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Damping Identification Analysis of Membrane Structures under the Wind Load by Wavelet Transform

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Abstract

In this paper, we take advantage of Wavelet Transform to identify damping ratios of membrane structures under wind action. Due to the light-weight and flexibility of membrane structures, they are very sensitive to the wind load, and show a type of fluid-structure interaction phenomenon simultaneously. In this study, we firstly obtain the responses of an air-supported membrane structure by ADINA with the consideration of this characteristic, and then conduct Wavelet Transform on these responses. Based on the Wavelet Transform, damping ratios could be obtained from the slope of Wavelet Transform in a semi-logarithmic scale at a certain dilation coefficient. According to this principle, damping ratios could eventually be obtained. There are two numerical examples in this study. The first one is a simulated signal, which is used to verify the accuracy of the Wavelet Transform method. The second one is an air-supported membrane structure under wind action, damping ratios obtained from this method is about 0.05~0.09. The Wavelet Transform method could be regarded as a very good method for the the damping analysis, especially for the large spatial structures whose natural frequencies are closely spaced.

Keywords : Wavelet Transform, Fluid-structure Interaction, Membrane Structures, Wind Action

1. INTRODUCTION

As a newly developed structural system, membrane structures attract more and more attention due to their advantages such as light weight, easily constructed, good aseismatic and economic property and so on. They are widely applied in the design of stadium, airport and other large spatial structural systems.

So far, studies on membrane structures have mainly been focused on the form finding, prestress determination, pattern cutting and the static analysis. The research on the dynamic analysis of membrane structures still concentrates on determination of wind pressure distribution. However, concrete dynamic characteristics of membrane structures are still unknown yet. Thus, it is very necessary to do the study on these aspects.

Because membrane structures are light weighted and flexible, they exhibit good earthquake resistant property. However, they are highly susceptible to the wind action simultaneously. Based on these properties, wind load is regarded as the dominant load in the dynamic analysis of membrane structures.

Different from rigid systems, when the membrane structure is subjected to the wind action, its response will also influence the wind field in reverse. This kind of aerodynamic phenomenon is termed “fluid-structure interaction” (“FSI” for short). Swaddiwudhipong (2002), Glück(2003), Teixeira(2005) have studied this phenomena by CFD analysis.

The structure of numerical wind tunnel is shown in Fig.1, which is an analytical model to simulate a real wind tunnel. It is regarded as a good tool to investigate the aerodynamic characteristic of membrane structures. In numerical wind tunnel, wind field is generated in fluid model and membrane structure is built in structure model, furthermore, fluid-structure interfaces should be also defined in both two models. These two models will be calculated separately in each time step, and the results will interact on each other through fluid-structure interface. Through this kind of iteration, fluid-structure interaction could be finally realized.

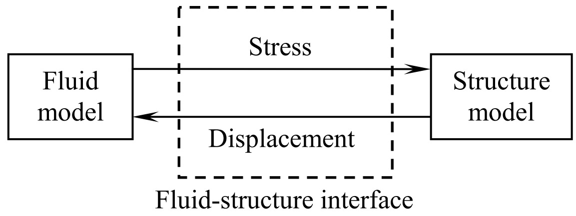


Figure 1. Fluid-structure interaction

Based on the results obtained from FSI calculations, aerodynamic property will be analyzed in this paper. According to Junji Katagiri’s paper (1998), aerodynamic damping could be obtained through three steps:

① Determining structural damping ratio ξ_s.

② Determining total damping ratio ξ_t when structure is subjected to wind action.

③ Aerodynamic damping ratio could be eventually obtained from the difference between structural damping ratio and total damping ratio.

Because the structural damping of the membrane structure is very small, it is usually neglected in real dynamic analysis. Therefore, the damping ratios obtained from the responses under the wind action could be considered as the aerodynamic damping ratios.

There are lots of methods for damping determining based on the responses such as displacements or accelerations. Among them, the well-known one is logarithmic decrement method, and it could be used to calculate damping ratio of a single D.O.F system easily. But when applied to a multi- D.O.F system, it should work with a band-pass filter which is used to extract response with a certain frequency. In terms of large spatial structures, their natural frequencies are very closely spaced, so it is very difficult to extract response with a certain natural frequency by band-pass filter. When dealing with this kind of problem, Wavelet Transform will show its superiority over logarithmic decrement method.

