Optimization of Fuzzy Set-Fuzzy Systems based on IG by Means of GAs with Successive Tuning Method

Keon-Jun Park*, Sung-Kwun Oh† and Hyun-Ki Kim*

Abstract – We introduce an optimization of fuzzy set-fuzzy systems based on IG (Information Granules). The proposed fuzzy model implements system structure and parameter identification by means of IG and GAs. The concept of information granulation was coped with to enhance the abilities of structural optimization of the fuzzy model. Granulation of information realized with C-Means clustering helps determine the initial parameters of the fuzzy model such as the initial apexes of the membership functions in the premise part and the initial values of polynomial functions in the consequence part of the fuzzy rules. The initial parameters are adjusted effectively with the help of the GAs and the standard least square method. To optimally identify the structure and the parameters of the fuzzy model we exploit GAs with successive tuning method to simultaneously search the structure and the parameters within one individual. We also consider the variant generation-based evolution to adjust the rate of identification of the structure and the parameters in successive tuning method. The proposed model is evaluated with the performance of the conventional fuzzy model.

Keywords: Fuzzy Set-Fuzzy Systems, Genetic Algorithms, Information Granules, Optimization, Successive Tuning Method

1. Introduction

Fuzzy modeling has been studied to deal with complex, ill-defined, and uncertain systems in numerous types of avenues. Researches on this process have been exploited for a long time. Linguistic modeling [2] and fuzzy relation equation-based approach [3] were proposed as primordial identification methods for fuzzy models. The general class of Sugeno-Takagi models [4] gave rise to more sophisticated rule-based systems where the rules come with conclusions forming local regression models. While appealing with respect to the basic topology (a modular fuzzy model composed of a series of rules) [5] these models still await formal solutions as far as the structure optimization of the model is concerned, say a construction of the underlying fuzzy sets-information granules being viewed as basic building blocks of any fuzzy model. Some enhancements to the model have been proposed by Oh and Pedrycz [6], yet the problem of finding "good" initial parameters of the fuzzy sets in the rules remains open.

This study concentrates on the central problem of fuzzy modeling that is a development of information granules-fuzzy sets. Taking into consideration the essence of the granulation process, we propose to cast the problem in the setting of clustering techniques and genetic algorithms. Information granulation with the aid of C-Means clustering helps determine the initial parameters of the fuzzy model such as the initial apexes of the membership functions in the premise part and the initial values of polynomial function in the consequence part. The initial parameters are tuned (adjusted) effectively by means of the genetic algorithms and the least square method.

To optimally identify the structure and the parameters of the fuzzy model we exploit GAs with successive tuning method to simultaneously search the structure and the parameters within one individual. Furthermore, we consider the variant generation-based evolution to adjust the rate of identification of the structure and the parameters in the successive tuning method. The proposed model is contrasted with the performance of conventional models in the literature.

2. Information Granules

Roughly speaking, information granules (IG) [7], [8] are viewed as related collections of objects (data point, in particular) drawn together by the criteria of proximity, similarity, or functionality. Granulation of information is an inherent and omnipresent activity of human beings carried out with the intent of gaining a better insight into a problem under consideration and arriving at its efficient solution. In particular, granulation of information is aimed at transforming the problem at hand into several smaller

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and therefore manageable tasks. In this way, we partition this problem into a series of well-defined subproblems (modules) of a far lower computational complexity than the original one. The form of information granulation themselves becomes an important design feature of the fuzzy model, which are geared toward capturing relationships between information granules.

Clustering is often regarded as a synonym of information granulation. The intent of clustering is to find a structure in the data and reveal clusters – information granules in the data set. The clustering algorithms have been used extensively not only to organize and categorize data, but they become useful in data compression and model construction. The C-Means clustering [9] has been applied to a variety of areas, including image and speech data compression and data preprocessing of system modeling.

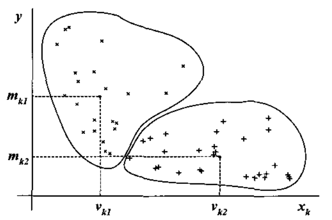
3. IG-based Fuzzy Set-Fuzzy Systems

The identification procedure for fuzzy models is usually split into the identification activities dealing with the premise and consequence parts of the rules. The identification completed at the premise level consists of two main steps. First, we select the input variables x_1, x_2, …, x_k of the rules. Second, we form fuzzy partitions of the spaces over which these individual variables are defined. The identification of the consequence part of the rules embraces two phases, namely 1) a selection of the consequence variables of the fuzzy rules, and 2) determination of the parameters of the consequence (conclusion part). And the least square error method is used at the parametric optimization of the consequence parts of the successive rules.

