valve point loading including network losses, ramp rates, and prohibited zones. But recent research has identified some deficiencies in GA performance as the crossover and mutation operations cannot ensure the better fitness of offspring because the chromosomes in the population have similar structures and their average fitness is high towards the end of the evolutionary process [5].

Recently, meta-heuristic techniques such as evolutionary programming (EP) [6], particle swarm optimization (PSO) [7], ant colony searching algorithm (ACSA), and Tabu search algorithm (TSA) have been given much attention by many researchers due to their

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ability in obtaining the near global optimal solution. In these methods, quality of solution depends on user defined factors. Improper selection of these factors may increase the computational time for achieving optimal solution.

From the literature survey, it is observed that most of the algorithms have some limitations to provide optimal solution with less computational time. Therefore, it is necessary to find suitable algorithms to solve economic dispatch problems more effectively. In ED problems, the power balance equation is a nonlinear equation in terms of lambda. Non linear equations with single variable can be solved by root finding techniques. In this contrast, root finding techniques have been proposed for solving ED problems. It is observed from the case studies that the root finding techniques provide the optimal solution with less computational time. Furthermore, the proposed methods provide a good qualitative solution over many existing algorithms including genetic algorithm, lambda iterative method, and particle swarm optimization method.

The main aim of the present paper is to develop a new method for solving the ED problem with generator constraints and transmission losses by employing the secant method [12]. It is a root finding method available in numerical methods [13]. This paper presents the secant method to solve the ED problem with the generator constraints and transmission losses. The proposed algorithm has been implemented in MATLAB on a Pentium III, 550 MHz personal computer with 256-MB RAM. The paper is organized into three sections. Statement and formulation of the ED problem with various constraints is introduced in Section II. The description of the secant method and its application for solving ED is presented in Section III. The simulation results of case studies on various power systems are presented in Section IV. The conclusions of the work are presented in the last section.

2. Economic Dispatch Problem

The ED problem is a non-linear programming optimization problem. The main objective is to minimize the total fuel cost at thermal plants subjected to various constraints such as generating constraints, prohibited zones and ramp rate limits.

The fuel cost function is considered as a quadratic function of real power generation and the ED problem can be formulated as follows:

$$F_i(P_i) = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \qquad (1)$$

Where

$F_i(P_i)$ Fuel cost of i^{th} generating unit

P_i Output power of i^{th} generating unit

$\alpha_i, \beta_i, \gamma_i$ Co-efficient of quadratic fuel cost function

The objective function is

$$\text{Minimize } F_t = \sum_{i=1}^{ng} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \qquad (2)$$

Where

F_t Total fuel cost

n_g Number of generating units

The objective function is subjected to equality and inequality constraints.

2.1 Equality constraint

It is given by the power balance equation.

$$\sum_{i=1}^{ng} P_i = P_D + P_L \qquad (3)$$

Where

P_D Power demand

P_L Incremental transmission loss

Where the transmission loss [10] is assumed as a quadratic function of output powers of generating units and given by

$$P_L = \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} P_i B_{ij} P_j + B_{i0} P_j + B_{00} \qquad (4)$$

Where

B_{ij}, B_{0i}, B_{00} B-loss coefficients

2.2 Generator constraints

Generator constraints are given by the minimum and maximum output power of each generating unit.

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad (5)$$

Where

$P_i^{\min}$ Minimum real output power of i^{th} generating unit

$P_i^{\max}$ Maximum real output power of i^{th} generating unit

2.3 Prohibited zones

Prohibited operating zones in the fuel cost function are either due to vibrations in the shaft bearing caused by a steam valve or due to the associated auxiliary equipment such as boiler or feed pumps. In practice, determination of the shape of the fuel cost curve in the neighborhood of a prohibited zone is difficult. Therefore, the best economy is achieved by avoiding the operation of generating units in these areas.