Staszewski(1997),(1998) initially took advantage of Wavelet Transform method to identify modal damping. From his study, we can see that the damping ratio of the system could be estimated from the slope of the straight line of the Wavelet Transform modulus for a given value of dilation a_0 in a semi-logarithmic scale. Kareem and Kijewski (2002) applied this method to the time-frequency analysis of wind effects on structures. They presented a signal padding method to solve the problem of end effects. Moreover, they introduced a basic procedure for modal identification from structural responses under the wind action. Firstly, we will utilize random decrement technique to transform random response to be free vibration response, then apply Wavelet Transform on these free vibration responses, modal damping ratios could be finally obtained. Lardies (2002), Meo (2006) also applied Wavelet Transform method to modal parameter identification of a TV tower and suspension bridge respectively..

In this paper, we introduce Wavelet Transform and its superiority over Fourier Transform simply in section 2 and section 3. We interpret how to use Wavelet Transform to extract modal damping ratios in section 4. Finally, we show two numerical examples in section 5. The first one is used to check the accuracy of this method. The second one is an air-supported membrane structure. We want to study the dynamic characteristic by identifying the damping ratios of this membrane structure under the wind action.

2. INTRODUCTION OF WAVELET TRANSFORM

Mathematically, Wavelet Transform is defined as inner products of the signal and a family of wavelets. Let $\varphi(t)$ be the mother wavelet, then the corresponding son wavelets could be generated by dilation and translation from mother wavelet (Charles (1992)).

$$\varphi_{a,b}(t) = \frac{1}{\sqrt{a}}\varphi\left(\frac{t-b}{a}\right), \quad a > 0,\ b \in R \tag{1}$$

Where, a – dilation coefficient;
b – translation coefficient.

The Continuous Wavelet Transform of a signal could be expressed mathematically as follows:

$$CWT_{\varphi}(a,b) = \langle s(t), \varphi_{a,b}(t)\rangle = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} s(t)\,\varphi_{a,b}^{*}(t)dt \tag{2}$$

Here, s(t) represents a signal and the asterisk means the complex conjugate of the original wavelet. The inner product is normalized by $\frac{1}{\sqrt{a}}$ to ensure that the integral energy given by each wavelet is independent from the dilation coefficient a. The coefficients obtained from Wavelet Transform could also be represented as a measure of the similitude between the signal and the wavelet at time t and scale a.

Morlet wavelet (1985) is one of the most widely used wavelet functions in wavelet analysis. Its function could be expressed as follows:

$$g(t) = e^{-\frac{1}{2}t^2}[cos(2\pi f_0 t) + i\ sin(2\pi f_0 t)] \tag{3}$$

Therefore,the Fourier Transform of the Morlet wavelet is

$$G(af) = e^{-2\pi^2(af-f_0)^2} \tag{4}$$

From Eq. (4), we can see that the Fourier Transform could reach its peak when the frequency f equals to $\frac{f_0}{a}$. Thus, we will take advantage of this characteristic of the Morlet wavelet to obtain the modal damping ratios later.

3. SUPERIORITY OF “WAVELET TRANSFORM” OVER “FOURIER TRANSFORM”

Fourier Transform could be expressed as Eq. (5).

$$f(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} s(t)e^{-i\omega t}\,dt \tag{5}$$

If function s(t) is considered as a signal with finite energy, then its Fourier Transform $f(\omega)$ represents the spectrum of this signal. And these two series $s(t)$ and $f(\omega)$ are interpretations of the signal in time domain and frequency domain respectively.

The inadequateness of “Fourier Transform” (As introduced in Charles (1992)):

(1) It should take an infinite amount of time to extract spectral information.
(2) It can not reflect the change of frequency with time, so it cannot be used to determine the time intervals which yield spectral information on desirable frequency band. In order to realize this function, it should work with band-pass filter.
(3) For high-frequency spectral information, the time interval should be relatively small to give better accuracy, and for low-frequency spectral information, the time interval should be relatively large to give complete information. This is impossible to realize in Fourier Transform.

Compared with Fourier Transform, the basic wavelet used in Wavelet Transform has a flexible time-frequency window and so-called zoom-in and zoom-out capability, so it could show its superiority over Fourier Transform on above aspects.

4. WAVELET TRANSFORM FOR SYSTEM IDENTIFICATION

A typical analytical signal which takes the form of a complex exponential function is given by

$$x(t) = A(t)e^{i\omega t} \tag{6}$$

Where, $A(t)$ is the time varying amplitude .
The Wavelet Transform of the signal $x(t)$ is

$$CWT_{\varphi}(a,b) = \langle x(t), \varphi_{a,b}(t) \rangle = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} A(t) e^{i\omega t} \cdot \varphi^* \left(\frac{t-b}{a} \right) dt \quad (7)$$

Applying Taylor's formula to $A(t)$ around time instant $t = b$, $A(t) = A(b) + o(A'(b))$. Then Eq. (8) could be written as:

$$CWT_{\varphi}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} (A(b) + o(A'(b)))\, e^{i\omega t} \cdot \varphi^* \left(\frac{t-b}{a} \right) dt \quad (8)$$

