

Single-phase Active Power Filter Based on Rotating Reference Frame Method for Harmonics Compensation

Jin-Sun Kim[†] and Young-Seok Kim*

Abstract – This paper presents a new control method of single-phase active power filter (APF) for the compensation of harmonic current components in nonlinear loads. To facilitate the possibility of complex calculation for harmonic current detection of the single phase, a single-phase system that has two phases was constructed by including an imaginary second-phase giving time delay to the load current. The imaginary phase, which lagged the load current T/4 (Here T is the fundamental cycle) is used in the conventional method. But in this proposed method, the new signal as the second phase is delayed by the filter. Because this control method is applied to a single-phase system, an instantaneous calculation was developed by using the rotating reference frames synchronized to source-frequency rather than by applying instantaneous reactive power theory that uses the conventional fixed reference frames.

The control scheme of single-phase APF for the current source with R-L loads is applied to a laboratory prototype to verify the proposed control method.

Keywords: harmonics, imaginary phase, instantaneous reactive power rotating reference frame, single-phase APF

1. Introduction

To supply superior quality power to the electric traction load and distributed groups of loads in intelligent building etc., researchers have studied and developed the single-phase active power filters [1], [2].

How to reduce the capacity of storage energy for the compensation of harmonics is important in the control schemes of active power filters. Therefore, Akagi's instantaneous reactive power compensation theory and others have been applied in various control methods [3]. However, this theory cannot be applied for the single-phase active power filter because the instantaneous power is instantly fed from one phase to the other phases of the power system [4].

Therefore, conventional detection approaches like the notch filter and FFT have difficulty in achieving steady state and dynamic characteristics simultaneously to meet the need of real time harmonic detection and compensation [5].

To improve these drawbacks, this paper presents a new approach for single-phase harmonic detection method, from which the fundamental component of the load current

can be derived by using the positive and negative components of the current phasors in the rotating reference frames.

2. Single-phase to Two-phase Transformation

The actual current $i_{L,Re}(\omega t)$ and the signal of current $i_{L,LPF}(\omega t)$ delayed by filtering with θ are expressed as follows:

$$i_{L,Re}(\omega t) = I_{Re1} \sin(\omega t - \varphi) + \sum_{n=2}^{\infty} I_{Re\ 2n-1} \sin[(2n-1)\omega t - \varphi_{2n-1}] \quad (1)$$

$$i_{L,LPF}(\omega t) = I_{LPF1} \sin(\omega t - \theta - \varphi) + \sum_{n=2}^{\infty} I_{LPF\ 2n-1} \sin[(2n-1)(\omega t - \theta) - \varphi_{2n-1}] \quad (2)$$

Suppose $i_{L,Re}(\omega t)$ is the component of the α axis and $i_{L,LPF}(\omega t)$ is the component of the β axis in two-phase co-ordinates as follows:

[†] Corresponding Author: Dept. of Electrical Engineering, Inha University, Korea. (jsk2473@chol.com)

* Dept. of Electrical Engineering, Inha University, Korea. (youngsk@inha.ac.kr)

$$\mathbf{i}_\alpha = \mathbf{i}_{L,Re}(\omega t), \quad \mathbf{i}_\beta = \mathbf{i}_{L,LPF}(\omega t) \quad (3)$$

From (3), we can obtain the orthogonal coordinate where \vec{i}_{pos} represents the instantaneous positive sequence component of the current vector with angular velocity of ω , \vec{i}_{neg} represents the instantaneous negative sequence component of the current vector with angular velocity of $-\omega$.

3. $\alpha - \beta$ TO $d_{pos} - q_{pos}$ Reference Frame

The component of the fundamental current is obtained by separation of the current phasor into ac and dc components in the synchronously rotating frames, called $d_{pos} - q_{pos}$ and $d_{neg} - q_{neg}$ reference frame, which rotate at the speed ω and $-\omega$, respectively, in Fig. 1. Because the harmonic components except the fundamental component of the current space phasor will become alternative components in the d-q reference frame, they will be cut off by the low-pass filter [6].

Then, the following equations are true only for the fundamental component ($i_{\alpha 1}, i_{\beta 1}$) of i_α and i_β involving the dc current in (3) after the coordinate transformation.

