레일리페이딩 환경에서 복호 후 재전송방식을 위한 부분적 릴레이 선택방식 연구

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Partial Relay Selection for Decode and Forward over Rayleigh Fading Channels

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Key Words : Decode-and-Forward; Symbol Error Rate (SER); Outage Probability (OP); Bit Error Rate (BER); Average Capacity

ABSTRACT

This paper provides closed form expressions for the evaluation of the end-to-end outage probability, symbol error rate, bit error rate and average capacity of the partial-based Decode-and-Forward (DF) relay selection scheme with an arbitrary number of relays. In a comparison with the performance of systems that exploit Amplify-and-Forward (AF), it can be seen that the performance of our proposed protocol converges to that of partial-based AF relay selection in high SNR regime. We also perform Monte-Carlo simulations to validate the analysis.

Ⅰ. Introduction

Achieving spatial diversity through the use of relaying transmission is a promising, recently evolved concept that serves as a substitute of the common diversity techniques, especially when transmitting or receiving from multiple antennas is unfeasible. In most recent publications on the cooperative diversity networks[1][2], a distributed relay selection is proposed for a two-hop AF (or DF) system that can obtain full diversity order, where the selected criterion is the best instantaneous SNR composed of the SNR across the two-hops. The only disadvantage of this system is the need for perfect time synchronization and centralized processing approach. In addition, in some resource-constrained wireless systems (especially, ad-hoc or wireless sensor networks), time synchronization and monitoring the connectivity among nodes requires feedback channels which mean frequent update, an extra computation burden and a high power consumption[3][4].

To overcome such problem, in [5], Krikidis et. al. proposed an AF relaying where the best relay is selected based on partial channel state information (i.e., only neighboring (1 hop) channel information is available to the nodes). Also in [5], the statistical behavior of the systems also was provided and confirmed by numerical results.

Besides the AF protocol, an important relaying scheme which also has attracted research interest is Decode-and-Forward. To the best of the authors' knowledge, there is no published work concerning the performance of fixed DF relays with partial relay selection operating over Rayleigh fading channels.

Motivated by all of the above, in this letter, we

※ 이 논문은 2007년도 정부(과학기술부)의 재원으로 한국과학재단의 지원을 받아 수행된 연구임(No. R01-2007-000-20400-0)

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논문번호 : KICS2009-03-109, 접수일자 : 2009년 3월 12일, 최종게재논문통보일자 : 2009년 6월 26일

investigate the performance of the partial-based DF relay selection scheme where the relay selection follows the best link between the source and the relay and the best relay only participates in the relaying. We also study the impact of signal processing techniques at the relays on the performance of the system by comparing a system that uses AF[5] to one that uses DF. In addition, a practical aspect of relay detection, i.e., without assuming that relays can perfectly detect the cyclic redundancy code (CRC) of received signals is considered in the paper.

The remaining of this paper is organized as follows. Section II presents the system model and performance analysis of the system including the closed form expressions of the outage probability, symbol error probability, bit error probability and spectral efficiency. Numerical results are performed in section III. Finally, the conclusions are given in section IV.

Ⅱ. System model

We consider a wireless relay networks consisting of one source (S), N relays R_i with $i=1,2,\cdots N$ and one destination (D). Each node is equipped with single antenna and operates in half-duplex mode. It is assumed that the destination is not able to receive signals from the source directly, which may result from high shadowing between the source and the destination. However, the relays can communicate with both the source and the destination.

The communication occurs in two hops. In the first hop, the source broadcasts the information to N relays. In the second hop, selection diversity is applied, i.e., only the relay whose link in the first hop has highest SNR to be chosen for forwarding the information to the destination.

It is assumed that every channel between the nodes experiences slow, flat, Rayleigh fading. Due to Rayleigh fading, the channel powers in two hops, denoted by $\alpha_{1,i}=|h_{SR_i}|^2$ and $\alpha_{2,i}=|h_{R_iD}|^2$ are independent and exponential random variables whose means are $\lambda_{1,i}$ and $\lambda_{2,i}$, respectively. The average transmit power for the source and the best relay in the first and second hop are denoted by ρ_1 and ρ_2, respectively. Let us define the instantaneous signal-to-noise ratio (SNR) for $S \to R_i$ and $R_i \to D$ links as $\gamma_{1,i}=\rho_1\alpha_{1,i}$ and $\gamma_{2,i}=\rho_2\alpha_{2,i}$, respectively.