By calculating Fourier Transform of $\varphi^* \left(\frac{t-b}{a} \right)$ and neglecting the term of high order which is small enough, we can obtain following equation according to the Parseval's theorem.

$$CWT_{\varphi}(a,b) = \sqrt{a} \cdot A(b) \cdot e^{i\omega b} \cdot \varphi_{a,b}^*(a\omega) \quad (9)$$

The dynamic equation of a multi-D.O.F system could be expressed as follows:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{P\} \quad (10)$$

Where, *[M]*, *[C]*, and *[K]* are mass, damping and stiffness matrices respectively, *P* is the external force term. Eq. (10) is a coupled equation which could not be solved directly. We should decouple it with modal matrix Φ obtained from eigenvalue analysis of the system. $[\bar{M}] = \Phi^T [M] \Phi$, $[\bar{C}] = \Phi^T [C] \Phi$, $[\bar{K}] = \Phi^T [K] \Phi$, $[\bar{P}] = \Phi^T [P]$, due to the disappearance of off-diagonal terms, the original equations could be transformed to be N uncoupled equations.

$$m_i \ddot{x}_i(t) + c_i \dot{x}_i(t) + k_i x_i(t) = p_i(t)$$

$$\text{for } i = 1,2,\cdots,\text{N} \quad (11)$$

The general form of solution of Eq. (11) could be given as:

$$x_i(t) = A_i e^{-\xi_i \omega_{n_i} t} sin(\omega_{d_i} t + \phi_i) \quad (12)$$

Where, ω_{n_i} is the natural frequency and ω_{d_i} is the damped natural frequency, ξ_i is the damping ratio.

Therefore, the impulse response function Eq. (13) could be given according to Eq. (12).

$$x(t) = \sum_{i=1}^{N} A_i e^{-\xi_i \omega_{n_i} t} sin(\omega_{d_i} t + \phi_i) \quad (13)$$

It could be simplified as follows.

$$x(t) = \sum_{i=1}^{N} A_i(t) e^{j\varphi_i(t)} \quad (14)$$

Where, $A_i(t) = A_i e^{-\xi_i \omega_{n_i} t}$, $\varphi_i(t) = \omega_{d_i} t + \phi_i$.

Then the Wavelet Transform of this response could be given by Eq. (15).

$$CWT_{\varphi}(a,b) = \langle x(t), \varphi_{a,b}(t) \rangle = \frac{1}{\sqrt{a}} \sum_{i=1}^{N} \int_{-\infty}^{+\infty} A_i(t) e^{j\varphi_i(t)}\, \varphi^* \left(\frac{t-b}{a} \right) dt \quad (15)$$

According to Eqs. (7), (8) and (9), the following equation could be obtained.

$$CWT_{\varphi}(a,b) = \sqrt{a} \sum_{i=1}^{N} A_i(b) \cdot e^{j\varphi_i(t)} \cdot \varphi_{a,b}^*(a\omega) \quad (16)$$

When the natural frequency of the signal ω_{d_i} is equal to the frequency of the son wavelet $\frac{\omega_0}{a}$, i.e. $a = a_0 = \frac{\omega_0}{\omega_{d_i}}$, the Wavelet Transform reaches its peak. The modulus of Wavelet Transform could be expressed as:

$$\left| CWT_{\varphi}(a_0, b) \right| = \sqrt{a_0} A_i e^{-\xi_i \omega_{n_i} b} \cdot \left| \varphi_{a,b}^*(a_0 \omega_{d_i}) \right| \quad (17)$$

Then applying logarithm to both sides of Eq. (17),

$$ln \left| CWT_{\varphi}(a_0, b) \right| = -\xi_i \omega_{n_i} b + ln\left(\sqrt{a_0} A_i \varphi_{a,b}^*(a_0 \omega_{d_i}) \right) \quad (18)$$

Thus, the damping ratio ξ_i of the ith mode could be finally obtained from the slope of logarithmic value of the Wavelet Transform modulus according to Eq. (18). Above introduced method has referenced Staszewski(1997), Kareem and Kijewski(2002)'s paper.

5. NUMERICAL EXAMPLE

5.1 A single DOF example

Firstly, we take the response of a single D.O.F system for instance. We want to introduce the process to obtain modal damping ratio by the Wavelet Transform method, and check the accuracy of this method.

$$\text{Signal: } z(t) = 100 e^{-0.1\pi t} \cdot cos(6.275t) \quad (19)$$

The continuous Wavelet Transform of this signal is shown as Fig.2.