To transform the $\alpha - \beta$ reference frame into the $d_{pos} - q_{pos}$ reference frame, which rotates at a certain speed, (3) can be expressed in the following matrix form

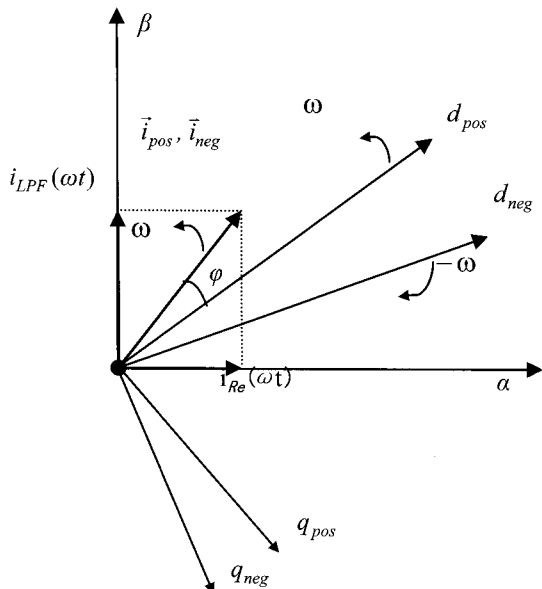


Fig. 1. The relationship between the $\alpha - \beta$ and $d - q$ reference frames.

$$\begin{aligned} \begin{bmatrix} i_{d pos} \\ i_{q pos} \end{bmatrix} &= \begin{bmatrix} \sin \omega t & -\cos \omega t \\ -\cos \omega t & -\sin \omega t \end{bmatrix} \begin{bmatrix} i_{\alpha 1} \\ i_{\beta 1} \end{bmatrix} \\ &= \begin{bmatrix} \sin \omega t \cdot i_{\alpha 1} - \cos \omega t \cdot i_{\beta 1} \\ -\cos \omega t \cdot i_{\alpha 1} - \sin \omega t \cdot i_{\beta 1} \end{bmatrix} \end{aligned} \quad (4)$$

The $i_{d pos}$ and $i_{q pos}$ can be split into two parts (dc values and ac values), respectively, as

$$\begin{aligned} i_{d pos} &= I_{Re1} \sin \omega t [\cos \varphi \sin \omega t - \sin \varphi \cos \omega t] \\ &\quad - I_{LPF} \cos \omega t [\cos(\varphi + \theta) \sin \omega t - \sin(\varphi + \theta) \cos \omega t] \\ &= \frac{1}{2} [I_{Re1} \cos \varphi + I_{LPF1} \sin(\varphi + \theta)] \\ &\quad - \frac{1}{2} [I_{Re1} \cos \varphi - I_{LPF1} \sin(\varphi + \theta)] \cos 2\omega t \\ &\quad - \frac{1}{2} [I_{Re1} \sin \varphi + I_{LPF1} \cos(\varphi + \theta)] \sin 2\omega t \\ &= \bar{i}_{d pos} + \tilde{i}_{d pos} \end{aligned} \quad (5)$$

Where,

$$\bar{i}_{d pos} = \frac{1}{2} [I_{Re1} \cos \varphi + I_{LPF1} \sin(\varphi + \theta)] \quad (6)$$

$$\begin{aligned} \tilde{i}_{d pos} &= -\frac{1}{2} [I_{Re1} \cos \varphi - I_{LPF1} \sin(\varphi + \theta)] \cos 2\omega t \\ &\quad - \frac{1}{2} [I_{Re1} \sin \varphi + I_{LPF1} \cos(\varphi + \theta)] \sin 2\omega t \end{aligned} \quad (7)$$

$$\begin{aligned} i_{q pos} &= -I_{Re1} \cos \omega t [\cos \varphi \sin \omega t - \sin \varphi \cos \omega t] \\ &\quad - I_{LPF} \sin \omega t [\cos(\varphi + \theta) \sin \omega t - \sin(\varphi + \theta) \cos \omega t] \\ &= \frac{1}{2} [I_{Re1} \sin \varphi - I_{LPF1} \cos(\varphi + \theta)] \\ &\quad - \frac{1}{2} [I_{Re1} \cos \varphi - I_{LPF1} \sin(\varphi + \theta)] \sin 2\omega t \\ &\quad + \frac{1}{2} [I_{Re1} \sin \varphi + I_{LPF1} \cos(\varphi + \theta)] \cos 2\omega t \\ &= \bar{i}_{q pos} + \tilde{i}_{q pos} \end{aligned} \quad (8)$$