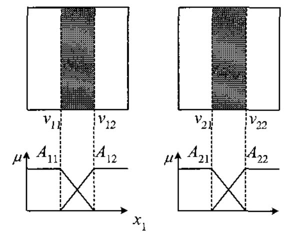
In this study, we carry out the modeling using characteristics of the experimental data set. Therefore, it is important to understand the nature of the data. The C-Means clustering addresses this issue. Subsequently we design the fuzzy model by considering the centers (prototypes) of clusters. In this manner the clustering helps us determine the initial parameters of the fuzzy model such as the initial apexes of the membership functions in the premise and the initial values of the polynomial function in the consequence.

3.1 Premise identification

In the premise part of the rules, we confine ourselves to a triangular type of membership function whose parameters are subject to some optimization. The C-Means clustering helps us organize the data into clusters so in this way we capture the characteristics of the experimental data. In the regions where some clusters of data have been identified, we end up with some fuzzy sets that help reflect the specificity of the data set. In the sequel, the modal values of the clusters are refined (optimized) using genetic optimization, and genetic algorithms (GAs), in particular.



(a) Clusters formed by C-Means clustering



(b) Respective fuzzy partition and resulting MFs with each input variable

Fig. 1. Identification of the premise part of the rules of the system

The identification of the premise part is completed in the following manner.

Given is a set of data $\mathbf{U}=\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k ; \mathbf{y}\}$, where $\mathbf{x}_k = [x_{1k}, \ldots, x_{mk}]^T$, $\mathbf{y} = [y_1, \ldots, y_m]^T$, k is the number of variables and , m is the number of data.

[Step 1] Arrange a set of data $\mathbf{U}$ into data set $\mathbf{X}_k$ composed of respective input data and output data.

$$\mathbf{X}_k=[\mathbf{x}_k ; \mathbf{y}] \tag{1}$$

[Step 2] Determine the centers (prototypes) $\mathbf{v}_{kg}$ with data set $\mathbf{X}_k$ using C-Means clustering algorithms.

[Step 2-1] Categorize data set $\mathbf{X}_k$ into c-clusters (in essence this is effectively the granulation of information).

[Step 2-2] Calculate the center vectors $\mathbf{v}_{kg}$ of each cluster.

$$\mathbf{v}_{kg} = \{v_{k1}, v_{k2}, \cdots, v_{kc}\} \tag{2}$$

[Step 3] Partition the corresponding input space using the prototypes of the clusters $\mathbf{v}_{kg}$. Associate each cluster with some meaning (semantics), say Small, Big, etc.

[Step 4] Set the initial apexes of the membership functions using the prototypes $\mathbf{v}_{kg}$.

3.2 Consequence identification

We can identify the structure of the consequence parts of rules by considering the initial center points of polynomial functions based upon the information granulation. Afterwards, those center points are immediately adjusted every evolution. The center vectors of input-output data including each rule are as follows.

[Step 1] Set the center vector V_{jk} of the input data from the apexes of the membership functions, that is

$$V_{jk} = v_{kc}, \quad j = 1,2,\ldots,n \tag{3}$$

[Step 2] Determine the center vector M_j by a weighted average between output data and firing strength $\hat{w}_{ji}$ of each rule, that is

$$M_j = \frac{1}{m}\sum_{i=1}^{m}\hat{w}_{ji}y_i \tag{4}$$

The identification of the conclusion parts of the rules deals with a selection of their structure that is followed by the determination of the respective parameters of the local functions occurring there.

The conclusion is expressed as follows.

$$R^j : If\ x_k\ is\ A_{kc}\ then\ y_j - M_j = f_j(x_1,\ldots,x_k) \tag{5}$$

Type 1 (Simplified Inference): $f_j = a_{j0}$

Type 2 (Linear Inference):

$$f_j = a_{j0} + a_{j1}(x_1 - V_{j1}) + \cdots + a_{jk}(x_k - V_{jk}) \tag{6}$$

Type 3 (Quadratic Inference):

$$\begin{aligned} f_j &= a_{j0} + a_{j1}(x_1 - V_{j1}) + \cdots + a_{jk}(x_k - V_{jk}) \\ &+ a_{j(k+1)}(x_1 - V_{j1})^2 + \cdots + a_{j(2k)}(x_k - V_{jk})^2 \\ &+ \cdots + a_{j((k+2)(k+1)/2)}(x_{k-1} - V_{j(k-1)})(x_k - V_{jk}) \end{aligned} \tag{7}$$