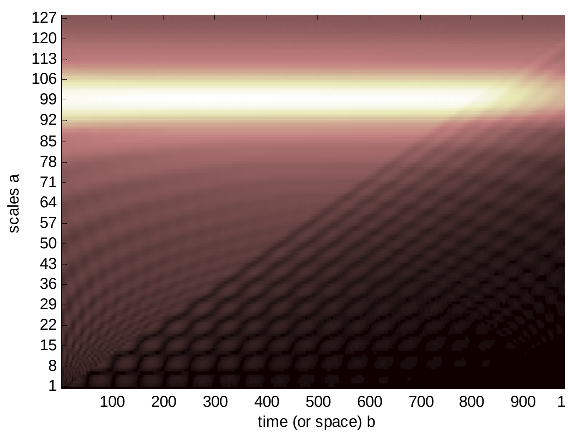
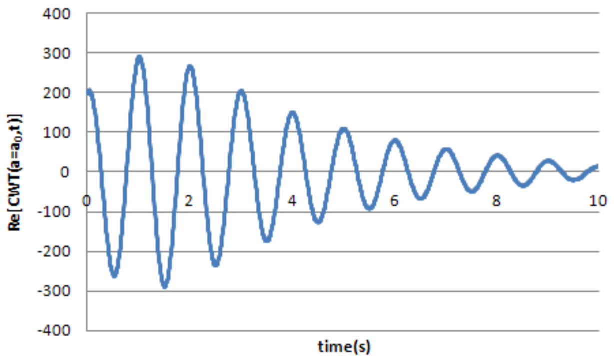
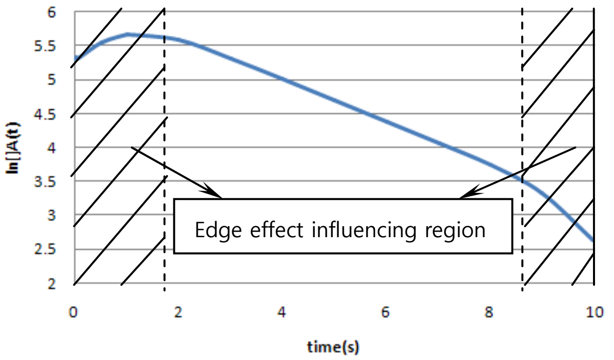


Figure 2. Continuous Wavelet Transform

From Fig.2, we can see that the energy is mainly concentrated where scale a is about 100. Therefore, the natural frequency could be obtained through the equation $\omega_d = \frac{\omega_0}{a_0 \Delta}$, where ω_d is the damped frequency of signal, ω_0 is the frequency of wavelet function and Δ is the sampling period.



(a) Real part of Wavelet Transform at $a=a_0$



(b) Logarithmic value of Wavelet Transform modulus

Figure 3. Wavelet Transform

The logarithmic value of Wavelet Transform modulus has been shown in Fig.3.

Due to the edge effect near the beginning and the ending part, there is a considerable error at these regions. Therefore, we only consider the central part and obtain modal damping ratio finally.

Through calculation, the slope of the line is -0.3087, then modal damping ratio $\xi_c = \frac{0.3087}{2*\pi} = 0.04913$. Compared with the exact value $\xi_e = 0.05$, the relative error is only about $\varepsilon = \frac{0.05-0.04913}{0.05} = 1.74\%$, so it could be regarded as a very good method.

As we know, we can also obtain modal damping ratio by the logarithmic decrement method. We could also get a relatively accurate result for this problem, but when applied into a multi-D.O.F system, it should work with a band-pass filter simultaneously. In terms of large spatial structures, their natural frequencies are very closely spaced, so it is very difficult to extract response with a single natural frequency by band-pass filter. Therefore, when dealing with this kind of problem, Wavelet Transform could show its advantages compared with the traditional method.

5.2 An air-supported membrane structure under wind action

5.2.1 Basic parameters

An air supported membrane structure is studied in this section. As we know, membrane structure does not have any stiffness to resist external force if we do not give it some initial prestress. In terms of air supported membrane structure, stiffness is provided by introducing internal pressure. According to experience, internal pressure for semi-spherical membrane structures should exceed $0.7q$, where q is the velocity pressure. Assuming design wind velocity is 30m/s, then the velocity pressure is about $\frac{1}{2}\rho V^2 = 542.25pa$, so the internal pressure can take a value a little bigger than $0.7q$, here we takes $400pa$.

Wind velocity $U(z)$ takes the power law model (Simiu and Scanlan (1986)). Its distribution along the height could be expressed as follows:

$$U(z) = U_{10}\left(\frac{z}{10}\right)^{0.22} \tag{20}$$

Material parameters of membrane and air are shown as Table 1.