Fuel cost function curve with prohibited zones is shown in Fig. 1. Prohibited zones divide the operating region between minimum and maximum generation limits into disjoint convex sub regions. The generation limits for units with prohibited zones are

$$\begin{aligned} &P_i^{\min} \le P_i \le P_{i,1}^d \\ &P_{i,j-1}^d \le P_i \le P_{i,j}^u \quad ,\mathrm{j}=2,3,\ldots n_i \\ &P_{i,n_i}^u \le P_i \le P_i^{\max} \end{aligned} \tag{6}$$

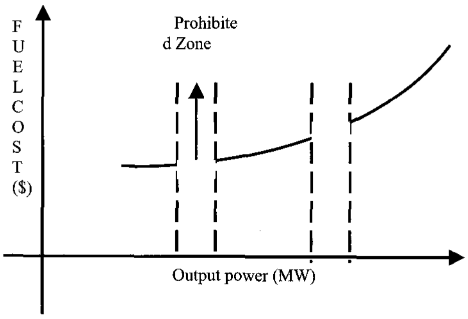

Fig. 1. Fuel cost function curve with prohibited operating zones

2.4 Ramp rate limits

The range of actual operation of online generating units is restricted by its ramp rate limits. These limits can impact the operation of the power system. The operational decision at the present hour may affect the operational decision at a later hour due to ramp rate limits. In actual operation, three possible situations exist due to variation in power demand from the present hour to the next hour. First, during the steady state operation, the operation of the online unit is in steady state condition. Second, when the power demand is increased, the power generation of the generating unit is also increased. Third, if the power demand is decreased then the power generation of the generating unit is also decreased.

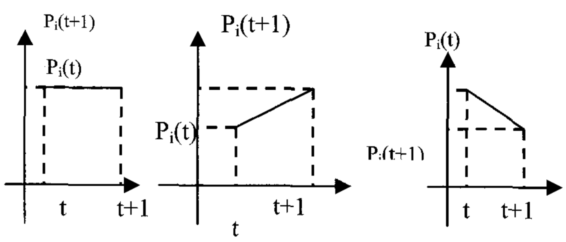

Fig. 2. Ramp rate limits of the generating units

Inequality constraints due to ramp rate limits of generating units are given as

A. when generation is increased

$$P_i - P_i^0 \le UR_i \tag{7}$$

B. when generation is decreased

$$P_i^0 - P_i \le DR_i \tag{8}$$

Finally generator constraints can be modified as

$$\begin{aligned} &\max(P_i^{\min}, P_{i,t}^o - DR_i) \le P_{i,t} \le \min(P_{i,t}^o + UR_i) \\ &i = 1,2,..\ to\ ng \quad \mathrm{t}=1,2,\ldots\mathrm{T} \end{aligned} \tag{9}$$

The formulation of Lagrange function for the ED problem is given by

$$\chi = F_T + \lambda \times \left(P_D + P_L - \sum_{i=1}^{ng} P_i \right) \tag{10}$$

The expressions of lambda and output power are

$$\lambda_i = \frac{\beta_i + (2 \times \gamma_i \times P_i)}{1 - \left(2 \times \sum_{i=1}^{ng} B_{ij} P_j + B_{i0} \right)} \tag{11}$$

$$P_i = \frac{\lambda_i \times \left(1 - B_{i0} - 2 \times \sum_{\substack{j=1 \\ i \ne j}}^{ng} B_{ij} P_j \right) - \beta_i}{2 \times (\gamma_i + \lambda_i B_{ii})} \tag{12}$$

Where

λ_i Incremental fuel cost of i^{th} generating unit

3. Secant Method for Economic Dispatch Problems

In this section, the secant method is presented to solve the ED problem. Two steps are involved in the proposed approach.

3.1 Selection of lambda values.

3.2 At required power demand, optimal lambda is evaluated by the secant method.

3.1 Selection of the Lambda values

The selection of lambda values is as follows,

(i) From Equation (11), initially lambda values are evaluated at minimum and maximum output powers of all generating units.

(ii) All the lambda values are arranged in ascending order and then minimum and maximum values of lambda are selected.

The power balance equation is written as a function of lambda. Therefore

$$f(\lambda) = \sum_{i=1}^{ng} P_i(\lambda) - P_D - P_L(\lambda) = 0 \quad (13)$$

From Equation (13), it is observed that $f(\lambda)$ is non linear in terms of lambda. The non linear equation with single variable can be solved by root finding techniques available in numerical methods. Here, secant method has been proposed to solve ED problems.

