

Noninformative priors for the ratio of the scale parameters in the half logistic distributions

Sang Gil Kang¹ · Dal Ho Kim² · Woo Dong Lee³

¹Department of Computer and Data Information, Sangji University

²Department of Statistics, Kyungpook National University

³Department of Asset Management, Daegu Haany University

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Abstract

In this paper, we develop the noninformative priors for the ratio of the scale parameters in the half logistic distributions. We develop the first and second order matching priors. It turns out that the second order matching prior matches the alternative coverage probabilities, and is a highest posterior density matching prior. Also we reveal that the one-at-a-time reference prior and Jeffreys' prior are the second order matching prior. We show that the proposed reference prior matches the target coverage probabilities in a frequentist sense through simulation study, and an example based on real data is given.

Keywords: Half logistic distribution, matching prior, reference prior, scale parameter.

1. Introduction

The half logistic distribution was introduced by Balakrishnan (1985) as a life testing model with increasing hazard rate. Balakrishnan and Puthenpura (1986) obtained the coefficient of the best linear unbiased estimators for the location and scale parameters based on complete and censored samples. Balakrishnan and Wong (1991) obtained the approximate maximum likelihood estimators for the location and scale parameters. Adatia (1997) derived the approximate best linear unbiased estimators of the parameters. Kang and Park (2005) derived the approximate maximum likelihood estimators of the scale parameter based on multiply type-II censored samples. Kim and Han (2010) obtained the maximum likelihood estimator and Bayes estimator for the scale parameter of the half-logistic distribution based on a progressively type II censored sample assuming a natural conjugate prior. However the problem of comparison of two scale parameters has not been considered yet. Thus we want to develop noninformative priors for inference of the ratio of two scale parameters.

Subjective priors are ideal when sufficient information from past experience, expert opinion or previously collected data exist. However, often even without adequate prior information,

¹ Associate professor, Department of Computer and Data Information, Sangji University, Wonju 220-702, Korea.

² Professor, Department of Statistics, Kyungpook National University, Daegu 702-701, Korea.

³ Corresponding author: Professor, Department of Asset Management, Daegu Haany University, Kyungsan 712-715, Korea. E-mail: wdlee@dhu.ac.kr

one can use Bayesian techniques efficiently with some noninformative or default priors. Once a noninformative prior is developed, then there is no necessity for exploring the effect of hyper parameters.

The notion of a noninformative prior has attracted much attention in recent years. There are two different notions of noninformative priors. One is a probability matching prior introduced by Welch and Peers (1963) which matches the posterior and frequentist probabilities of confidence intervals. Interest in such priors has been revived with the work of Stein (1985) and Tibshirani (1989). Among others, we may cite the work of Mukerjee and Dey (1993), DiCiccio and Stern (1994), Datta and Ghosh (1995a,b, 1996), Mukerjee and Ghosh (1997).

The other is the reference prior introduced by Bernardo (1979) which maximizes the Kullback-Leibler divergence between the prior and the posterior. Ghosh and Mukerjee (1992), and Berger and Bernardo (1989,1992) give a general algorithm to derive a reference prior by splitting the parameters into several groups according to their order of inferential importance. This approach is very successful in various practical problems (Kang, 2011; Kim *et al.*, 2009). Quite often reference priors satisfy the matching criterion described earlier.

The outline of the remaining sections is as follows. In Section 2, we develop first order and second order probability matching priors. We reveal that the second order matching prior is a highest posterior density (HPD) matching prior or matches the alternative coverage probabilities up to the second order. Also we derive the reference priors for the parameter. It turns out that the one-at-a-time reference prior and Jeffreys' prior are the second order matching prior. Section 3 devotes to show that the propriety of the posterior distribution for the general prior including the reference prior and the matching prior. In Section 4, simulated frequentist coverage probabilities under the proposed prior and an example are given.