Table 1. Air and membrane properties

Air properties	value	Membrane properties	value
Velocity V_{10}	30m/s	Young's modulus E	300Mpa
Kinematic viscosity μ	$1.5*10^{-5}$Pa·s	Poisson's ratio ν	0.15
Density ρ	1.205 kg/m^3	Density ρ	1100kg/m^3

The membrane model and the fluid model are shown as Fig.4 and Fig.5. The radius of the membrane structure is 10m, and the size of the fluid model is 250m×100m×50m.

Point N_1 (0, 0, 10) and point N_2 (-5.212, -5.378, 5.632) are chosen for the following analysis. Point N_1 locates at the peak of the membrane structure, and point N_2 exists in the middle of the surface. Their positions are shown in Fig. 4.

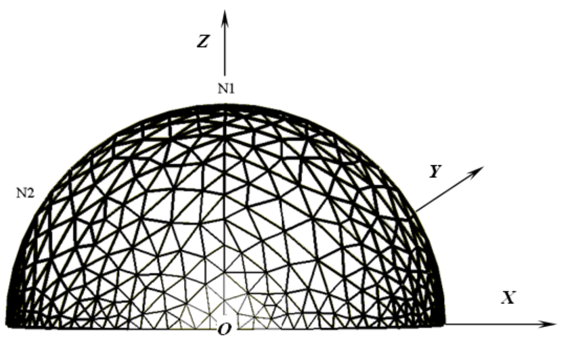


Figure 4. Structure model (728 elements)

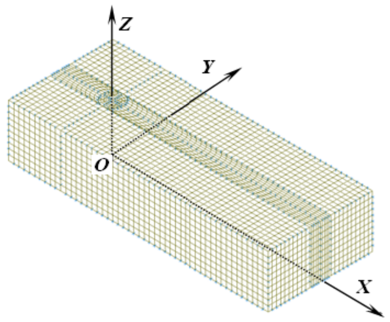


Figure 5. Fluid model (15720 elements)

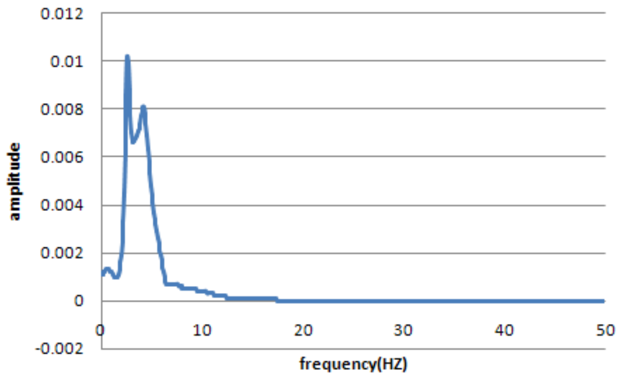


Figure 7. Fourier Transform

5.2.2 Modal analysis

Modal analysis of the membrane structure is carried out in ADINA (see ADINA manual).

From Table 2, we can see that the natural frequencies are very close. Therefore, it is very difficult to utilize band-pass filter to extract response with a single frequency.

Table 2. Natural frequencies

Mode number	Natural frequency (Hz)	Mode number	*Natural frequency (Hz)*
1	1.76634	2	1.76634
3	2.36897	4	2.54268
5	2.62630	6	2.86291
7	2.86291	8	3.11456
9	3.16721	10	3.16721
⋮	-	⋮	-
⋮	-	18	4.27899
⋮	-	⋮	-

5.2.3 Damping ratios obtained from responses

1) Wavelet Transform of the displacement in the z direction

Displacement in the z direction of point N_1 and its Fourier Transform are shown in Fig.6 and Fig.7. From the Fourier Transform of displacement, we can obtain the dominant frequencies are $f_1 = 2.584Hz$ and $f_2 = 4.248Hz$, the corresponding modes are 4th mode and 18th mode.

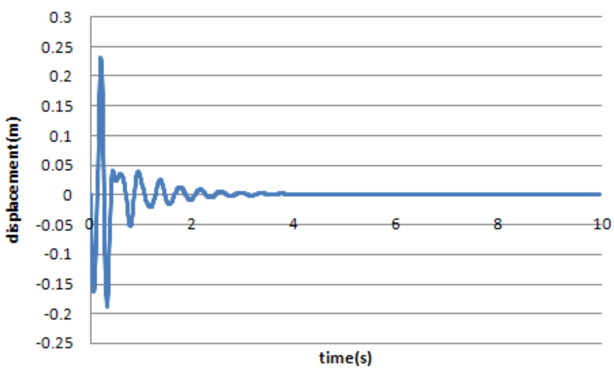


Figure 6. Displacement in the z direction

The further calculation could be obtained from Wavelet Transform. $f_{d_i} = \frac{f_0}{a_0 \Delta}$, where, f_0 is the frequency of the mother Wavelet function, f_{d_i} is the damped natural frequency of the system. When the damping ratio is less than 0.2, we could regard that the damped natural frequency is approximately equal to the natural frequency. The coefficients of Wavelet Transform are shown in Fig.8. Energy is mainly concentrated on the scales of $a_{01} = 48$ and $a_{02} = 77.5$. The corresponding frequencies are $f_1 = 4.2105\text{Hz}$, $f_2 = 2.5806\text{Hz}$, which are close to the results obtained from the Fourier Transform.