In this letter, we deal with a network in which relays are grouped into a cluster where the chosen criterion is based on average SNRs. Furthermore, the cluster has been selected by a long-term routing process to perform the communication between the source and the destination. Hence this system model ensures that all channels from $S \to R_i$ and $R_i \to D$ have the same average channel power, i.e., $\overline{\gamma_{1,i}}=\rho_1\lambda_{1,i}=\overline{\gamma_1}$ and $\overline{\gamma_{2,i}}=\rho_2\lambda_{2,i}=\overline{\gamma_2}$ for all i.

It is assumed that the receivers at the destination and relays have perfect channel state information but no transmitter channel state information is available at the source and relays.

This paper focuses on decode-and-forward protocol which is one of the simple cooperative communications protocols[6]. It is noticed that the relay can forward incorrectly decoded signals to the destination. Hence, according to [7, Property 1], for any modulation scheme the dual-hop $S \to R_i \to D$ channel can be modeled as an equivalent single hop whose output SNR γ_{eq} can be tightly approximated in the high SNR regime as follows:

$$\gamma_{eq}=\min\{\beta_1,\beta_2\} \tag{1}$$

where β_1 and β_2 denote the instantaneous SNRs of the links from the source to the best relay and from the best relay to the destination, respectively, with β_1 is defined as

$$\beta_1=\max_{i=1\cdots N}\gamma_{1,i} \tag{2}$$

If the branches from the source are independently faded, then order statistics give the cumulative distribution function (CDF) as

$$F_{\beta_1}(\gamma)=\Pr(\gamma_{1,i}\le\gamma,\ldots,\gamma_{1,N}\le\gamma)=\prod_{i=1}^{N}F_{\gamma_{1,i}}(\gamma) \tag{3}$$

where $F_{\gamma_{1,i}}(\gamma)$ is the corresponding CDF of $\gamma_{1,i}$. We know that the joint probability density function (pdf) of β_1 is given by differentiating (3) as

$$\begin{aligned} f_{\beta_1}(\gamma) &= \frac{\partial}{\partial\gamma}\prod_{i=1}^{N} F_{\gamma_{1,i}}(\gamma) \\ &= \sum_{i=1}^{N} f_{\gamma_{1,i}}(\gamma) \prod_{k=1,k\neq i}^{N} \Pr(\gamma_{1,k} \le \gamma) \end{aligned} \tag{4}$$

where, for all i, $\gamma_{1,i}$ are independent and identically distributed (i.i.d.) exponential random variables $(\overline{\gamma_{1,i}} = \overline{\gamma_1})$ then

$$f_{\gamma_{1,i}}(\gamma) = \frac{1}{\overline{\gamma_1}} e^{-\gamma/\overline{\gamma_1}},\ \Pr(\gamma_{1,i} \le \gamma) = 1 - e^{-\gamma/\overline{\gamma_1}} \tag{5}$$

Substituting (5) into (4), we obtain:

$$\begin{aligned} f_{\beta_1}(\gamma) &= \frac{N}{\overline{\gamma_1}} e^{-\gamma/\overline{\gamma_1}} \left(1 - e^{-\gamma/\overline{\gamma_1}}\right)^{N-1} \\ &= \sum_{i=1}^{N} (-1)^{i-1} \binom{N}{i} \frac{i}{\overline{\gamma_1}} e^{-i\gamma/\overline{\gamma_1}} \end{aligned} \tag{6}$$

The corresponding CDF of β_1 is obtained by integration of (6) between 0 and γ as

$$F_{\beta_1}(\gamma) = \sum_{i=1}^{N} (-1)^{i-1} \binom{N}{i} \left(1 - e^{-i\gamma/\overline{\gamma_1}}\right) \tag{7}$$

In addition, in a flat Rayleigh fading channel, the pdf and CDF of β_2 can be expressed as follows:

$$f_{\beta_2}(\gamma) = \frac{1}{\overline{\gamma_2}} e^{-\gamma/\overline{\gamma_2}}\ ,\ F_{\beta_2}(\gamma) = 1 - e^{-\gamma/\overline{\gamma_2}} \tag{8}$$

Under the assumption that two hops are subject to independent fading, the joint pdf of γ_{eq} is given by [8, p. 194, eq. (6-81)]:

$$\begin{aligned} f_{\gamma_{eq}} &= \frac{\partial}{\partial\gamma}\left[F_{\beta_1}(\gamma) + F_{\beta_2}(\gamma) - F_{\beta_1}(\gamma)F_{\beta_2}(\gamma)\right] \\ &= f_{\beta_1}(\gamma) + f_{\beta_2}(\gamma) - F_{\beta_1}(\gamma) f_{\beta_2}(\gamma) - f_{\beta_1}(\gamma) F_{\beta_2}(\gamma) \end{aligned} \tag{9}$$

Substituting (6)-(7) & (8) into (9), the pdf of γ_{eq} can be obtained as

$$f_{\gamma_{eq}} = \sum_{i=1}^{N} (-1)^{-1} \binom{N}{i} C_i e^{-C_i\gamma} \tag{10}$$

where $C_i = i/\overline{\gamma_1} + 1/\overline{\gamma_2}$.