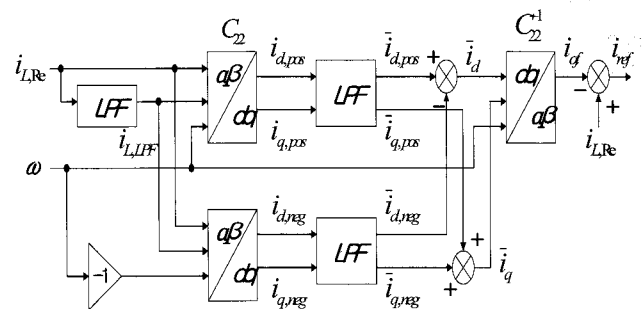


Fig. 2. Detecting algorithms of harmonics.

where,

$$\bar{i}_{q\ pos} = \frac{1}{2} [I_{Rel} \sin \varphi + I_{LPFI} \cos(\varphi + \theta)] \quad (9)$$

$$\begin{aligned} \tilde{i}_{d\ pos} = & -\frac{1}{2} [I_{Rel} \cos \varphi - I_{LPFI} \sin(\varphi + \theta)] \cos 2\omega t \\ & + \frac{1}{2} [I_{Rel} \sin \varphi + I_{LPFI} \cos(\varphi + \theta)] \cos 2\omega t \end{aligned} \quad (10)$$

Be sure that the symbols used in your equation have been defined before the equation appears or immediately following.

It is strongly encouraged that the authors use SI units only.

4. $\alpha - \beta$ TO $d_{neg} - q_{neg}$ Reference Frame

If the difference between actual phase and fictitious phase is not 90° exactly, interference between other components of the axes may occur [7]. The frequency components produced by heterodyne signal mixing will be twice as many as the fundamental component. These harmonic components will not be extracted by low-pass filters which implement the function to extract only dc quantities with low cut-off frequency.

To separate the components of heterodyne signal mixing from fundamental load current in $\alpha - \beta$ to $d_{pos} - q_{pos}$ reference frame, the synchronous $\alpha - \beta$ to $d_{neg} - q_{neg}$ reference frame is provided. In this case the unit vector is generated by $-\omega$ as shown in Fig. 1 and the components of heterodyne signal mixing are transformed into dc quantities. The dc quantities can be extracted by low-pass filters. The extracted dc quantities in $d_{neg} - q_{neg}$ reference frame are subtracted from dc quantities of positive sequence controller output and inverse transformed fundamental component of load current as revealed in Fig. 2.

To transform the $\alpha - \beta$ reference frame into the $d_{neg} - q_{neg}$ reference frame, which rotates at the speed $-\omega$, (3) can be expressed in the following matrix form

$$\begin{aligned} \begin{bmatrix} i_{d\ neg} \\ i_{q\ neg} \end{bmatrix} &= \begin{bmatrix} \sin(-\omega t) & -\cos(-\omega t) \\ -\cos(-\omega t) & -\sin(-\omega t) \end{bmatrix} \begin{bmatrix} i_{\alpha 1} \\ i_{\beta 1} \end{bmatrix} \\ &= \begin{bmatrix} -\sin \omega t \cdot i_{\alpha 1} - \cos \omega t \cdot i_{\beta 1} \\ -\cos \omega t \cdot i_{\alpha 1} + \sin \omega t \cdot i_{\beta 1} \end{bmatrix} \end{aligned} \quad (11)$$