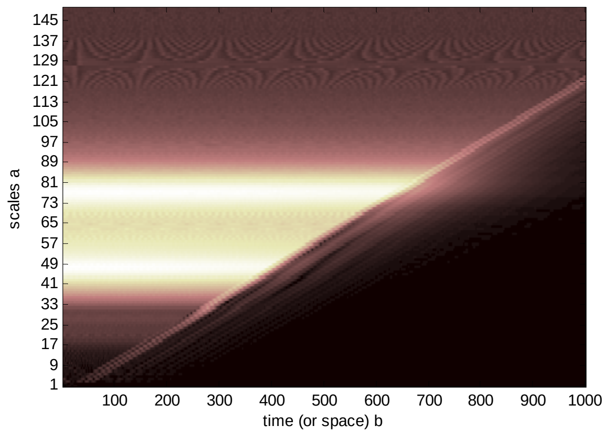
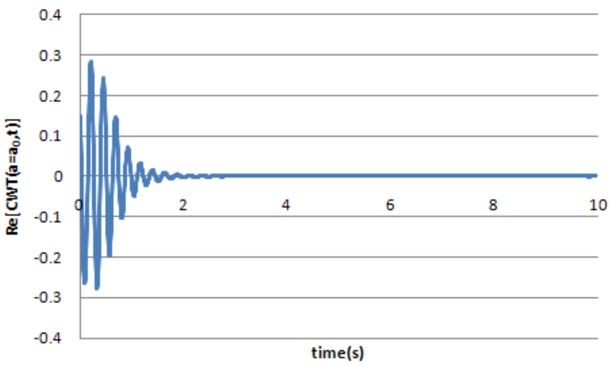
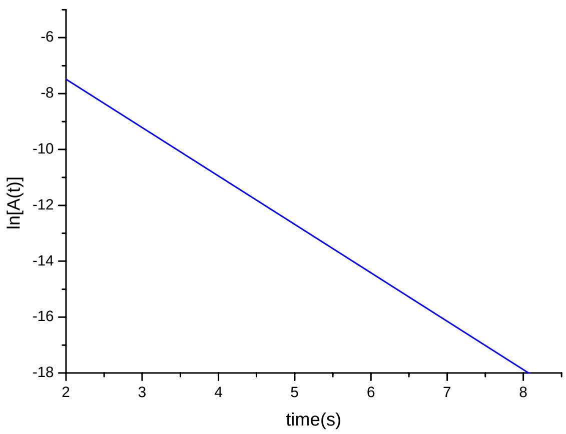


Figure 8. Coefficients of Wavelet Transform

Based on the Wavelet Transform, we can obtain the logarithmic of Wavelet Transform as shown in Fig.9 and Fig.10. Through calculating the slope of these straight lines, which are the opposite numbers of products of damping ratio and natural frequency, we can obtain damping ratios of these two modes eventually, as shown in Table 3.