Type 4 (Modified Quadratic Inference):

$$\begin{aligned} f_j &= a_{j0} + a_{j1}(x_1 - V_{j1}) + \cdots + a_{jk}(x_k - V_{jk}) \\ &+ a_{j(k+1)}(x_1 - V_{j1})(x_2 - V_{j2}) + \cdots \\ &+ a_{j(k(k+1)/2)}(x_{k-1} - V_{j(k-1)})(x_k - V_{jk}) \end{aligned} \tag{8}$$

The calculations of the numeric output of the model, based on the activation (matching) levels of the rules there, rely on the expression

$$y^* = \sum_{j=1}^{n}\hat{w}_{ji}y_j = \sum_{j=1}^{n}\hat{w}_{ji}(f_j(x_1,\ldots,x_k) + M_j) \tag{9}$$

If the input variables of the premise and parameters are given in consequence parameter identification, the optimal consequence parameters that minimize the assumed performance index can be determined. In what follows, we define the performance index as the mean squared error (MSE).

$$PI = \frac{1}{m}\sum_{i=1}^{m}(y_i - y_i^*)^2 \tag{10}$$

4. Optimization of IG-based Fuzzy Set-Fuzzy Systems

The need to solve optimization problems arises in many fields and is especially dominant in the engineering environment. There are several analytic and numerical optimization techniques yet one can easily encounter problems that are not well handled by them. The standard gradient-based optimization techniques might not be effective in the context of rule-based systems given their nonlinear character (in particular the form of the membership functions) and modularity of the systems. This forces us to explore other optimization techniques such as genetic algorithms. It has been demonstrated that genetic algorithms [10] are useful in a global optimization of such problems given their ability to efficiently use historical information to produce new and improved solutions. GAs are shown to support robust search in complex search spaces. In particular, they are stochastic and less likely to get trapped in local minima as we can witness quite often when dealing with gradient-descent techniques.

In this study, we exploit a successive tuning method to successively identify the structure and the parameters of the fuzzy model. This method is to simultaneously search the structure and the parameters within one individual. Using genetic algorithms with a successive tuning method, we determine such a structure as the number of input variables, input variables being selected, and the number of membership functions standing in the premise and the type of polynomial in the conclusion. As well, the membership parameters of the premise part are genetically optimized. Figure 2 shows an arrangement of the content of the string to be used in genetic optimization to successively identify the fuzzy model. Here, parentheses signify the number of chromosomes for each parameter.

To effectively identify the structure and the parameters of the fuzzy model we also consider the variant generation-based evolution, which is an evolutionary method that separates structure chromosomes and parameter ones and conducts an operation (crossover and mutation operation) for selected chromosomes for some generations. A variant generation-based evolution is literally assigned variant generation across generation to adjust the rate of structure and parameter operation, respectively (refer to Fig. 3).

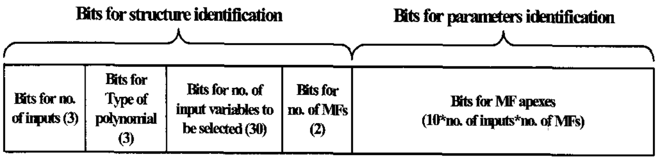


Fig. 2. Data structure of genetic algorithms used for the optimization

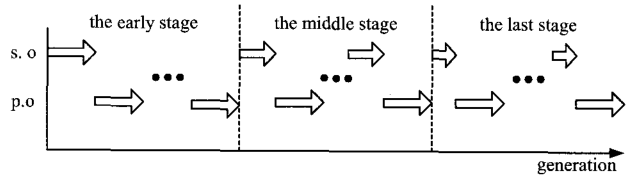


Fig. 3. Variant-generation evolution process for structure and parameters operation (s.o: structure operation, p.o: parameter operation)