The $i_{d\ neg}$ and $i_{q\ neg}$ can be divided into two parts (dc

values and ac values), respectively, as

$$\begin{aligned} i_{d\ neg} &= -I_{Rel} \sin \omega t [\cos \varphi \sin \omega t - \sin \varphi \cos \omega t] \\ &\quad - I_{LPFI} \cos \omega t [\cos(\varphi + \theta) \sin \omega t - \sin(\varphi + \theta) \cos \omega t] \\ &= -\frac{1}{2} [I_{Rel} \cos \varphi - I_{LPFI} \sin(\varphi + \theta)] \\ &\quad + \frac{1}{2} [I_{Rel} \cos \varphi + I_{LPFI} \sin(\varphi + \theta)] \cos 2\omega t \\ &\quad + \frac{1}{2} [I_{Rel} \sin \varphi - I_{LPFI} \cos(\varphi + \theta)] \sin 2\omega t \\ &= \bar{i}_{d\ neg} + \tilde{i}_{d\ neg} \end{aligned} \quad (12)$$

where,

$$\bar{i}_{d\ neg} = -\frac{1}{2} [I_{Rel} \cos \varphi - I_{LPFI} \sin(\varphi + \theta)] \quad (13)$$

$$\begin{aligned} \tilde{i}_{d\ negs} &= \frac{1}{2} [I_{Rel} \cos \varphi - I_{LPFI} \sin(\varphi + \theta)] \cos 2\omega t \\ &\quad + \frac{1}{2} [I_{Rel} \sin \varphi + I_{LPFI} \cos(\varphi + \theta)] \sin 2\omega t \end{aligned} \quad (14)$$

$$\begin{aligned} i_{q\ neg} &= -I_{Rel} \cos \omega t [\cos \varphi \sin \omega t - \sin \varphi \cos \omega t] \\ &\quad + I_{LPFI} \sin \omega t [\cos(\varphi + \theta) \sin \omega t - \sin(\varphi + \theta) \cos \omega t] \\ &= \frac{1}{2} [I_{Rel} \sin \varphi + I_{LPFI} \cos(\varphi + \theta)] \\ &\quad - \frac{1}{2} [I_{Rel} \cos \varphi + I_{LPFI} \sin(\varphi + \theta)] \sin 2\omega t \\ &\quad + \frac{1}{2} [I_{Rel} \sin \varphi - I_{LPFI} \cos(\varphi + \theta)] \cos 2\omega t \\ &= \bar{i}_{q\ pos} + \tilde{i}_{q\ pos} \end{aligned} \quad (15)$$

where,

$$\bar{i}_{q\ neg} = \frac{1}{2} [I_{Rel} \sin \varphi + I_{LPFI} \cos(\varphi + \theta)] \quad (16)$$

$$\begin{aligned} \tilde{i}_{q\ negs} &= -\frac{1}{2} [I_{Rel} \cos \varphi + I_{LPFI} \sin(\varphi + \theta)] \sin 2\omega t \\ &\quad + \frac{1}{2} [I_{Rel} \sin \varphi - I_{LPFI} \cos(\varphi + \theta)] \cos 2\omega t \end{aligned} \quad (17)$$

5. The Decision of Reference

To obtain the fundamental component of load current, two synchronously rotating reference frames were constructed, and the process to obtain the dc component of d-axis and q-axis in the d-q reference frame was obtained.

Using (6) and (13), the dc component of the d-axis can be expressed as

$$\bar{i}_d = \bar{i}_{d\ pos} - \bar{i}_{d\ neg} = I_{Rel} \cos \varphi \quad (18)$$

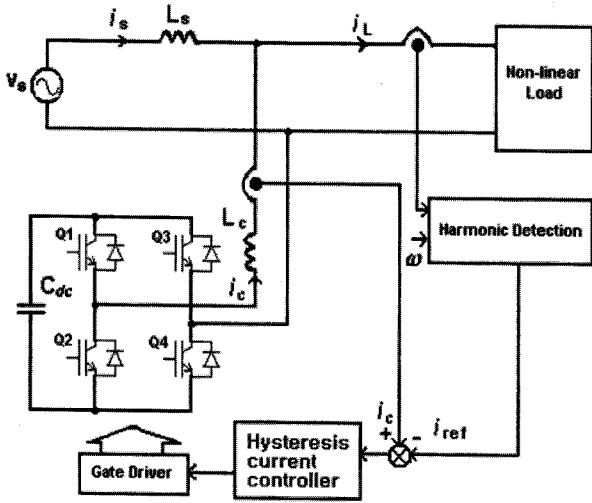


Fig. 3. Configuration for single-phase parallel active power filter.

Using (9) and (16), the dc component of q-axis can be expressed as

$$\bar{i}_q = \bar{i}_{q\ pos} + \bar{i}_{q\ neg} = I_{Re1} \sin \varphi \quad (19)$$

However, the real load current $i_{L,Re}$ is valid in the actual compensation, and the fundamental component of the load current is the component of the α -axis, which corresponds to the dc component of the current phasor in the d-q reference frame.