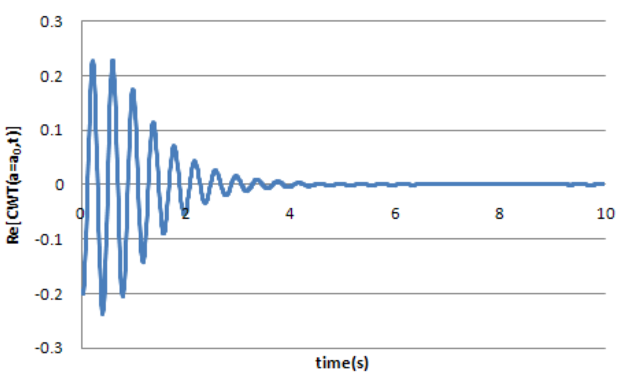


(a) Real part of Wavelet Transform

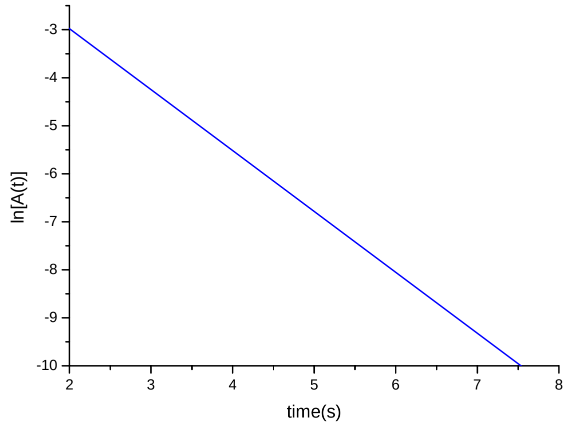


(b) Logarithmic value of Wavelet Transform modulus

Figure 9. Wavelet Transform at $a_{01} = 48$



(a) Real part of Wavelet Transform



(b) Logarithmic value of Wavelet Transform modulus

Figure 10. Wavelet Transform modulus at $a_{02} = 77.5$

Table 3. Damping ratio obtained from Wavelet Transform

Scale factor	Corresponding natural frequency	Damping ratio
48	4.1667	0.06620
77.5	2.5806	0.07842

2) Wavelet Transform of displacement in the y direction

Displacement in the y direction of point N_2 and its Fourier Transform are shown in Fig.11 and Fig.12.

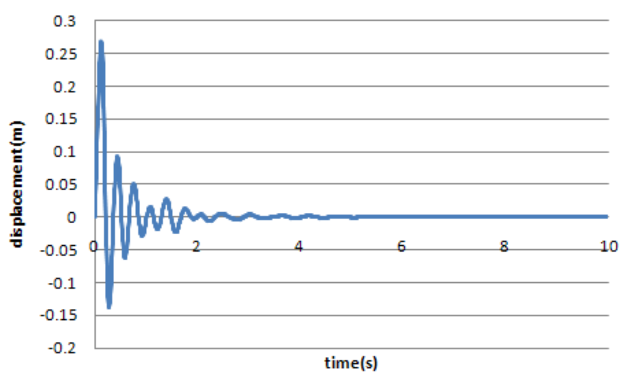


Figure 11. Displacement in the y direction

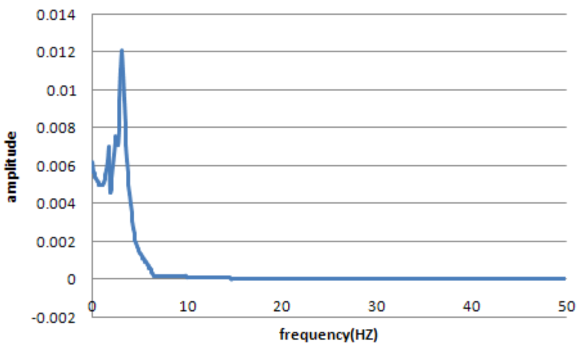


Figure 12. Fourier Transform

From the Fourier Transform of displacement, we can obtain the dominant frequencies are $f_3 = 1.757$Hz and $f_4 = 3.222$Hz, and the corresponding modes are 1st (2nd) mode and 9th (10th) mode.

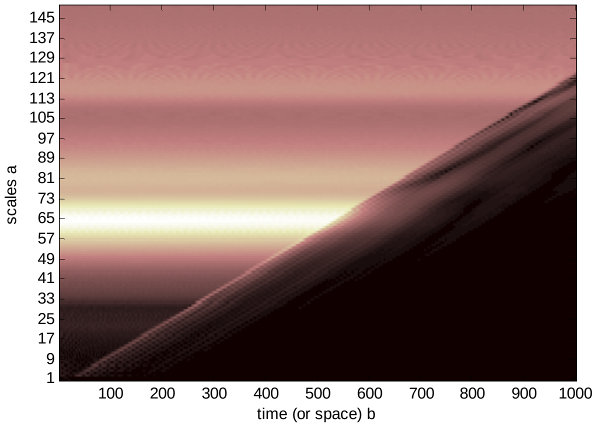
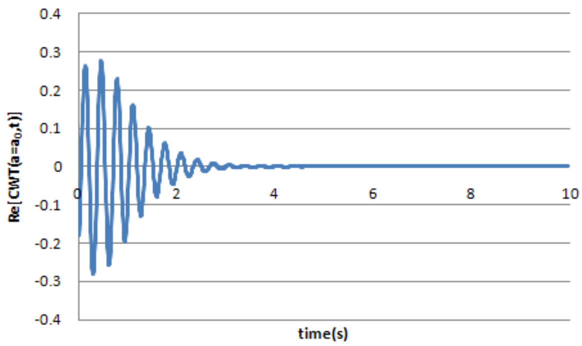