3.2 Secant method

The secant method [12, 13] is a root finding algorithm that uses succession of roots of secant lines to better approximate the root of a function. This method assumes that the function is approximately linear in the local region of interest and uses the zero crossing over the line connecting the limits of the interval as the new reference point. The next iteration starts from evaluating the function at the new reference point and then forms another line. The process is repeated until the root is found. Geometrically, Newton method uses the tangent line and secant method approximates the tangent line by secant line. It has super linear convergence and it will converge within five iterations if the guess value is correct.

The intersection of the straight line with the x-axis can be obtained by using linear interpolation with the following points.

x	x_1	x_3	x_2
$f(x)$	$f(x_1)$	0	$f(x_2)$

$$\frac{x_3 - x_2}{x_2 - x_1} = \frac{0 - f(x_2)}{f(x_2) - f(x_1)}$$

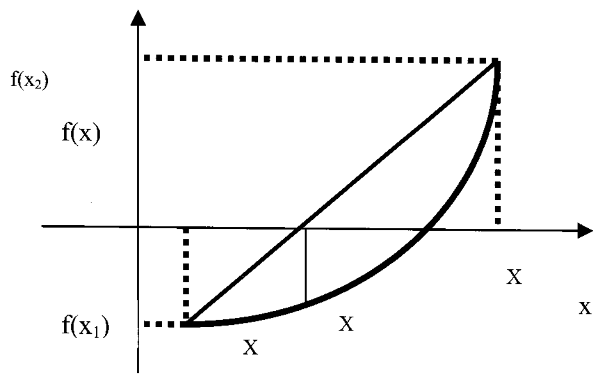

Fig. 3. Graphical representation of the secant method

$$\Rightarrow x_3 = x_2 - f(x_2) \; \frac{x_2 - x_1}{f(x_2) - f(x_1)}$$

The next guess is then obtained from the straight line through two points $[x_2, f(x_2)]$ and $[x_3, f(x_3)]$. In general, the guessed value is calculated from the two previous points $[x_{k-1}, f(x_{k-1})]$ and $[x_k, f(x_k)]$ as

$$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k) \quad (14)$$

Application of the secant method for solving economic dispatch problem is as follows.

Values of $x_{k-1}, x_k, f(x_{k-1})$ and $f(x_k)$ are selected as,

$$x_{k-1} = \lambda_{\min} \;\&\; f(x_{k-1}) = \sum_{i=1}^{ng} P_i(\lambda_{\min}) - P_D - P_L \quad (15)$$

$$x_k = \lambda_{\max} \;\&\; f(x_k) = \sum_{i=1}^{ng} P_i(\lambda_{\max}) - P_D - P_L \quad (16)$$

If P_i violates the generator limits, the generating limits are set as

$$if\ P_i \ge P_i^{\max}\ then\ P_i = P_i^{\max}$$
$$if\ P_i \le P_i^{\min}\ then\ P_i = P_i^{\min}$$

Similarly if P_i is within the prohibited zone, then the output power of generating units is set as the average

Modification of generator limits for ED problem with ramp rate limits

In the first hour, the generating limits are considered as follows,

$$P_{i\min\_ramp} = \max(P_i^{\min}, P_{i,t}^0 - DR_i) \quad (17)$$

$$P_{i\max\_ramp} = \min(P_i^{\max}, P_{i,t}^0 + UR_i) \quad (18)$$

For the next hour, optimal solution of the first hour becomes the initial output powers for the second hour. The generator limits are followed by the ramp rate limits for all power demands.

From (14), optimal lambda value is evaluated by secant method at required power demand. The chief advantage of this method is that it converges super linearly to find the root of the polynomial.