2. The noninformative priors

Suppose that X and Y are independently distributed random variables according to the half-logistic distribution $\mathcal{HL}(\sigma_1)$ with the scale parameter σ_1 , and the half-logistic distribution $\mathcal{HL}(\sigma_2)$ with the scale parameter σ_2 . Then the probability density functions of half logistic distributions of X and Y are given by

$$f(x|\sigma_1) = \frac{2}{\sigma_1} \frac{\exp\{-x/\sigma_1\}}{[1 + \exp\{-x/\sigma_1\}]^2}, x \geq 0, \sigma_1 > 0, \quad (2.1)$$

and

$$f(y|\sigma_2) = \frac{2}{\sigma_2} \frac{\exp\{-y/\sigma_2\}}{[1 + \exp\{-y/\sigma_2\}]^2}, y \geq 0, \sigma_2 > 0, \quad (2.2)$$

respectively.

Let X_1, X_2, \dots, X_{n_1} be a random sample of size n_1 from $\mathcal{HL}(\sigma_1)$ and Y_1, Y_2, \dots, Y_{n_2} be a random sample of size n_2 from $\mathcal{HL}(\sigma_2)$, respectively. Let $\mathbf{x} = (x_1, x_2, \dots, x_{n_1})$ be observations of X_1, X_2, \dots, X_{n_1} and $\mathbf{y} = (y_1, y_2, \dots, y_{n_2})$ be observations of Y_1, Y_2, \dots, Y_{n_2} , respectively. We want to make a Bayesian inference about the ratio of scale parameters based on objective priors.

2.1. The probability matching priors

For a prior π , let $\theta_1^{1-\alpha}(\pi; \mathbf{X})$ denote the $(1 - \alpha)$ th posterior quantile of θ_1 , that is,

$$P^\pi[\theta_1 \leq \theta_1^{1-\alpha}(\pi; \mathbf{X}) | \mathbf{X}] = 1 - \alpha, \tag{2.3}$$

where $\boldsymbol{\theta} = (\theta_1, \dots, \theta_t)^T$ and θ_1 is the parameter of interest. We want to find priors π for which

$$P[\theta_1 \leq \theta_1^{1-\alpha}(\pi; \mathbf{X}) | \boldsymbol{\theta}] = 1 - \alpha + o(n^{-r}). \tag{2.4}$$

for some $r > 0$, as n goes to infinity. Priors π satisfying (2.4) are called matching priors. If $r = 1/2$, then π is referred to as a first order matching prior, while if $r = 1$, π is referred to as a second order matching prior.

In order to find such matching priors π , let

$$\theta_1 = \frac{\sigma_2}{\sigma_1} \text{ and } \theta_2 = \sigma_1^{n_1} \sigma_2^{n_2}.$$

With this parametrization, the likelihood function of parameters (θ_1, θ_2) for the models (2.1) and (2.2) is given by

$$\begin{aligned} &L(\theta_1, \theta_2) \\ &\propto \theta_2^{-1} \prod_{i=1}^{n_1} \left[1 + \exp \left(-\theta_1^{\frac{n_2}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} x_i \right) \right]^{-2} \prod_{i=1}^{n_2} \left[1 + \exp \left(-\theta_1^{\frac{-n_1}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} y_i \right) \right]^{-2} \\ &\times \exp \left\{ -\theta_1^{\frac{n_2}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} \sum_{i=1}^{n_1} x_i - \theta_1^{\frac{-n_1}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} \sum_{i=1}^{n_2} y_i \right\}. \end{aligned} \tag{2.5}$$

Based on (2.5), the Fisher information matrix is given by

$$\mathbf{I}(\theta_1, \theta_2) = \begin{pmatrix} \frac{n_1 n_2 (3 + \pi^2)}{9(n_1 + n_2)} \theta_1^{-2} & 0 \\ 0 & \frac{3 + \pi^2}{9(n_1 + n_2)^2} \theta_2^{-2} \end{pmatrix}. \tag{2.6}$$

From the above Fisher information matrix \mathbf{I} , θ_1 is orthogonal to θ_2 in the sense of Cox and Reid (1987). Following Tibshirani (1989), the class of first order probability matching prior is characterized by

$$\pi_m^{(1)}(\theta_1, \theta_2) \propto \theta_1^{-1} d(\theta_2), \tag{2.7}$$

where $d(\theta_2) > 0$ is an arbitrary function differentiable in its argument.