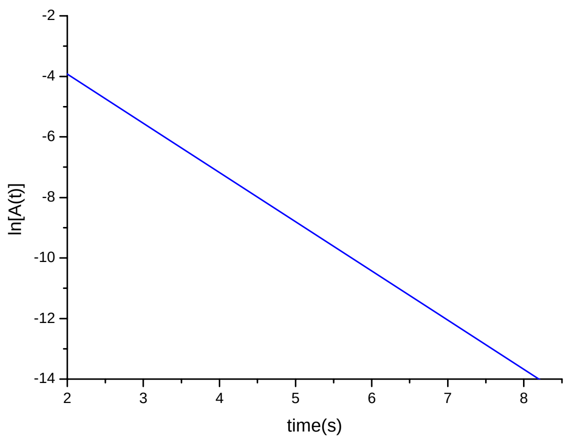
Figure 13. Coefficients of Wavelet Transform

The coefficients of Wavelet Transform are shown in Fig.13. Energy is mainly concentrated on the scale of $a_{03} = 65$ and $a_{04} = 118$. And the corresponding frequencies are $f_1 = 3.0769$Hz and $f_2 = 1.6949$Hz, which are close to the results obtained from the Fourier Transform.

Based on the Wavelet Transform, we can obtain the logarithmic value of Wavelet Transform as shown in Fig.14 and Fig.15. Calculating the slopes of these straight lines, which are the opposite numbers of products of damping ratio and natural frequency. Eventually, we can obtain damping ratios of these two modes as shown in Table 4.

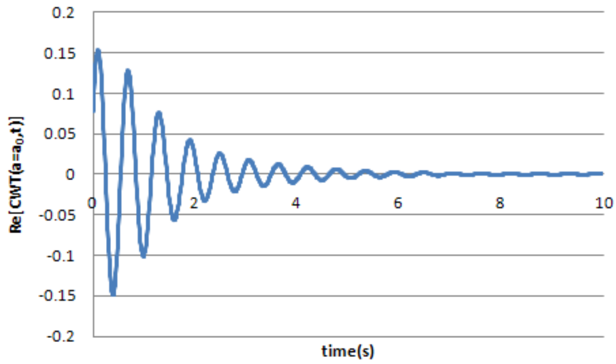


(a) Real part of Wavelet Transform

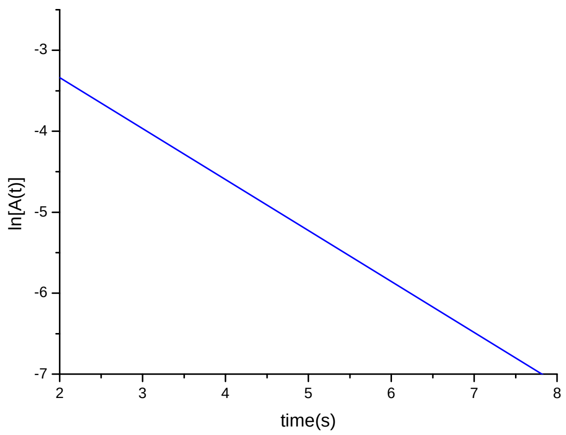


(b) Logarithmic of Wavelet Transform modulus

Figure 14. Logarithmic value of Wavelet Transform modulus at $a_{03} = 65$



(a) Real part of Wavelet Transform



(b) Logarithmic of Wavelet Transform modulus

Figure 15. Logarithmic of Wavelet Transform modulus at $a_{04} = 118$

Table 4. Damping ratios obtained from the Wavelet Transform

Scale factor	Corresponding natural frequency	Damping ratio
65	3.0769	0.08410
118	1.6949	0.05863

6. CONCLUSIONS

In this paper, we take advantage of the Wavelet Transform method to identify the damping ratios of the membrane structures based on the responses under wind action. Compared with the widely used logarithmic decrement method, this method is more applicable to structures in which natural frequencies are very closely spaced.

Wavelet Transform could be regarded as a measure of similitude between the signal and its wavelet function. Based on this property, we could get damping ratios from the slope of Wavelet Transform modulus in a semi-logarithmic scale at a certain dilation coefficient.

We utilize numerical wind tunnel to obtain real structural responses under wind action. In order to consider fluid-structure interaction, wind field and structure are built in the fluid domain and the structure domain respectively. The stresses and displacements obtained from these two domains will interact on the fluid-structure interface. Then next iteration starts similarly. The structural responses could be obtained through these iterations finally.

According to the responses of the membrane structure obtained from numerical wind tunnel, we could calculate the damping ratios based on the above Wavelet Transform method. The values of damping ratio obtained are in the range between 0.05 and 0.09, which obviously could not be neglected in the dynamic analysis.

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