It is worth remarking that the pdf of γ_{eq} agrees exactly with that in [5, eq. (13)] for partial-based AF relay selection. Therefore, it is expected that partial-based DF relay selection and partial-based AF relay selection have the same high SNR performance, especially considering their outage probability, average SER, average BER and achievable spectral efficiency, which are investigated as follows:

2.1 Outage Probability

The outage event occurs when $\frac{1}{2}\log_2(1+\gamma_{\mathrm{eq}})$ is less than the end-to-end spectral efficiency R in bps/Hz. Note that the ratio 1/2 is included to reflect that the source-to-destination information transmission via relays takes place in two time slots. Then, the outage probability of DF relay system is derived by integrating the pdf of γ_{eq} as

$$\begin{aligned} P_o &= \Pr\left[\frac{1}{2}\log_2(1+\gamma_{\mathrm{eq}}) < \mathrm{R}\right] \\ &= \Pr(\gamma_{eq} < 2^{2R} - 1) \\ &= \int_0^{2^{2R}-1} f_{\gamma_{eq}}(\gamma)\, d\gamma \end{aligned} \tag{11}$$

By interchanging the integral and summation, (11) can be expressed as

$$P_o = \sum_{i=1}^{N} (-1)^{i-1} \binom{N}{i} \left[(1 - e^{-C_i(2^{2R}-1)}\right] \tag{12}$$

2.2 Symbol Error Rate

Using moment generating function (MGF) approach[9], we derive the closed-form expression of SER for DF relay systems. In this paper, we consider only M-ary phase shift keying signals (M-PSK) for SER and square M-QAM for BER with illustrative purpose. However, the closed-form expression of SER and BER for other modulation schemes can be obtained in the same manner. In particular, the SER of our relay selection scheme for M-PSK modulation can be given by

$$P_s = \frac{1}{\pi}\int_0^{\pi-\pi/M} M_{\gamma_{eq}}\left(-\frac{\log_2(M)g_{MPSK}}{\sin^2\theta}\right)d\theta \quad (13)$$

where $g_{MPSK}=\sin^2(\pi/M)$ and $M_{\gamma_{eq}}(\gamma)$ is the MGF of γ_{eq} defined as $M_{\gamma_{eq}}=E_{\gamma_{eq}}\{e^{s\gamma_{eq}}\}$.

Taking the expectation with respect to the random variable γ_{eq} whose pdf is given in (10) yields the result of $M_{\gamma_{eq}}(s)$and then substituting it into (13), we obtain the closed-form expression for SEP after some simple manipulation as follows [9, p. 142, eq. (5.79)]:

$$\begin{aligned}P_s &= \sum_{i=1}^{N}(-1)^{i-1}\binom{N}{i}\frac{1}{\pi}\int_0^{\pi-\pi/M}\frac{\sin^2\theta}{\sin^2+a_i}d\theta \\ &= \sum_{i=1}^{N}(-1)^{i-1}\binom{N}{i}\frac{M-1}{M}\times \\ &\left[1-\sqrt{\frac{a_i}{a_i+1}}\left(\frac{M}{(M-1)\pi}\right)\times\left[\frac{\pi}{2}+\tan^{-1}\left(\sqrt{\frac{a_i}{1+a_i}}\cot\left(\frac{\pi}{\mathrm{M}}\right)\right)\right]\right]\end{aligned} \quad (14)$$

where $a_i=g_{MPSK}C_i^{-1}\log_2M$.