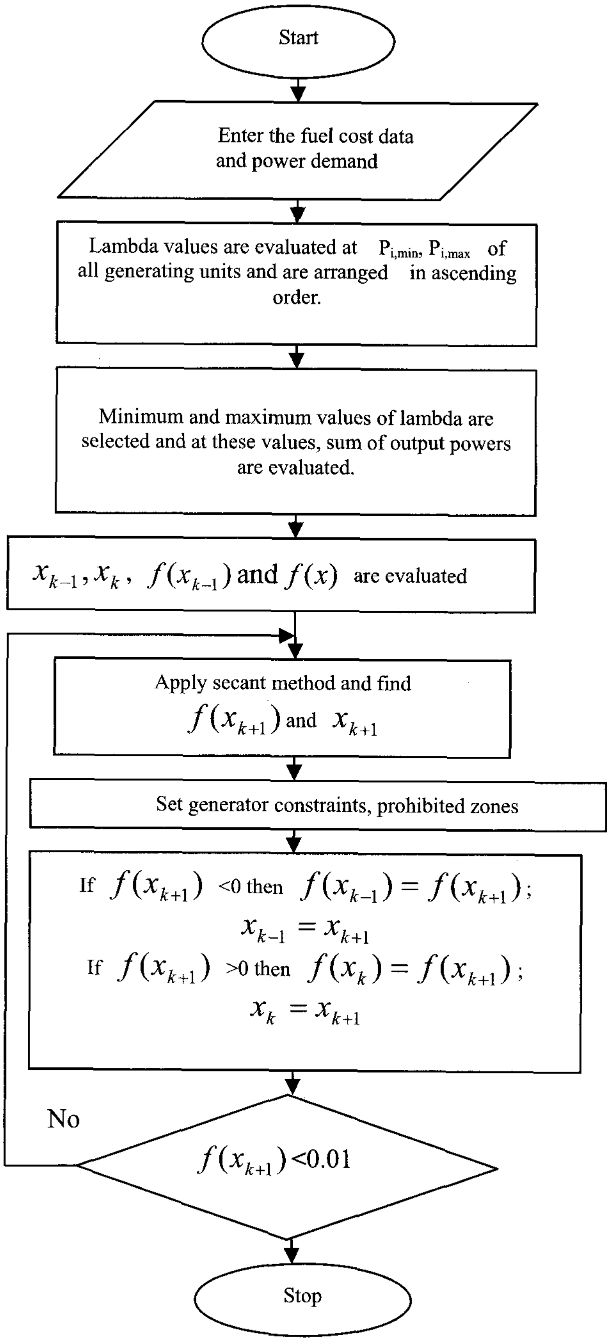

Fig. 4. Flowchart of the proposed algorithm

4. Case Studies and Simulation Results

The proposed algorithm was implemented in MATLAB and executed on a Pentium III, 550 MHz personal computer with 256 MB RAM to solve the ED problem of a power system having 6, 15, and 40 generating units with generator constraints and transmission losses. The simulation results obtained from the proposed method were compared in terms of the solution quality and computational efficiency with the conventional lambda iterative method, heuristic method such as genetic algorithm, and meta-heuristic method such as particle swarm optimization.

During the execution of the conventional lambda iterative method, the initial lambda is selected as the lowest lambda value among all lambda values of the generating units at their minimum and maximum output powers. While executing the lambda iterative method and proposed method, lambda is taken as the control parameter.

4.1 Case 1

In this case, the system contains six thermal units [7]. The fuel cost data of six thermal units is given in Table 1. In this case, simulation results in terms of output powers of the proposed method are compared with the conventional lambda iterative method.

Initially the values of lambda are computed for all generators at their minimum and maximum output powers and then arranged in ascending order and finally minimum and maximum lambda values are selected. The output powers are evaluated at these lambda values by incorporating the generating limits and also the values of $f(x_{k-1}), f(x)$ are computed at the given power demand. Finally, the optimal lambda value is evaluated by the secant method. Table 2 lists the statistical results of lambda iterative method and secant method that involved generation cost and computational time.

From Table 2, it is clear that the proposed method yields a qualitative solution of fuel cost identical to the lambda iterative method.