Therefore, the reverse-transformation of co-ordinates is needed and expressed by the following matrix forms

$$\begin{bmatrix} i_{\alpha f} \\ i_{\beta f} \end{bmatrix} = \begin{bmatrix} \sin \omega t & -\cos \omega t \\ -\cos \omega t & -\sin \omega t \end{bmatrix}^{-1} \begin{bmatrix} \bar{i}_d \\ \bar{i}_q \end{bmatrix} \quad (20)$$

where,

$$\begin{aligned} i_{\alpha f} &= \sin \omega t \cdot \bar{i}_d - \cos \omega t \cdot \bar{i}_q \\ &= I_{Re1} \cos \varphi \cdot \sin \omega t - I_{Re1} \sin \varphi \cdot \cos \omega t \end{aligned} \quad (21)$$

Finally, the current compensation reference can be obtained by subtracting the fundamental component from the load currents as

$$i_{ref} = i_{L,Re} - i_{\alpha f} \quad (22)$$

6. Experimental Results

Before the experiment, simulation was carried out with the simulation package (PSIM) and under the condition of the system parameters of Table 1. Fig. 4 shows the source current, reference current, compensation current, and supply voltage before and after compensation. The result of this simulation indicated that the source current THD was improved from 18.4% to 3.1%. This result satisfies the regulations of IEEE std. 519.

Abreast with this simulation, experimental results are presented to validate the proposed scheme. In Fig. 3, the single-phase parallel active power filter, which is connected in parallel with the non-linear loads, are represented, and Table 2 shows the system parameters of the active power filter system. The input voltage of the load V_s is 110V, 60Hz, and the source inductance is 0.1mH.

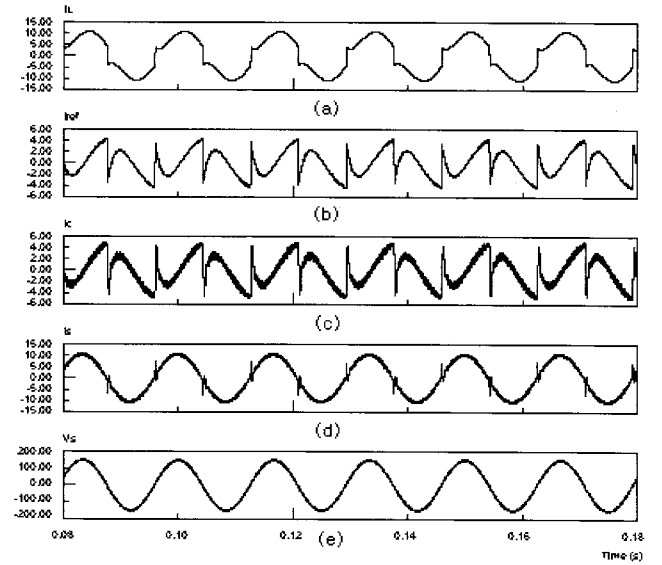


Fig. 4. The simulation results of (a) source current, (b) reference current, (c) compensation current, (d) source current after compensation, and (e) supply voltage.

Table 1. System parameters of single-phase active power under simulation

Supply voltage (V_s), Frequency	110 V, 60Hz
Source inductor (L_s)	0.1 mH
Load inductor	15 mH
Load resistance	12.8 Ω
Inverter dc-link capacitor (C_{dc})	4700 μF
LC-filter inductor (L_s)	1.8 mH

Table 2. System parameters of single-phase active power under experimentation

Supply voltage (V_S), Frequency	110 V, 60Hz
Source inductor (L_S)	0.1 mH
Load inductor	35 mH
Load resistance	15 Ω
Inverter dc-link capacitor (C_{dc})	2350 μF
LC-filter inductor (L_S)	1.8 mH

The converter with the RL load is used as the harmonic current source, and the values of load are resistance $R=15 \Omega$ and $L=35\text{mH}$. The inverter dc-link capacitor C_{dc} is 2350 μF .