The class of prior given in (2.7) can be narrowed down to the second order probability matching priors as given in Mukerjee and Ghosh (1997). A second order probability matching prior is of the form (2.7), and d must satisfy an additional differential equation (2.10) of Mukerjee and Ghosh (1997), namely

$$\frac{1}{6} d(\theta_2) \frac{\partial}{\partial \theta_1} \{ I_{11}^{-\frac{3}{2}} L_{1,1,1} \} + \frac{\partial}{\partial \theta_2} \{ I_{11}^{-\frac{1}{2}} L_{112} I^{22} d(\theta_2) \} = 0, \tag{2.8}$$

where

$$\begin{aligned} L_{1,1,1} &= E \left[\left(\frac{\partial \log L}{\partial \theta_1} \right)^3 \right] = \frac{n_1 n_2 (n_1 - n_2) \pi^2}{3(n_1 + n_2)^2} \theta_1^{-3}, \\ L_{112} &= E \left[\frac{\partial^3 \log L}{\partial \theta_1^2 \partial \theta_2} \right] = \frac{n_1 n_2 \pi^2}{6(n_1 + n_2)^2} \theta_1^{-2} \theta_2^{-1}, \\ I_{11} &= \frac{n_1 n_2 (3 + \pi^2)}{9(n_1 + n_2)} \theta_1^{-2}, I^{22} = \frac{9(n_1 + n_2)^2}{3 + \pi^2} \theta_2^2. \end{aligned}$$

Then (2.8) simplifies to

$$\frac{\partial}{\partial \theta_2} \left\{ \frac{\pi^2 (n_1 n_2)^{\frac{1}{2}}}{(3 + \pi^2)^{\frac{3}{2}} (n_1 + n_2)^{\frac{1}{2}}} \theta_1^{-1} \theta_2 d(\theta_2) \right\} = 0. \quad (2.9)$$

Hence the set of solution of (2.9) is of the form $d(\theta_2) = \theta_2^{-1}$. Thus the resulting second order probability matching prior is

$$\pi_m^{(2)}(\theta_1, \theta_2) \propto \theta_1^{-1} \theta_2^{-1}. \quad (2.10)$$

Remark 2.1 There are alternative ways through which matching can be accomplished. Datta, Ghosh and Mukerjee (2000) provided a theorem which establishes the equivalence of second order matching priors and HPD matching priors (DiCiccio and Stern, 1994; Ghosh and Mukerjee, 1995) within the class of first order matching priors. The equivalence condition is that $I_{11}^{-3/2} L_{111}$ dose not depend on θ_1 . Since

$$L_{111} = E \left[\frac{\partial^3 \log L}{\partial \theta_1^3} \right] = \frac{n_1 n_2 [6(n_1 + n_2) + (3n_1 + n_2)\pi^2]}{6(n_1 + n_2)^2} \theta_1^{-3},$$

$I_{11}^{-3/2} L_{111}$ does not depend on θ_1 . Therefore the second order probability matching prior (2.10) is a HPD matching prior. Also

$$\begin{aligned} L_{11,1} &= E \left[\frac{\partial^2 \log L}{\partial \theta_1^2} \frac{\partial \log L}{\partial \theta_1} \right] = c_1 \theta_1^{-3}, \\ L_{11,2} &= E \left[\frac{\partial^2 \log L}{\partial \theta_1^2} \frac{\partial \log L}{\partial \theta_2} \right] = c_2 \theta_1^{-2} \theta_2^{-1} \end{aligned}$$

and $d(\theta_2) = \theta_2^{-1}$, where c_1 and c_2 are constants. Then

$$\begin{aligned} \frac{\partial}{\partial \theta_2} \{L_{112} I^{22} I_{11}^{-1/2} d(\theta_2)\} &= 0, \frac{\partial}{\partial \theta_2} \{L_{11,2} I^{22} I_{11}^{-1/2} d(\theta_2)\} = 0, \\ \frac{\partial}{\partial \theta_1} \{I_{11}^{-3/2} L_{111}\} &= 0, \frac{\partial}{\partial \theta_1} \{I_{11}^{-3/2} L_{11,1}\} = 0. \end{aligned}$$

Therefore the second order matching prior (2.10) matches the alternative coverage probabilities (Mukerjee and Reid, 1999).