Table 1. Fuel cost data of 6 generating unit system

Unit	α_i (\$)	β_i (\$/MW)	γ_i (\$/MW^2)	P_{min} (MW)	P_{max} (MW)
1	240	7	0.007	100	500
2	200	10	0.0095	50	200
3	220	8.5	0.009	80	300
4	200	11	0.009	50	150
5	220	10.5	0.008	50	200
6	190	12	0.0075	50	120

Table 2. The optimum solution of each unit by lambda iterative method and proposed method

Output powers (MW)	Lambda iterative method	Proposed
P1	446.7073	446.7073
P2	171.258	171.258
P3	264.105	264.105
P4	125.216	125.216
P5	172.118	172.118
P6	083.593	083.593
Fuel cost ($)	15275.93	15275.93
Computational time (Sec)	0.031	0.016

4.2 Case 2

In this case, the proposed method was applied to solve the ED problem with transmission losses for the 6 generating unit system, which is presented in Case 1. The incremental transmission loss is represented as a quadratic form in terms of output powers. B_{mn} Coefficients [7] are given as

$$B_{ij} = 10^{-3}\begin{bmatrix} 1.7 & 1.2 & 0.7 & -0.1 & -0.5 & -2.0 \\ 1.2 & 1.4 & 0.9 & 0.1 & -0.6 & -0.1 \\ 0.7 & 0.9 & 3.1 & 0.0 & -1.0 & -0.6 \\ -0.1 & 0.1 & 0.0 & 0.24 & -0.6 & -0.8 \\ -0.5 & -0.6 & -0.1 & -0.6 & 12.9 & -0.2 \\ -2.0 & -1.0 & -0.6 & -0.8 & -0.2 & 15.0 \end{bmatrix}$$

$$B_{oi} = 10^{-3} \bullet \left[-0.3908 \ -1.297 \ 7.047 \ 0.591 \ 2.161 \ -6.635\right]$$

$$B_{00} = 0.056$$

Power demand = **1263** MW.

Table 3 lists the statistical results that involved generation cost of various methods such as particle swarm optimization, genetic algorithm, lambda iterative method, and the proposed method.

It is clear from Table 3 that the proposed approach provides a qualitative solution compared to modern heuristic methods such as particle swarm optimization and genetic algorithm reported in [7].

Table 3. Optimal solution of 6 generating unit system by various methods

Power out puts (MW)	PSO [7]	GA [7]	Lambda iterative method	Proposed method
P1	447.49	474.80	447.4	447.4
P2	173.32	178.63	173.24	173.24
P3	263.47	262.20	263.38	263.38
P4	139.05	134.28	138.98	138.98
P5	165.47	151.90	165.39	165.39
P6	87.128	74.181	87.052	87.052
Power (MW)	1276.0	1276.0	1275.44	1275.44
Loss (MW)	12.958	13.021	12.4449	12.444
Fuel Cost ($/h)	15450	15459	15442.39	15442.3

4.3 Case 3

In this case, the proposed method has been tested on the 15 unit system with transmission losses, ramp rate limits, and prohibited zones. The data was obtained from [8]. Here the main aim is to enlighten the effectiveness of the proposed method compared to the lambda iterative method in terms of the solution quality, convergence characteristics, and computational efficiency to solve large scale ED problems with all constraints for a mixed generating unit system. The optimal solutions by various methods are shown in Table 4.

Table 4. The optimum solution of each unit by various methods

Output Powers (MW)	GA [7]	PSO [7]	Lambda iterative method	Proposed method
P1	415.3108	439.1162	455	455
P2	359.7206	407.9727	455	455
P3	104.425	119.6327	130	130
P4	74.9853	129.9925	130	130
P5	380.2844	151.0681	305	305
P6	426.7902	459.9978	287.1	287.1
P7	341.3164	425.5601	366.51	366.51
P8	124.7867	98.5699	162.88	162.88
P9	133.1445	113.4936	25	25
P10	89.2567	101.1142	146.48	146.48
P11	60.0572	33.9116	80	80
P12	49.9998	79.9583	69.657	69.656
P13	38.7713	25.0042	25	25
P14	41.9425	41.414	15	15
P15	22.6445	35.614	15	15
Power (MW)	2668.2782	2662.44	2667.62	2667.6245
Loss (MW)	38.2782	32.4306	37.6244	37.6245
Fuel cost ($/Hr)	33113	32858	32857.4	32857.4
Time(Sec)	-	-	0.52	0.26