As the controller, M67 DSP board is used in the experiments. The M67 DSP board communicates with the PC via the PCI slot and emulator. The load current and the inverter dc-link voltage are transformed into the inner value of $\pm 10 \text{ V}$ by the sensing circuits which consist of PT and CT. These transformed load current and inverter dc-link voltage are transformed into 16-bit digital values, which are input to the DSP. These input values are used to calculate the current compensation reference, which is transformed into an analog reference signal by the digital-to-analog converter. Next, the gate signal is generated by comparing the analog reference signal and the actual current signal.

This generated gate signal drives the voltage source inverter with the protection circuit through the dead-time circuit and the gate driver, which consists of the IGBT drive IC. The switching frequency is 20 kHz.

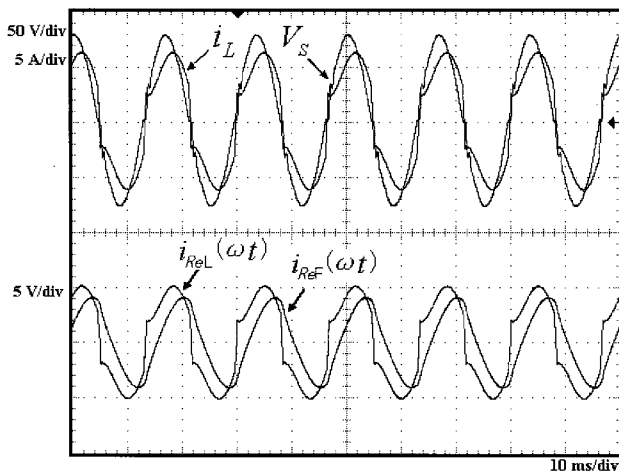
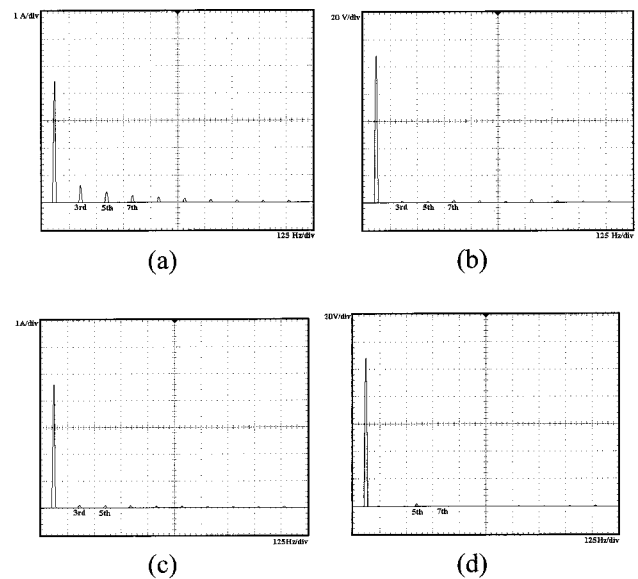
**Fig. 5.** Source voltage and load current waveforms and their waveforms by the low pass filtering current before compensation.

Fig. 5 shows the experimental waveforms before compensation. The source voltages and currents were distorted by the harmonic current source, and the power factor is about 0.941 lagging. The fictitious current waveform $i_{L,LPF}(\omega t)$, whose phase was delayed by the low pass filter, is shown.

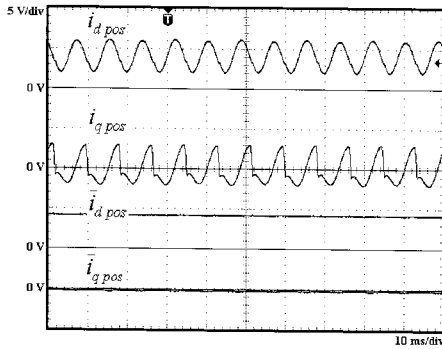
Fig. 6 indicates the source current and source voltage spectra before and after compensation. The source current THD was changed from 19.1% to 4.1% and source voltage THD was changed from 3.8% to 2.1% respectively. From these waveforms, the harmonics components were eliminated by APF.

Fig. 7 shows the experimental waveforms after compensation by the active power filter. Fig. 7-(a) and Fig. 7-(b) display the positive and negative sequence components of the current phasors in the d-q reference frame, respectively. These waveforms indicate the procedure to obtain the fundamental component of the load current in relation with (5), (6), (8), (9), (12), (13), (15) and (16). Fig. 7-(c) shows that the reference can be derived by subtracting the fundamental components from the load current signal.