2.3 Bit Error Rate

The BER of the system over Rayleigh fading channels for M-ary square quadrature amplitude (M-QAM) modulation ($M=4^m$ with $m=1,2,\ldots$) with Gray mapping can be given as [10]

$$P_b=\int_0^{\infty}\sum_{j=1}^{\log_2\sqrt{M}}\sum_{k=0}^{\nu_j}\phi_k^j\,\mathrm{erfc}(\sqrt{\omega_k\gamma})f_{\gamma_{eq}}(\gamma)d\gamma \quad (15)$$

where

$$\nu_j=(1-2^{-j})\sqrt{M}-1,\ \omega_k=\frac{(2k+1)^2 3\log_2M}{2M-2},$$

$$\phi_k^j=(-1)^{\left\lfloor\frac{k2^{j-1}}{\sqrt{M}}\right\rfloor}\frac{\left(2^{j-1}-\left\lfloor\frac{k2^{j-1}}{\sqrt{M}}+\frac{1}{2}\right\rfloor\right)}{\sqrt{M}\log_2\sqrt{\mathrm{M}}}.$$

Furthermore, we define $\lfloor . \rfloor$ and erfc(.) as the floor and complementary error function, respectively. Substituting (10) into (15) and taking the integral with respect to γ, we achieve the closed-form expression for BER as follows:

$$P_b=\sum_{j=1}^{\log_2\sqrt{M}}\sum_{k=0}^{\nu_j}\phi_k^j\sum_{i=1}^{N}(-1)^{i-1}\binom{N}{i}\left(1-\sqrt{\frac{\omega_kC_i^{-1}}{1+\omega_kC_i^{-1}}}\right) \quad (16)$$

Although the expression for P_b in (16) enables numerical evaluation of the system performance, this expression does not offer insight into the effect of the different parameters (e.g., the number of relays (N), average channel power of the links in the network) that influence the system performance. Aiming at expressing P_b in an asymptotic form in such a way we can see the behavior of the system as $\mathrm{SNR}\rightarrow\infty$. To do so, applying [11, p. 14, eq. (3.5.8)], (16) can be easily rewritten and approximated as

$$\begin{aligned}P_b &= \sum_{j=1}^{\log_2\sqrt{M}}\sum_{k=0}^{\nu_j}\phi_k^j\sum_{i=1}^{N}(-1)^{i-1}\binom{N}{i}\left[1-\left(1+\frac{1}{\omega_kC_i^{-1}}\right)^{-\frac{1}{2}}\right] \\ &\approx \sum_{j=1}^{\log_2\sqrt{M}}\sum_{k=0}^{\nu_j}\phi_k^j\sum_{i=1}^{N}(-1)^{i-1}\binom{N}{i}\left(\frac{i}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right)\end{aligned} \quad (17)$$

However, from binomial theorem [11, p.10], we know that: $\sum_{i=1}^{N}(-1)^{i-1}\binom{N}{i}i=\begin{cases}1, N=1\\0, N>1\end{cases}$. Hence, (17) can be expressed as follows:

$$P_b=\begin{cases}\displaystyle\sum_{j=1}^{\log_2\sqrt{M}}\sum_{k=0}^{\nu_j}\frac{\phi_k^j}{2\omega_k}\left(\frac{1}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right) & ,N=1\\ \displaystyle\sum_{j=1}^{\log_2\sqrt{M}}\sum_{k=0}^{\nu_j}\frac{\phi_k^j}{2\omega_k\overline{\gamma}_2} & ,N>1\end{cases} \quad (18)$$

From (18), it can clearly be seen that $\overline{\gamma}_1$ does not involved in the evaluation of P_b in case of $N>1$ and the diversity order of the system is equal to one regardless of any values of N. Moreover, at sufficiently high SNR regime and having more than one relay helping the source, the performance of the system depends only the link from the relay to the destination. It means that neither number of relays nor average channel power of links between source and the relays does take effect in the performance of the system.

2.4 Average Capacity

Average capacity is one of the important information theoretic measures of the system. Here, the average capacity of the system is considered and can be obtained by averaging the instantaneous capacity over the fading distribution as follows:

$$\begin{aligned} C &= E_{\gamma_{eq}}\left[\frac{B}{2}\log_2(1+\gamma_{eq})\right] \qquad (19) \\ &= \frac{B}{2\ln 2}\sum_{i=1}^{N}(-1)^{i-1}\binom{N}{i}e^{C_i}\Gamma(0,C_i) \end{aligned}$$

where B is the channel bandwidth in Hz and $\Gamma(a,x)=\int_x^{\infty} t^{a-1}e^{-t}dt$ is an incomplete Gamma function [11, p. 260, eq. (6.5.3)].

Ⅲ. Numerical Results

Using the analysis presented in Section II, many performance evaluations will be presented and will be compared with simulation results. For simplicity, we assumed that the average transmit SNRs for all transmit nodes in two hops are equal, i.e., $\rho_1=\rho_2$. We consider the system with 1, 3 and 5 relays with $\lambda_{1,i}=2$ and $\lambda_{2,i}=1$ for all i.