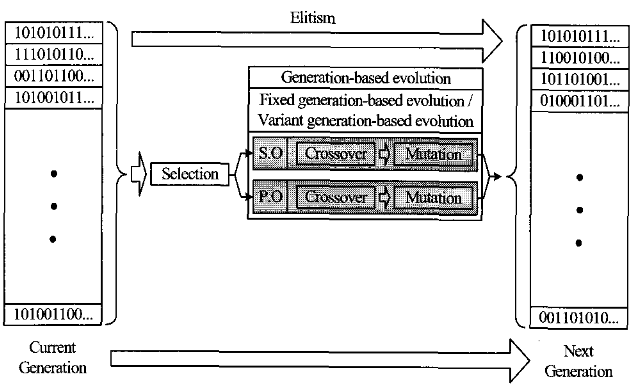


Fig. 4. Procedure for the successive generation of GAs

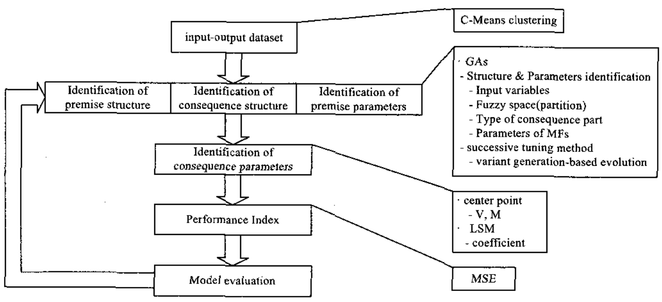


Fig. 5. General flow for optimization of IG-based Fuzzy Set-Fuzzy Systems

Fig. 4 is a schematic diagram illustrating how to produce the next generation from the current one. The general flow for optimization of the proposed model is revealed in Fig. 5.

5. Experimental Studies

This section includes comprehensive numeric study illustrating the design of the proposed fuzzy model.

For the genetic optimization of the fuzzy model, the underlying genetic algorithm uses a binary bit string, roulette-wheel in the selection operator, one-point crossover in the crossover operator, and invert in the mutation operator. We also apply elitism to keep the best individual across generations. Table 1 summarizes the list of parameters used in the genetic optimization.

We consider a nonlinear static system.

$$y=(1+x_1^{-2}+x_2^{-1.5})^2, \quad 1\le x_1,x_2\le 5 \tag{11}$$

This nonlinear static equation is widely used to evaluate the modeling performance of the fuzzy model. This system represents the nonlinear characteristic. Using Eq. (11), 50 input-output data are generated: the inputs are generated randomly and the corresponding output is then computed through the above relationship. To come up with a quantitative evaluation of the fuzzy model, we use the standard MSE performance index.

We carried out the structure and identification of the parameters on the basis of the experimental data using GAs. Table 2 summarizes the performance index for the Max-Min-based and IG-based fuzzy models. From Table 1 it is clear that the performance of the IG-based fuzzy model is

Table 1. Initial parameters of the genetic optimization of the fuzzy model

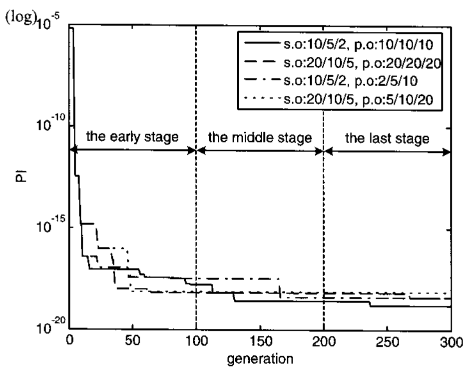
	Parameter	Value
GA	Max. generation number	300
	Population size	150
	Crossover rate	0.65
	Mutation rate	0.1
	Variant generation (the early/the middle/the last)	s.o: 10/5/2 p.o: 10/10/10 s.o: 20/10/5 p.o: 20/20/20 s.o: 10/5/2 p.o: 2/5/10 s.o: 20/10/5 p.o: 5/10/20
Fuzzy model	Max. input number	1≤l≤Max(2)
	Polynomial type (Type T)	1≤T≤4
	MFs type	Triangular
	No. of MFs	2~5

l, Max,T: integer

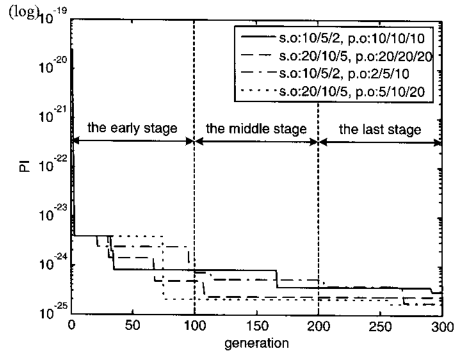
better than that of the Max-Min-based fuzzy model. Those models came to have the same structure with five membership functions per each input variable and Type 3. The best performance is PI = $1.663e^{-25}$ by means of the variant generation-based evolution with s.o: 20/10/5, p.o: 20/20/20, so we selected this model.