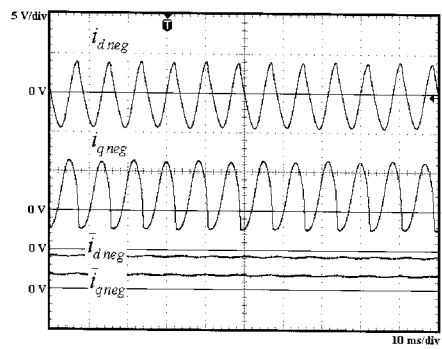
These experiment results are presented in Fig. 7-(d), and the source voltage and current waveforms with a power factor of about 0.942 lagging was improved.

**Fig. 6.** Load current and source voltage spectra before and after compensation

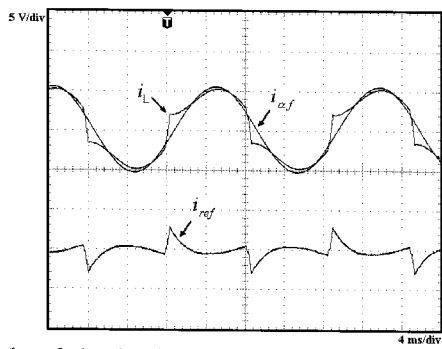
- (a) Load current i_L spectra before compensation
- (b) Supply voltage V_S spectra before compensation
- (c) Load current i_L spectra after compensation
- (d) Supply voltage V_S spectra after compensation



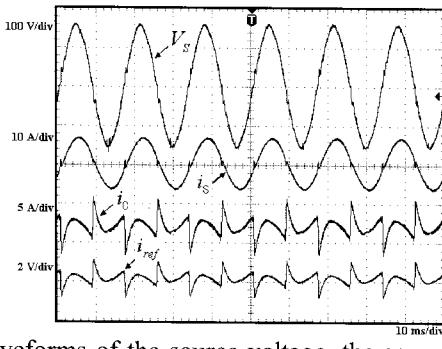
(a) The positive components of the current phasor in $d_{pos} - q_{pos}$ coordinates



(b) The negative components the current phasor in $d_{neg} - q_{neg}$ coordinates



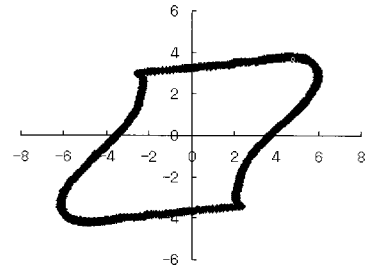
(c) Signals of the load, the fundamental component, and the reference.



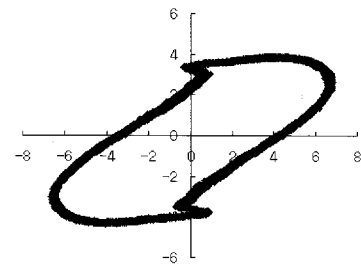
(d) Waveforms of the source voltage, the source current, the compensation current, and the reference.

Fig. 7. Experimental waveforms after compensation by active power filter

Fig. 8 displays the loci of the current vector in the α - β coordinate. In this figure, the harmonic compensation effect is represented by the ellipse. The compensated waveform should give a neat circle, but the compensated source current THD is about 4%, so the waveform of Fig. 7-(a) is not a perfect ellipse.

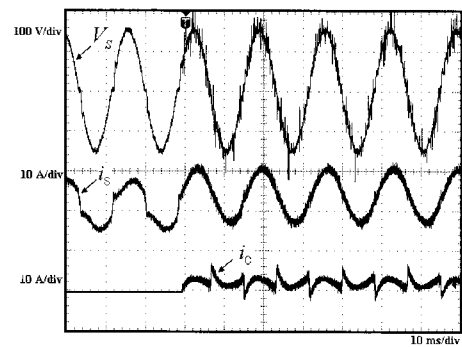


(a) Before compensation

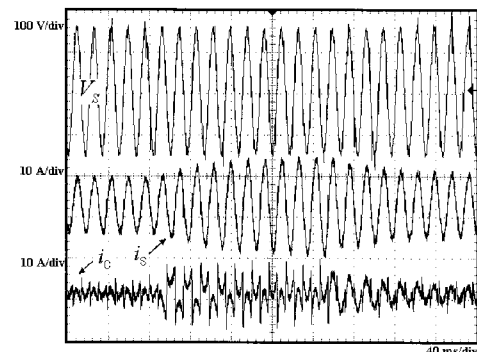


(b) After compensation

Fig. 8. Loci of current vector in α - β co-ordinates.