2.2. The reference priors

Reference priors introduced by Bernardo (1979), and extended further by Berger and Bernardo (1992) have become very popular over the years for the development of noninformative priors. In this section, we derive the reference priors for different groups of ordering of (θ_1, θ_2) . Then due to the orthogonality of the parameters, following Datta and Ghosh (1995b), choosing rectangular compacts for each θ_1 and θ_2 when θ_1 is the parameter of interest, the reference priors are given by as follows.

For the likelihood (2.5), if θ_1 is the parameter of interest, then the reference prior distributions for group of ordering of $\{(\theta_1, \theta_2)\}$ is

$$\pi_1(\theta_1, \theta_2) \propto \theta_1^{-1}\theta_2^{-1}.$$

For group of ordering of $\{\theta_1, \theta_2\}$, the reference prior is

$$\pi_2(\theta_1, \theta_2) \propto \theta_1^{-1}\theta_2^{-1}.$$

Remark 2.2 From the above reference priors, we know that the one-at-a-time reference prior π_2 and Jeffreys' prior π_1 are the second order matching prior, and all priors are the same.

3. Implementation of the Bayesian procedure

We investigate the propriety of posteriors for a general class of priors which includes the reference prior and the matching prior. We consider the class of priors

$$\pi(\theta_1, \theta_2) \propto \theta_1^{-a}\theta_2^{-b}, \tag{3.1}$$

where $a > 0$ and $b > 0$. The following general theorem can be proved.

Theorem 3.1 The posterior distribution of (θ_1, θ_2) under the prior π , (3.1), is proper if $bn_1 - a + 1 > 0$ and $bn_2 + a - 1 > 0$.

Proof: Note that the joint posterior for θ_1 and θ_2 given \mathbf{x} and \mathbf{y} is

$$\begin{aligned} &\pi(\theta_1, \theta_2|\mathbf{x}, \mathbf{y}) \\ &\propto \theta_1^{-a}\theta_2^{-b-1} \exp \left\{ -\theta_1^{\frac{n_2}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} \sum_{i=1}^{n_1} x_i - \theta_1^{\frac{-n_1}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} \sum_{i=1}^{n_2} y_i \right\} \\ &\times \prod_{i=1}^{n_1} \left[1 + \exp \left(-\theta_1^{\frac{n_2}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} x_i \right) \right]^{-2} \prod_{i=1}^{n_2} \left[1 + \exp \left(-\theta_1^{\frac{-n_1}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} y_i \right) \right]^{-2}. \end{aligned} \tag{3.2}$$

Then we get

$$\begin{aligned} \pi(\theta_1, \theta_2|\mathbf{x}, \mathbf{y}) &\leq \theta_1^{-a}\theta_2^{-b-1} \exp \left\{ -\theta_1^{\frac{n_2}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} \sum_{i=1}^{n_1} x_i - \theta_1^{\frac{-n_1}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} \sum_{i=1}^{n_2} y_i \right\} \\ &\equiv \pi'(\theta_1, \theta_2|\mathbf{x}, \mathbf{y}). \end{aligned} \tag{3.3}$$