From Fig. 1 to Fig. 4, the performances of the partial-based relay selection with difference diversity processing techniques at relays (AF [5] or DF) are illustrated and compared.

Fig. 1 shows the outage probability as a function of SNR for the system at the end-to-end spectral efficiency $R=1$ bps/Hz. Figs. 2 and 3 show the average SER for 8-PSK and average BER for 16-QAM of the relaying system with different numbers of cooperative nodes. As shown in the figures, the performance of the system with the improvement of the outage probability, average SER or average BER is not be proportional to the number of relays. In addition, the performance of the proposed protocol operating with more than two relays $(N>1)$ will converge at high SNR region and will improve only around 3 dB compared with that of conventional relaying link $(N=1)$.

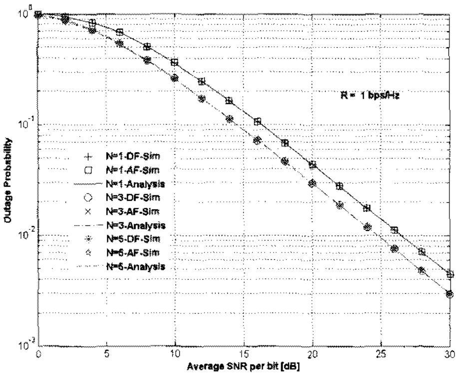

Fig. 1. Outage probability for partial-based relay selection system (R=1bps/Hz, $\lambda_{1,i}=2, \lambda_{2,i}=1$ for all i).

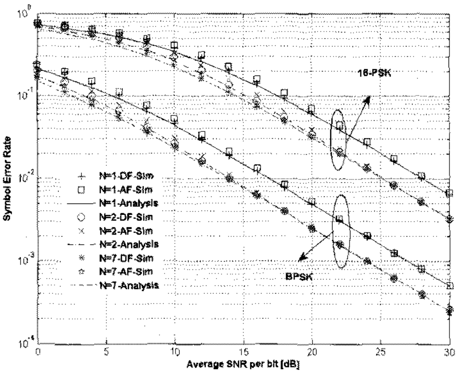

Fig. 2. SER for partial-based relay selection system $(\lambda_{1,i}=\lambda_{2,i}=1$ for all $i)$.

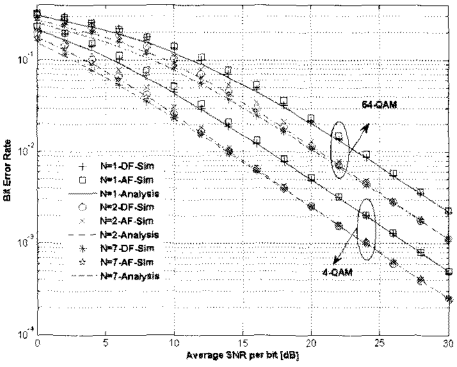

Fig. 3. BER for partial-based relay selection system $(\lambda_{1,i}=\lambda_{2,i}=1$ for all $i)$.

The average capacity for the system is also illustrated in Fig. 4. Our analytical results and the simulation results are in excellent agreement in

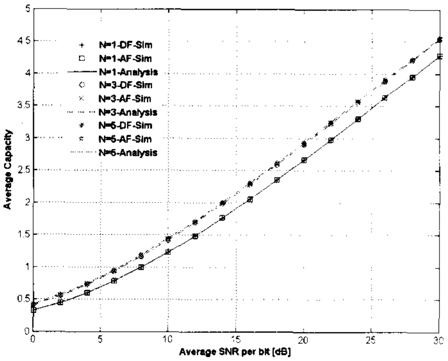

Fig. 4. Average capacity for partial-based relay selection system ($\lambda_{1,i} = 2, \lambda_{2,i} = 1$ for all i).

high SNR regime. It is obvious to observe that under same channel conditions, the large SNR performance of the partial-based DF relay selection system is identical to that of the partial-based AF relay selection system.

Ⅳ. Conclusion

This paper presented the partial-based DF relay selection and derived its closed-form expressions of outage probability, average SER, average BER, and average capacity over Rayleigh fading channels. Simulation results are in excellent agreement with the derived results. Our analytical expressions are general and offer a convenient way to evaluate the system performance without time-consuming computer simulations. Moreover, our analysis reveals an interesting result for this relaying protocol: the performance of partial-based DF relay selection converges to that of partial-based AF relay selection in high SNR regime.

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