(a) Waveforms of supply voltage, source current, and compensation current when APF is started.



(b) Waveforms of supply voltage, source current, and compensation current when the load is changed as ($20\Omega \Rightarrow 10\Omega \Rightarrow 20\Omega$).

Fig. 9. Waveforms during transient states

Fig. 9 indicates the transient waveforms when the active power filter was started and the load was changed ($20\ \Omega \Rightarrow 10\ \Omega \Rightarrow 20\ \Omega$). These results indicate that harmonics were instantaneously compensated even though the load was strongly varying with time.

7. Conclusion

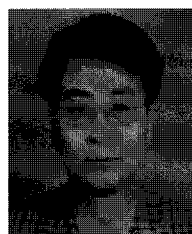
A control scheme of a single-phase active power filter based on rotating reference frame method to compensate harmonics was proposed in this paper. This method can perform the instantaneous calculation (what calculation?) without the delay of phase $T/4$. Especially, under transient states, the proposed control method can calculate the current compensation reference accurately. The THD of the source current was reduced from 19% to 4%, which satisfies the regulations of IEEE std. 519. These experimental results verified the effectiveness of the control algorithm.

Acknowledgements

This work was supported by Inha University Research Grant.

References

- [1] Gengyin Li, Chengyong Zhao, Ming Zhou, Guangkai Li and Zhiye Chen, "A predicted control scheme of single phase active power filter in electric traction systems", 2000, Advances in Power System Control, Operation and Management, APSCOM-00 International Conference, Vol. 1, pp. 101 – 104.
- [2] S. Serena, Q. Chongming, M. Qiao and Smedley, "A single-phase active power filter with double-edge integration control", Nov.-Dec. 2001, IECON '01. The 27th Annual Conference of the IEEE, pp. 949 – 953.
- [3] H. Akagi, Y. Kanazawa and A. Nabae, "Instantaneous Reactive Power Compensators Comprising Switching Device without Energy Storage Components," 1984, *IEEE Trans. Ind. Applicat.*, May.-Jun. Vol. IA-20, No. 3, pp. 625-630.
- [4] M. Aredes, J. Hafner and K. Heumann, "Three-phase four-wire shunt active filter control strategies," Mar. 1997, *IEEE Trans. Power Electr.*, Vol. 12, No. 2, pp. 311-318.
- [5] Jinjun Liu, Jun Yang and Zhaoan Wang, "A new approach for single-phase harmonic current detecting and its application in a hybrid active power system", 1999, Industrial Electronics Society, IECON '99 Proceedings, The 25th Annual Conference of the IEEE, Vol. 2, pp. 849 – 854
- [6] M. Saitou, N. Matsui, and T. Shimizu, "A control strategy of single-phase active filter using a novel d-q transformation", Oct. 2003, Industry Applications Conference, 2003. 38th IAS Annual Meeting. Conference Record, Vol. 2, pp. 1222 – 1227.
- [7] A. Sannino and J. Svensson, "Static series compensator for voltage sag mitigation supplying nonlinear loads", Jan. 2002, Power Engineering Society Winter Meeting, 2002. IEEE, Vol. 2, pp. 1147 – 1152.



Jin-Sun Kim

He received his B.S degree in Electrical Engineering from Inha University, Korea in 2005 and M.S. and Ph.D. degrees in Electrical Engineering from Inha University, Korea, in 1986 and 1988, respectively.

His research interests are power electronics, electrical machines, power supply, and electrical circuits.



Young-Seok Kim

He received his B.S degree in Electrical Engineering from Inha University, Korea in 1977, and M.S. and Ph.D. degrees in Electrical Engineering from Nagoya University, Japan in 1984 and 1987, respectively.

In 1989, he joined Inha University in Korea where he is currently a Professor in the School of Electrical Engineering. His Current research interests Include sensorless motor drives, active power filters and power converting for renewable energy.