Thus integrating with respect to θ_2 in (3.3), we can get

$$\begin{aligned} \pi'(\theta_1|\mathbf{x}, \mathbf{y}) &\propto \theta_1^{-bn_2-a} \left[\sum_{i=1}^{n_1} x_i + \theta_1^{-1} \sum_{i=1}^{n_2} y_i \right]^{-b(n_1+n_2)} \\ &= \left(\sum_{i=1}^{n_1} x_i \right)^{-b(n_1+n_2)} \int_0^\infty \theta_1^{-a+bn_1} \left(\theta_1 + \frac{\sum_{j=1}^{n_2} y_j}{\sum_{i=1}^{n_1} x_i} \right)^{-b(n_1+n_2)} d\theta_1. \end{aligned} \quad (3.4)$$

Letting $z = \theta_1/(\theta_1 + k)$, where $k = \sum_{j=1}^{n_2} y_j / \sum_{i=1}^{n_1} x_i$, the above integration results in beta function. Thus the integration in (3.4) is finite if $bn_1 - a + 1 > 0$ and $bn_2 + a - 1 > 0$. This completes the proof. \square

Theorem 3.2 Under the prior (3.1), the marginal posterior density of θ_1 is given by

$$\begin{aligned} &\pi(\theta_1|\mathbf{x}, \mathbf{y}) \\ &\propto \int_0^\infty \theta_1^{-a} \theta_2^{-b-1} \exp \left\{ -\theta_1^{\frac{-n_2}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} \sum_{i=1}^{n_1} x_i - \theta_1^{\frac{-n_1}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} \sum_{i=1}^{n_2} y_i \right\} \\ &\times \prod_{i=1}^{n_1} \left[1 + \exp \left(-\theta_1^{\frac{-n_2}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} x_i \right) \right]^{-2} \prod_{i=1}^{n_2} \left[1 + \exp \left(-\theta_1^{\frac{-n_1}{n_1+n_2}} \theta_2^{\frac{-1}{n_1+n_2}} y_i \right) \right]^{-2} d\theta_2. \end{aligned} \quad (3.5)$$

Therefore we have the marginal posterior density of θ_1 , and so it is easy to compute the marginal moment of θ_1 . In Section 4, we investigate the frequentist coverage probabilities for the reference prior.

4. Numerical studies

We evaluate the frequentist coverage probability by investigating the credible interval of the marginal posterior density of θ_1 under the reference prior given in Section 3 for several configurations (σ_1, σ_2) and (n_1, n_2) . That is to say, the frequentist coverage of a $(1-\alpha)$ th posterior quantile should be close to $1-\alpha$. This is done numerically. Table 4.1 gives numerical values of the frequentist coverage probabilities of 0.05 (0.95) posterior quantiles for the our prior. The computation of these numerical values is based on the following algorithm for any fixed true (λ_1, λ_2) and any prespecified probability value α . Here α is 0.05 (0.95). Let $\theta_1^\pi(\alpha|\mathbf{X}, \mathbf{Y})$ be the posterior α -quantile of θ_1 given \mathbf{X} and \mathbf{Y} . That is, $F(\theta_1^\pi(\alpha|\mathbf{X}, \mathbf{Y})|\mathbf{X}, \mathbf{Y}) = \alpha$, where $F(\cdot|\mathbf{X}, \mathbf{Y})$ is the marginal posterior distribution of θ_1 . Then the frequentist coverage probability of this one sided credible interval of θ_1 is

$$P_{(\theta_1, \theta_2)}(\alpha; \theta_1) = P_{(\theta_1, \theta_2)}(0 < \theta_1 \leq \theta_1^\pi(\alpha|\mathbf{X}, \mathbf{Y})). \quad (4.1)$$

The computed $P_{(\theta_1, \theta_2)}(\alpha; \theta_1)$ when $\alpha = 0.05(0.95)$ is shown in Table 4.1. In particular, for fixed n and (σ_1, σ_2) , we take 10,000 independent random samples of $\mathbf{X} = (X_1, \dots, X_{n_1})$ and $\mathbf{Y} = (Y_1, \dots, Y_{n_2})$ from the half logistic distributions, respectively.

In Table 4.1, we can observe that the reference prior meets very well the target coverage probabilities even for the small sample sizes. Also note that the results of table are not much sensitive to the change of the values of (θ_1, σ_1) . Thus we recommend to use the reference prior.