Table 2. Performance index of IG-based fuzzy model

Identification		No. of MFs	Type	PI
Max/Min _FIS	s.o: 10/5/2 p.o: 10/10/10	5+5	Type 3	$1.581e^{-19}$
	s.o: 20/10/5 p.o: 20/20/20	5+5	Type 3	$3.865e^{-19}$
	s.o: 10/5/2 p.o: 2/5/10	5+5	Type 3	$4.123e^{-19}$
	s.o: 20/10/5 p.o: 5/10/20	5+5	Type 3	$7.515e^{-19}$
IG_FIS	s.o: 10/5/2 p.o: 10/10/10	5+5	Type 3	$2.944e^{-25}$
	s.o: 20/10/5 p.o: 20/20/20	5+5	Type 3	$2.288e^{-25}$
	s.o: 10/5/2 p.o: 2/5/10	5+5	Type 3	$1.756e^{-25}$
	s.o: 20/10/5 p.o: 5/10/20	5+5	Type 3	$1.663e^{-25}$



(a) Max_Min-based FIS



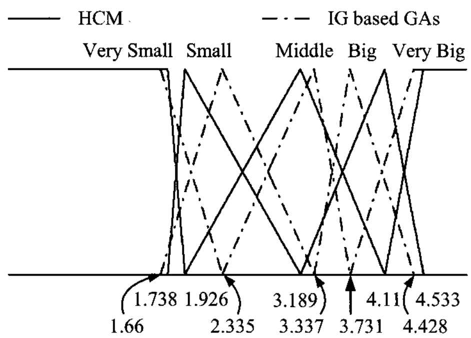
(b) IG-based FIS

Fig. 6. Minimization of performance index for IG-based fuzzy model

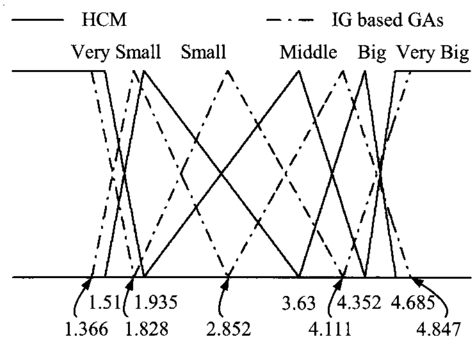
Fig. 6 shows the minimization of the performance index produced in successive generations of the genetic optimization.

The initial and optimized parameters of membership functions for the selected model, as mentioned above, are shown in Fig. 7.

The identification error (performance index) of the proposed model is also compared with the performance of some other models, as indicated in Table 3. The proposed fuzzy model outperforms several previous fuzzy models known in the literature.



(a) x_1



(b) x_2

Fig. 7. Initial and optimized parameter membership functions for the IG-based fuzzy model in case of the variant generation-based evolution with s.o: 20/10/5, p.o: 20/20/20

Table 3. Comparative analysis of selected models

Model		No. of rules	PI
Sugeno and Yasukawa [11]		6	0.079
Gomez-Skarmeta et al. [12]		5	0.070
Kim et al. [13]		3	0.019
Kim et al. [14]		3	0.0089
Oh et al. [15]	Basic PNN		0.0212
	Modified PNN		0.0041
Park et al. [16]	BFPNN	9	0.0033
	MFPNN	9	0.0023
Our Model		10	$1.663e^{-25}$

6. Conclusions

In this paper, we have developed a comprehensive framework for fuzzy set-fuzzy systems based on IG and an optimization by means of GAs. The underlying idea deals with an optimization of information granules by exploiting techniques of clustering and genetic algorithms. We defined some initial membership functions and the polynomial functions by means of information granulation realized with the C-Means clustering. The genetic algorithm was used afterwards to tune the initial values of the membership functions. Genetic algorithms with successive tuning method and variant generation-based evolution were also used for further structural and parametric optimization of the fuzzy model. The experimental studies show that the model is compact (comes realized in the form of a small number of rules) and the conversion is quite fast while its performance is better than some other models previously discussed in the literature. While the detailed discussion has been exclusively focused on triangular fuzzy sets, the developed methodology applies equally well to other classes of fuzzy sets as well as various types of nonlinear local models. The proposed models scale up quite easily and do not suffer from the curse of dimensionality encountered in some other architecture of rule-based systems.

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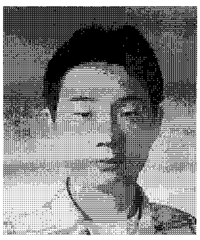

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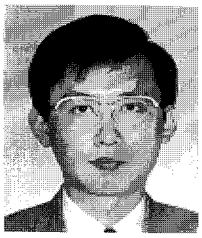

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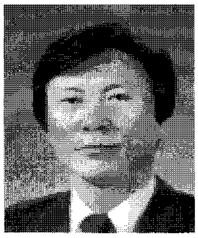

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