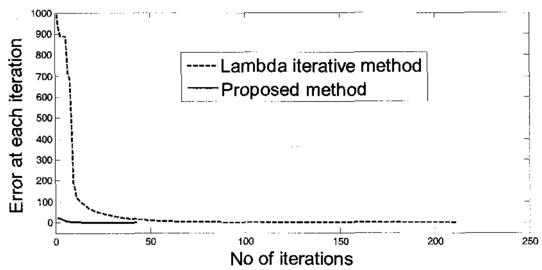

Fig. 5. Plot between no. of iterations versus error at each iteration for the 15 unit system at Power Demand of 2630 MW

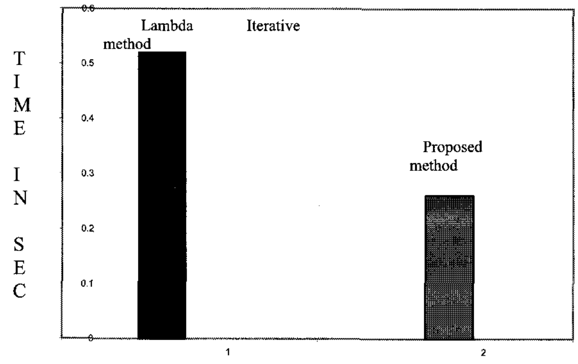

Fig. 6. Computational time of lambda iterative method and proposed method

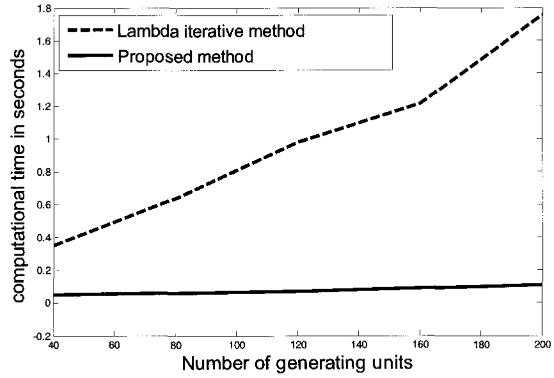

Fig. 7. Comparison of computational time by lambda iterative method and proposed method for 40-200 units

From Fig. 5 and Fig. 6, it is clear that the proposed method has better convergence and computational characteristics. These characteristics are important for large scale ED problems in real time operation of the power system.

4.4 Case 4

In this case, the proposed algorithm and Lambda iterative algorithm have both been applied to a 40 generating unit [4] system and the results are presented in Table V. In addition, the proposed algorithm has been applied to 120 and 200 generating units by considering three and five times the fuel cost data of 40 generating units.

It is observed from Fig. 7 that when dimensionality of the problem increases, lambda iterative method takes more computational time, whereas the proposed method can offer optimal solution in almost fixed time. The computational time characteristic of the proposed method and lambda iterative method for 40, 120 and 200 units systems is shown in Fig. 7.

Table 5. Simulation results of lambda iterative method and secant method

Units	40		200	
Power Demand (MW)	10500		52500	
Methods	Lambda iterative	Proposed	Lambda iterative	Proposed
Optimal lambda ($/MW)	16.257399	16.2574	16.257399	16.257399
fuel cost ($/hr)	143926.42	143926.42	719673.368	719673.38
Time	0.351	0.05	1.753	0.11

5. Conclusion

This paper has suggested secant method for solving the ED problem of a power system having 6, 15, and 40 generating units with the generator constraints and transmission losses. A salient feature of the proposed method is that it gives quality solution with less computational time compared to the lambda iterative method. As dimensionality of a problem increases, lambda iterative method takes greater computational time, whereas the proposed method can offer optimal solution in almost fixed time.

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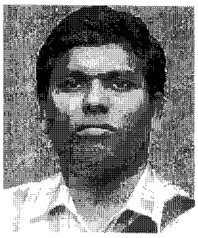

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