Table 4.1 Frequentist coverage probability of 0.05 (0.95) posterior quantiles of θ_1

| θ_1 | σ_1 | n_1 | n_2 | π_r |
|------------|------------|-------|-------|---------------|
| 0.1 | 0.1 | 5 | 5 | 0.048 (0.951) |
| | | 5 | 10 | 0.053 (0.949) |
| | | 10 | 10 | 0.053 (0.949) |
| | | 10 | 15 | 0.054 (0.950) |
| | 1.0 | 5 | 5 | 0.044 (0.945) |
| | | 5 | 10 | 0.050 (0.948) |
| | | 10 | 10 | 0.052 (0.952) |
| | | 10 | 15 | 0.046 (0.949) |
| | 10.0 | 5 | 5 | 0.049 (0.947) |
| | | 5 | 10 | 0.051 (0.953) |
| | | 10 | 10 | 0.051 (0.949) |
| | | 10 | 15 | 0.049 (0.951) |
| 1.0 | 0.1 | 5 | 5 | 0.051 (0.949) |
| | | 5 | 10 | 0.048 (0.953) |
| | | 10 | 10 | 0.048 (0.948) |
| | | 10 | 15 | 0.051 (0.948) |
| | 1.0 | 5 | 5 | 0.050 (0.948) |
| | | 5 | 10 | 0.052 (0.951) |
| | | 10 | 10 | 0.048 (0.952) |
| | | 10 | 15 | 0.049 (0.950) |
| | 10.0 | 5 | 5 | 0.049 (0.949) |
| | | 5 | 10 | 0.046 (0.950) |
| | | 10 | 10 | 0.046 (0.950) |
| | | 10 | 15 | 0.047 (0.949) |
| 10.0 | 0.1 | 5 | 5 | 0.047 (0.951) |
| | | 5 | 10 | 0.052 (0.949) |
| | | 10 | 10 | 0.046 (0.951) |
| | | 10 | 15 | 0.051 (0.949) |
| | 1.0 | 5 | 5 | 0.050 (0.951) |
| | | 5 | 10 | 0.052 (0.947) |
| | | 10 | 10 | 0.046 (0.947) |
| | | 10 | 15 | 0.049 (0.947) |
| | 10.0 | 5 | 5 | 0.043 (0.950) |
| | | 5 | 10 | 0.049 (0.948) |
| | | 10 | 10 | 0.052 (0.950) |
| | | 10 | 15 | 0.051 (0.951) |

Example 4.1 This example is taken from Balakrishnan and Puthenpura (1986). The data is failure times, in minutes, for a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress. For this data, Balakrishnan and Puthenpura (1986) concluded that the half logistic distribution fits the data better than an exponential distribution. For estimation of ratio of scale parameters, we randomly divided this data into two groups. The data sets are given by

Group 1: 21.8, 70.7, 151.9, 75.3, 12.3, 28.6.

Group 2: 24.4, 138.6, 95.5, 98.1, 43.2, 46.9.

For this data, the maximum likelihood estimate (MLE) of θ_1 is 1.1826 and the corresponding 95% asymptotic confidence interval of θ_1 is (0.0635, 2.3016). Bayes estimate and the 95% credible interval based on the reference prior are 1.3431 and (0.4384, 3.1832), respectively. The estimate based on the reference prior and the MLE give almost same results, but the confidence intervals based on the MLE and the reference prior give some different results.

5. Concluding remarks

In the half logistic models, we have found the second order matching prior and the reference prior for the ratio of the scale parameters. We revealed that the second order matching prior is a HPD matching prior and matches the alternative coverage probabilities up to the second order. It turns out that the reference prior and Jeffreys' prior are the second order matching prior. As illustrated in our numerical study, the reference prior meets very well the target coverage probabilities. Thus we recommend the use of the reference prior for Bayesian inference of the ratio of the scale parameters in two independent half logistic distributions.

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